Computer Science C63 March 6, 2024
Scarborough Campus University of Toronto

Homework Assignment #5
Due: March 13, 2024, by 11:59 pm

• ***You must submit your assignment through the Crowdmark system.*** *You will receive by email an invitation through which you can submit your work in the form of separate PDF docucments with your answers to each question of the assignment. To work with a partner, you and your partner must form a group on Crowdmark. Crowdmark does not enforce a limit on the size of groups.* ***The course policy that limits the size of each group to at most two remains in effect:*** *submissions by groups of more than two persons will not be graded.*
• *To minimize bias in grading, Crowdmark does not reveal your name to the grader. Please do not subvert this feature by including your name(s) on the files you submit.*
• *It is your responsibility to ensure that the files you submit are legible. To this end, I encourage you to learn and use the LaTex typesetting system, which is designed to produce high-quality documents that contain mathematical notation. You are not required to produce the files you submit using LaTex; you may produce it any way you wish, as long as the resulting document is legible.*
• *By virtue of submitting this assignment you (and your partner, if you have one) acknowledge that you are aware of the policy on homework collaboration for this course.*[a]
• *For any question, you may use facts previously proved in this course, its prerequisites, or in the assigned sections of the textbook.*
• *Unless we explicitly state otherwise, you should justify your answers. Your paper will be graded based on the correctness a of your answers, and the clarity, precision, and conciseness of your presentation.*

[a] "In each homework assignment you may collaborate with at most one other student who is currently taking CSCC63. If you collaborate with another student on an assignment, you and your partner must submit only one copy of your solution, with both of your names. The solution will be graded in the usual way and both partners will receive the same mark. Collaboration involving more than two students is not allowed. **For help with your homework you may consult only the instructor, TAs, your homework partner (if you have one), your textbook, and your class notes. You may not consult any other source.**"

Question 1. (10 marks) A 3-CNF propositional formula F is ***uniform*** if and only if every clause contains only positive or only negative literals, i.e., it is of the form $(x \vee y \vee z)$ or $(\neg x \vee \neg y \vee \neg z)$, where x, y, z are propositional variables. Prove that the satisfiability problem even for uniform 3-CNF formulas is **NP**-complete.

Hint: Add new variables and new clauses to make the given 3-CNF formula uniform.

Question 2. (25 marks) The Set Packing problem, SetPack, is defined as follows:

Instance: $\langle U, \mathcal{C}, t\rangle$, where $U = \{a_1, a_2, \dots, a_n\}$ is a finite set (the "universe"), $\mathcal{C} = \{A_1, A_2, \dots, A_m\}$ is a collection of subsets of U, (i.e., $A_j \subseteq U$, for $1 \le j \le m$), and $t \in \mathbb{Z}^+$ is a positive integer (the "target").

Question: Is there a $\mathcal{C}' \subseteq \mathcal{C}$ consisting of t pairwise disjoint sets (i.e., for any $j \neq j'$ such that $A_j, A_{j'} \in \mathcal{C}'$, $A_j \cap A_{j'} = \varnothing$)? Such a $\mathcal{C}'$ is called a ***packing*** of $\mathcal{C}$.

a. Prove that SetPack $\in$ **NP**.

b. Give a polytime mapping-reduction of SetPack to CNF-Sat, and prove that it is correct. (Since SetPack $\in$ **NP**, the Cook-Levin Theorem implies that such a reduction exists. The question here is not to argue that such a reduction exists, but to show one explicitly.)

Hint: The formula the mapping reduction constructs from the given instance $\langle U, \mathcal{C}, t\rangle$ of SetPack could involve variables x_j^k, for $1 \le j \le m$ and $1 \le k \le t$, with the following intended meaning: x_j^k is true if and only if A_j is the k-th set in the packing $\mathcal{C}'$ of $\mathcal{C}$. (So, think of the packing $\mathcal{C}'$ as a sequence, rather than a set, of sets from $\mathcal{C}$.) Write your formula in a modular way with subformulas expressing specific facts, analogous to (but much simpler than) the way we constructed the formula in the proof of the Cook-Levin Theorem. In your answer be sure to explain the role of each piece of your formula.