Computer Science C73 November 4, 2026
Scarborough Campus University of Toronto

Homework Assignment #5
Due: November 18, 2026, by 11:59 pm

- ***You must submit your assignment through the Crowdmark system.*** *You will receive by email an invitation through which you can submit your work in the form of separate PDF docucments with your answers to each question of the assignment. To work with a partner, you and your partner must form a group on Crowdmark.* ***The group must be formed before either partner submits <u>any</u> answer.***
- *To minimize bias in grading, Crowdmark does not reveal your name to the grader. Please do not subvert this feature by including your name(s) on the files you submit.*
- *The PDF files you submit must be legible without undue effort by the grader. To this end, I encourage you to learn and use the LaTex typesetting system, which is designed to produce high-quality documents that contain mathematical notation. You are not required to produce the PDF files you submit using LaTex; you may produce them any way you wish, as long as the resulting document is legible.*
- *By virtue of submitting this assignment you (and your partner, if you have one) acknowledge that you are aware of the policy on homework collaboration for this course.*[a]
- *For any question, you may use data structures and algorithms previously described in class, or in prerequisites of this course, without describing them. You may also use any result that we covered in class, or is in the course notes by referring to it.*
- *Unless we explicitly state otherwise, you may describe algorithms in high-level pseudocode or point-form English, whichever leads to a clearer and simpler description. Do not provide executable code.*
- *Unless we explicitly state otherwise, you should justify your answers. Your paper will be graded based on* ***the correctness and efficiency of your answers, and the clarity, precision, and conciseness of your presentation****.*

[a] "For help with your homework you may consult only the instructor, TAs, your homework partner (if you have one), the CSCC73 Course Notes, and your personal class notes. You may not consult any other source, including AI-based tools."

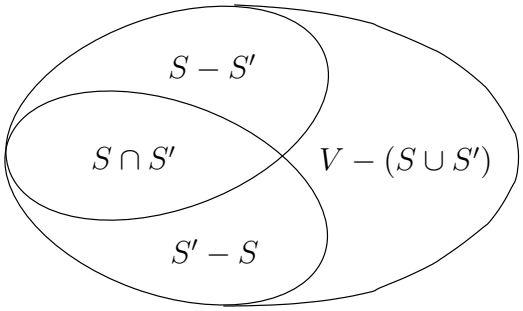
Question 1. (15 marks) Let $\mathcal{F} = (G, s, t, c)$ be a flow graph with integer capacities and f be a flow in $\mathcal{F}$. A ***cycle of flow*** f is a cycle of G every edge of which carries some traffic; that is, a path $u_1, u_2, \ldots, u_k$ of G such that $u_1 = u_k$ and $f(u_i, u_{i+1}) > 0$, for each i such that $1 \leq i < k$. A flow f is called ***acyclic*** if it has no cycle.

a. Suppose we use the version of the Ford-Fulkerson algorithm that always chooses an augmenting path with the fewest edges. Give an example flow network in which this algorithm ***necessarily*** produces a cyclic flow. Your example should be as simple as you can. You should identify the augmenting paths that the algorithm chooses in your example flow network and the cycle of the maximum flow that it creates.

b. Prove that for any flow network $\mathcal{F}$ and any flow f in $\mathcal{F}$, there is an acyclic flow of $\mathcal{F}$ with the same value as f. You may ***not*** assume that f is an integral flow.

Question 2. (15 marks) Let $\mathcal{F} = (G, s, t, c)$ be a flow network and (S, T), (S', T') be ***minimum*** cuts of $\mathcal{F}$. Prove that $(S \cap S', T \cup T')$ and $(S \cup S', T \cap T')$ are also minimum cuts of $\mathcal{F}$.

Hint: Prove both facts together. In thinking about this problem you may find the following diagram useful:



Question 3. (25 marks) Let $\mathcal{F} = (G, s, t, c)$ be a flow network with integer capacities, f be an ***integral*** maximum flow in $\mathcal{F}$, and e be an edge of the graph G. Let $\mathcal{F}^+ = (G, s, t, c^+)$ be the flow graph obtained from $\mathcal{F}$ by increasing the capacity of e by one unit, and $\mathcal{F}^- = (G, s, t, c^-)$ be the flow graph obtained from $\mathcal{F}$ by decreasing the capacity of e by one unit. (That is, for every edge $e' \neq e$ of G, $c^+(e') = c^-(e') = c(e')$, $c^+(e) = c(e) + 1$, and $c^-(e) = c(e) - 1$.[†])

a. Prove that the value of a maximum flow in $\mathcal{F}^+$ is $\mathcal{V}(f)$ or $\mathcal{V}(f) + 1$, and the value of a maximum flow in $\mathcal{F}^-$ is $\mathcal{V}(f)$ or $\mathcal{V}(f) - 1$.

b. Give an algorithm that, given $\mathcal{F}$, f, and e, finds a maximum flow in $\mathcal{F}^+$. Your algorithm should be ***significantly faster*** than recomputing a maximum flow of $\mathcal{F}^+$ from scratch. Justify the correctness of your algorithm and analyze its running time.

c. Do as in part (b), but for $\mathcal{F}^-$. In this case you may find it convenient to assume that G has no cycles. If the correctness of your algorithm depends on this assumption then, for full marks, you must state explicitly where it is needed.

[†]You may assume that $c(e) > 1$, so $c^-(e) > 0$.