Computer Science C63 — January 15, 2025
Scarborough Campus — University of Toronto

Homework Assignment #2
Due: January 22, 2025, by 11:59 pm

- ***You must submit your assignment through the Crowdmark system.*** *You will receive by email an invitation through which you can submit your work in the form of separate PDF docucments with your answers to each question of the assignment. To work with a partner, you and your partner must form a group on Crowdmark. Crowdmark does not enforce a limit on the size of groups.* ***The course policy that limits the size of each group to at most two remains in effect:*** *submissions by groups of more than two persons will not be graded.*
- *To minimize bias in grading, Crowdmark does not reveal your name to the grader. Please do not subvert this feature by including your name(s) on the files you submit.*
- *It is your responsibility to ensure that the files you submit are legible. To this end, I encourage you to learn and use the LaTex typesetting system, which is designed to produce high-quality documents that contain mathematical notation. You are not required to produce the files you submit using LaTex; you may produce it any way you wish, as long as the resulting document is legible.*
- *By virtue of submitting this assignment you (and your partner, if you have one) acknowledge that you are aware of the policy on homework collaboration for this course.*[a]
- *For any question, you may use facts previously proved in this course, its prerequisites, or in the assigned sections of the textbook.*
- *Unless we explicitly state otherwise, you should justify your answers. Your paper will be graded based on the correctness a of your answers, and the clarity, precision, and conciseness of your presentation.*

[a] "In each homework assignment you may collaborate with at most one other student who is currently taking CSCC63. If you collaborate with another student on an assignment, you and your partner must submit only one copy of your solution, with both of your names. The solution will be graded in the usual way and both partners will receive the same mark. Collaboration involving more than two students is not allowed. **For help with your homework you may consult only the instructor, TAs, your homework partner (if you have one), your textbook, and your class notes. You may not consult any other source.** *Nota Bene:* 'Other source' includes, but is not limited to, the use of AI-based tools, even for (allegedly) 'just improving' your work. What you submit must be entirely your own creation."

Question 1. (15 marks) Describe a TM that decides the following language L over the alphabet $\{0, 1, \#\}$:

$$L = \{\#x : \ x \in \{0,1\}^* \text{ and } x \text{ contains exactly twice as many 1s as 0s}\}.^1$$

For example, #, #101, #010111 are strings in L, while #001, #01, #1001111, 100111 are strings not in L.

First describe your TM in point-form English, explaining how it works informally but clearly. Then describe all the components of the TM, giving the transition function in graphical form as in Example 3.9 of your textbook, and relate the states of your TM to your point-form English description. For example, you might describe the role of a certain state q as "keep moving left skipping over symbols X and Y until we find symbol Z".

In https://mustafaquraish.github.io/TMSim/ you can find a nice TM simulator, courtesy of Mustafa Qureish (former UTSC student). Feel free to use this to "test" your TM, but don't submit its description in the format required by the simulator. The grader will rely only on your description to be convinced of its correctness. It is therefore important that your TM be as simple as possible, and your description of it be very clear. Overly complicated TMs and descriptions that the grader is unable to follow will receive few (possibly zero) marks. This is as much an exercise in TM design as it is in clear communication. You do not have the prove that your TM is correct, but it should be obvious that it is.

[1] The initial # only helps mark the beginning of the input, which simplifies the task a little.

Question 2. (25 marks)

a. You are observing the operation of a Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_0, h_A, h_R)$ on some input x of length n, and on every single move you have observed so far the machine has moved to the right. You know everything about this machine except the transition function δ and you also know the input string x. Is there a number of such moves after which you can safely conclude that the machine will never halt on that input? Justify your answer.

b. For the purposes of this question let us call a Turing machine M ***special*** if it has the following property: When started on any input of length n, M first makes n moves, each of them to the right, and after that the machine never moves to the left of cell $n + 1$ (where the leftmost tape cell is cell number 1).

Prove that a language L is regular if and only if L is recognized by a special Turing machine.