Computer Science C63 January 17, 2024
Scarborough Campus University of Toronto

Homework Assignment #2
Due: January 24, 2024, by 11:59 pm

• ***You must submit your assignment through the Crowdmark system.*** *You will receive by email an invitation through which you can submit your work in the form of separate PDF docucments with your answers to each question of the assignment. To work with a partner, you and your partner must form a group on Crowdmark. Crowdmark does not enforce a limit on the size of groups.* ***The course policy that limits the size of each group to at most two remains in effect:*** *submissions by groups of more than two persons will not be graded.*
• *To minimize bias in grading, Crowdmark does not reveal your name to the grader. Please do not subvert this feature by including your name(s) on the files you submit.*
• *It is your responsibility to ensure that the files you submit are legible. To this end, I encourage you to learn and use the LaTex typesetting system, which is designed to produce high-quality documents that contain mathematical notation. You are not required to produce the files you submit using LaTex; you may produce it any way you wish, as long as the resulting document is legible.*
• *By virtue of submitting this assignment you (and your partner, if you have one) acknowledge that you are aware of the policy on homework collaboration for this course.*[a]
• *For any question, you may use facts previously proved in this course, its prerequisites, or in the assigned sections of the textbook.*
• *Unless we explicitly state otherwise, you should justify your answers. Your paper will be graded based on the correctness a of your answers, and the clarity, precision, and conciseness of your presentation.*

[a] "In each homework assignment you may collaborate with at most one other student who is currently taking CSCC63. If you collaborate with another student on an assignment, you and your partner must submit only one copy of your solution, with both of your names. The solution will be graded in the usual way and both partners will receive the same mark. Collaboration involving more than two students is not allowed. **For help with your homework you may consult only the instructor, TAs, your homework partner (if you have one), your textbook, and your class notes. You may not consult any other source.**"

Question 1. (15 marks) Describe a (deterministic, single-tape) TM that computes the partial function $f : \{0,1\}^* \to \{0,1\}^*$ that maps the binary representation of a positive integer n to the binary representation of $2n - 1$. This function is partial because it is not defined if the input string is empty or if the input string starts with a 0. (We consider binary representations with no leading 0s; this makes the binary representation of each positive integer unique. Note that this means that the output of this TM must also have no leading 0s.)

First describe your TM in point-form English, explaining how it works informally but clearly. Then describe all the components of the TM, giving the transition function in graphical form as in Example 3.9 of your textbook, and relate the states of your TM to your point-form English description. For example, you might describe the role of a certain state q as "keep moving left skipping over symbols X and Y until we find symbol Z".

In https://mustafaquraish.github.io/TMSim/ you can find a nice TM simulator, courtesy of Mustafa Qureish (former UTSC student). Feel free to use this to "test" your TM, but don't submit its description in the format required by the simulator. The grader will rely only on your description to be convinced of its correctness. It is therefore important that your TM be as simple as possible, and your description of it be very clear. Overly complicated TMs and descriptions that the grader is unable to follow will receive few (possibly zero) marks. This is as much an exercise in TM design as it is in clear communication. You do not have the prove that your TM is correct, but it should be obvious that it is.

Hint: Make sure to read the document entitled "Turing machines as function computers". Think about how the binary representation of n relates that that of $2n$ and then to that of $2n-1$. Try some examples to verify your conclusion.

Question 2. (25 marks)

a. Vassos has been observing the operation of a Turing Machine with five states and four tape symbols on some input. After 53 million moves he notices that the TM never moved its tape head past the 10th (tenth) cell, and exclaims in exasperation: "I've seen enough, this thing will never halt!" Is he right? Justify your answer.

b. Let M be a Turing machine with the following property: There is a constant c such that for every input string x, M does not move its tape head to the right of cell c. Prove that $\mathcal{L}(M)$ is a regular language.

c. Prove that a language L is regular if and only if it is recognized by a Turing machine that always moves its tape head to the right.