Computer Science C63 | January 8, 2025
Scarborough Campus | University of Toronto

Homework Assignment #1
Due: January 15, 2025, by 11:59 pm

• ***You must submit your assignment through the Crowdmark system.*** *You will receive by email an invitation through which you can submit your work in the form of separate PDF docucments with your answers to each question of the assignment. To work with a partner, you and your partner must form a group on Crowdmark. Crowdmark does not enforce a limit on the size of groups.* ***The course policy that limits the size of each group to at most two remains in effect:*** *submissions by groups of more than two persons will not be graded.*
• *To minimize bias in grading, Crowdmark does not reveal your name to the grader. Please do not subvert this feature by including your name(s) on the files you submit.*
• *It is your responsibility to ensure that the files you submit are legible. To this end, I encourage you to learn and use the LaTex typesetting system, which is designed to produce high-quality documents that contain mathematical notation. You are not required to produce the files you submit using LaTex; you may produce it any way you wish, as long as the resulting document is legible.*
• *By virtue of submitting this assignment you (and your partner, if you have one) acknowledge that you are aware of the policy on homework collaboration for this course.*[a]
• *For any question, you may use facts previously proved in this course, its prerequisites, or in the assigned sections of the textbook.*
• *Unless we explicitly state otherwise, you should justify your answers. Your paper will be graded based on the correctness a of your answers, and the clarity, precision, and conciseness of your presentation.*

[a] "In each homework assignment you may collaborate with at most one other student who is currently taking CSCC63. If you collaborate with another student on an assignment, you and your partner must submit only one copy of your solution, with both of your names. The solution will be graded in the usual way and both partners will receive the same mark. Collaboration involving more than two students is not allowed. **For help with your homework you may consult only the instructor, TAs, your homework partner (if you have one), your textbook, and your class notes. You may not consult any other source.** *Nota Bene:* 'Other source' includes, but is not limited to, the use of AI-based tools, even for (allegedly) 'just improving' your work. What you submit must be entirely your own creation."

Question 1. (20 marks) Prove each of the following facts.[1]

(a) If $f\colon A \to B$ is an onto (surjective) function and A is countable then B is also countable.

(b) If $f\colon A \to B$ is an one-to-one (injective) function and A is uncountable then B is also uncountable.

(c) The set $\mathcal{P}$ of sets of natural numbers that are powers of 2 (i.e., $\mathcal{P} = \{A\colon A \subseteq \mathbb{N}$, and for every $n \in A$ there is some $k \in \mathbb{N}$ such that $n = 2^k\}$) is uncountable.

(d) If A is uncountable and B is countable, then $A - B$ is uncountable. ($A - B = \{x\colon x \in A$ and $x \notin B\}$.)

[**Continued on the next page**]

[1] Keep in mind (here and in all subsequent assignments) that once you have proved a fact, you can use it in subsequent proofs without re-proving it. If you find that you are repeating an argument you have seen before, it is worth checking whether you can apply the result in which that argument was used, or "factor" the common argument as a lemma and use it in both results, rather than repeating the same argument twice. This makes your proofs shorter, clearer, and more elegant. This is analogous to how you (should) develop your programs! Factor common functionalities into subroutines and reuse, rather than repeat code.

Question 2. (20 marks) Let $f : \mathbb{N} \to \mathbb{N}$ be a function. We say that

- f is ***increasing*** if for every $i \in \mathbb{N}$, $f(i) \le f(i+1)$.
- f is ***decreasing*** if for every $i \in \mathbb{N}$, $f(i) \ge f(i+1)$.
- f is ***monotone*** if it is increasing or it is decreasing.

For each set defined below, state whether it is countable or not, and justify your answer.

(a) The set of functions from $\{0, 1\}$ to $\mathbb{N}$.

(b) The set of increasing functions from $\mathbb{N}$ to $\mathbb{N}$.

(c) The set of decreasing functions from $\mathbb{N}$ to $\mathbb{N}$.

(d) The set of functions from $\mathbb{N}$ to $\mathbb{N}$ that are monotone.

(e) The set of functions from $\mathbb{N}$ to $\mathbb{N}$ that are ***not*** monotone.