Computer Science C63 | January 24, 2024
Scarborough Campus | University of Toronto

Homework Assignment #3
Due: January 31, 2024, by 11:59 pm

• ***You must submit your assignment through the Crowdmark system.*** *You will receive by email an invitation through which you can submit your work in the form of separate PDF docucments with your answers to each question of the assignment. To work with a partner, you and your partner must form a group on Crowdmark. Crowdmark does not enforce a limit on the size of groups.* ***The course policy that limits the size of each group to at most two remains in effect:*** *submissions by groups of more than two persons will not be graded.*
• *To minimize bias in grading, Crowdmark does not reveal your name to the grader. Please do not subvert this feature by including your name(s) on the files you submit.*
• *It is your responsibility to ensure that the files you submit are legible. To this end, I encourage you to learn and use the LaTex typesetting system, which is designed to produce high-quality documents that contain mathematical notation. You are not required to produce the files you submit using LaTex; you may produce it any way you wish, as long as the resulting document is legible.*
• *By virtue of submitting this assignment you (and your partner, if you have one) acknowledge that you are aware of the policy on homework collaboration for this course.*[a]
• *For any question, you may use facts previously proved in this course, its prerequisites, or in the assigned sections of the textbook.*
• *Unless we explicitly state otherwise, you should justify your answers. Your paper will be graded based on the correctness a of your answers, and the clarity, precision, and conciseness of your presentation.*

[a] "In each homework assignment you may collaborate with at most one other student who is currently taking CSCC63. If you collaborate with another student on an assignment, you and your partner must submit only one copy of your solution, with both of your names. The solution will be graded in the usual way and both partners will receive the same mark. Collaboration involving more than two students is not allowed. **For help with your homework you may consult only the instructor, TAs, your homework partner (if you have one), your textbook, and your class notes. You may not consult any other source.**"

Question 1. (20 marks) A ***Turing machine with rewind (TMR)*** is defined as an ordinary (deterministic, single-tape) Turing machine except that, on a "left" move, instead of the tape head moving one cell to the left, it moves all the way to the leftmost cell of the tape (it "rewinds"). Show how to simulate a TM M using a TMR M'. That is, given M explain how to construct a TMR M' so that, for any input string x, if M accepts (respectively, rejects, or loops on) x, so does M'. Describe your TMR in point-form English, explaining clearly how it operates and how it manages the information on its tape.

You do not need to prove that your TMR correctly simulates the given TM, but your description should be clear enough and contain enough detail that its correctness should be obvious to the reader.

Question 2. (20 marks) Show that the set of decidable languages is closed under union and intersection ***using the ordered enumerator characterization of decidable languages***. These facts were shown in the Week 2 tutorial. Here you are asked to prove the same results but using a different approach, based on enumerators, which are discussed in your Week 3 tutorial and on pages 180-181 of your textbook. Your proofs should not make any reference, direct or indirect, to Turing machine deciders or recognizers; only to Turing machine enumerators and ordered enumerators.

Question to think about, but not to include with your answers: Use the enumerator characterization of ***recognizable*** languages to show that the set of recognizable languages is closed under union and intersection.

Question 3. (20 marks) The ***symmetric difference*** of two sets K and L, denoted $K \oplus L$, is the set of elements that belong to exactly one of K or L:

$$K \oplus L = (K \cup L) - (K \cap L).$$

Prove that if K and L are decidable, then $(K \oplus L)^*$ is decidable. You may find it useful to use nondeterministic, multitape TMs in your proof. If you do so, you may describe the NTM at a high level explaining what it "guesses", but you must also describe it at a more concrete level explaining the "guesses" in terms of nondeterministic choices. For guidance on such descriptions refer to Week 3 notes and tutorial.

Question to think about, but not to include with your answers: Is it the case that if K and L are recognizable, then $(K \oplus L)^*$ is recognizable?