Computer Science C63 — January 10, 2024
Scarborough Campus — University of Toronto

Homework Assignment #1
Due: January 17, 2023, by 11:59 pm

• ***You must submit your assignment through the Crowdmark system.*** *You will receive by email an invitation through which you can submit your work in the form of separate PDF docucments with your answers to each question of the assignment. To work with a partner, you and your partner must form a group on Crowdmark. Crowdmark does not enforce a limit on the size of groups.* ***The course policy that limits the size of each group to at most two remains in effect:*** *submissions by groups of more than two persons will not be graded.*
• *To minimize bias in grading, Crowdmark does not reveal your name to the grader. Please do not subvert this feature by including your name(s) on the files you submit.*
• *It is your responsibility to ensure that the files you submit are legible. To this end, I encourage you to learn and use the LaTex typesetting system, which is designed to produce high-quality documents that contain mathematical notation. You are not required to produce the files you submit using LaTex; you may produce it any way you wish, as long as the resulting document is legible.*
• *By virtue of submitting this assignment you (and your partner, if you have one) acknowledge that you are aware of the policy on homework collaboration for this course.*[a]
• *For any question, you may use facts previously proved in this course, its prerequisites, or in the assigned sections of the textbook.*
• *Unless we explicitly state otherwise, you should justify your answers. Your paper will be graded based on the correctness a of your answers, and the clarity, precision, and conciseness of your presentation.*

[a] "In each homework assignment you may collaborate with at most one other student who is currently taking CSCC63. If you collaborate with another student on an assignment, you and your partner must submit only one copy of your solution, with both of your names. The solution will be graded in the usual way and both partners will receive the same mark. Collaboration involving more than two students is not allowed. **For help with your homework you may consult only the instructor, TAs, your homework partner (if you have one), your textbook, and your class notes. You may not consult any other source.**"

Question 1. (20 marks) Prove each of the following facts.[1]

(a) If A is a set and S is a sequence of elements from A in which every element of A appears at least once, then A is countable. (Note that S is an enumeration of A with possible repetition.)

(b) If A is a countable set and $f : A \to B$ is a function, then the image of f is countable. (The image of f is the set $\{b : \ b = f(a) \text{ for some } a \in A\}$.)

(c) If A is an uncountable set and $f : A \to B$ is a one-to-one function, then B is uncountable.

(d) The set of all functions from finite subsets of $\mathbb{N}$ to $\mathbb{N}$ is countable. (Hint: First think about why the set of functions from $\{0, 1\}$ to $\mathbb{N}$ is countable.)

Question 2. (20 marks) For each set defined below, state whether it is countable or not, and justify[2] your answer.

[1] Keep in mind (here and in all subsequent assignments) that once you have proved a fact, you can use it in subsequent proofs without re-proving it. If you find that you are repeating an argument you have seen before, it is worth checking whether you can apply the result in which that argument was used, rather than repeating the argument. This makes your proofs shorter, clearer, and more elegant. This feature of mathematics is something that should appeal to you as computer scientists: This is how you (should) develop your programs!
[2] "Justify" means "prove", but sometimes it signals that a short argument suffices.

(a) The set of square matrices of integers.

(b) The set of functions from $\mathbb{N} \times \mathbb{N}$ to $\mathbb{N}$.

(c) The set of unbounded functions from $\mathbb{N}$ to $\mathbb{N}$. A function $f : \mathbb{N} \to \mathbb{N}$ is ***unbounded*** if for every $m \in \mathbb{N}$ there is some $n \in \mathbb{N}$ such that $f(n) \geq m$; that is, the function takes on arbitrarily large values.

(d) The set of non-regular languages over alphabet Σ. (Recall that an alphabet Σ is a <u>finite</u> set of symbols; a string over Σ is a <u>finite</u> sequence of symbols from Σ; and a language over Σ is a (possibly infinite) set of strings over Σ.)