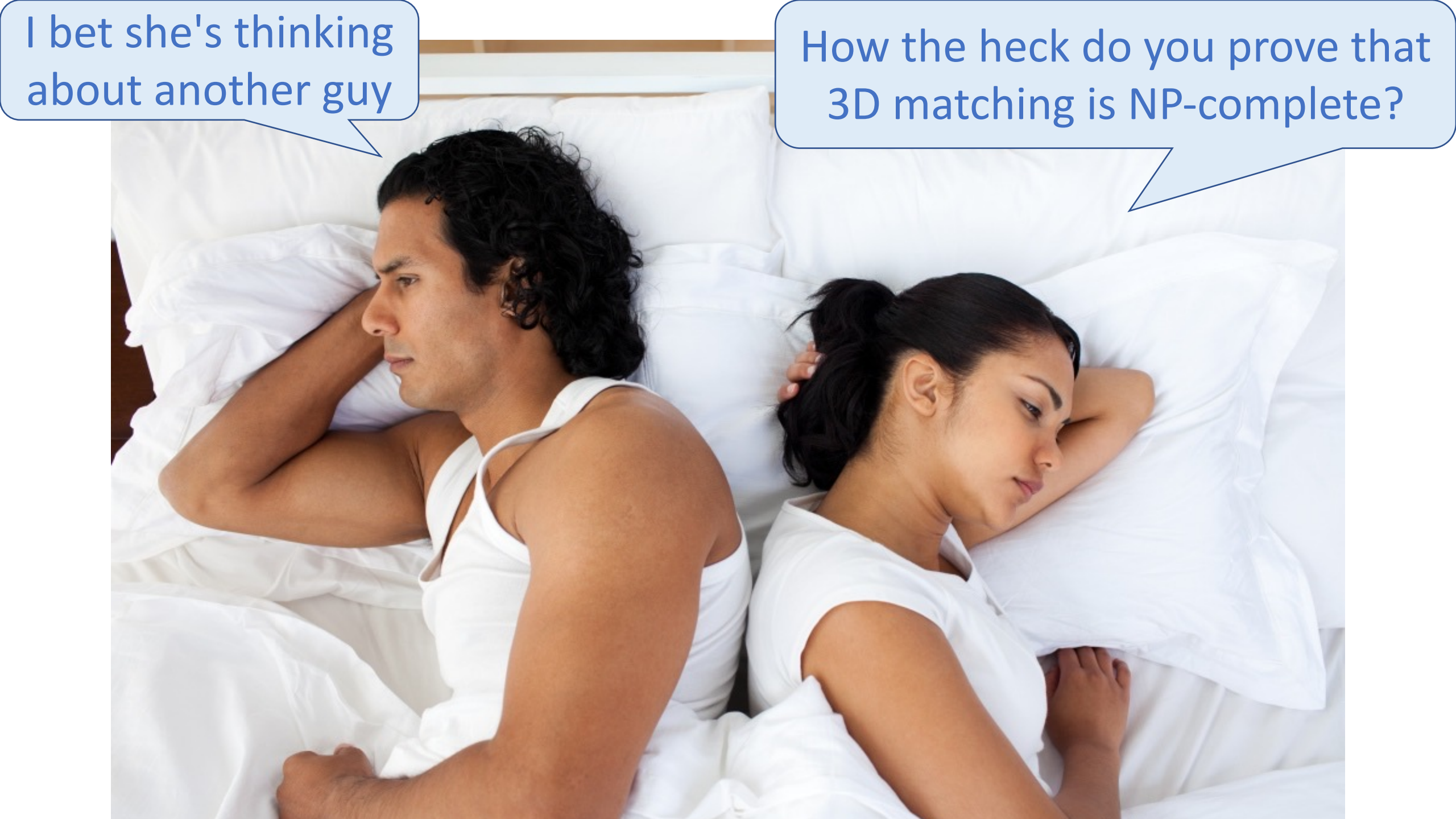

I bet she's thinking about another guy
How the heck do you prove that 3D matching is NP-complete?

3-dimensional matching

$A = \{a, b, c, d\}$
$B = \{w, x, y, z\}$
$C = \{1, 2, 3, 4\}$

$T = \{(a, x, 1), (a, z, 4),$
$(b, x, 2), (b, x, 3),$
$(c, w, 3), (c, x, 3),$
$(d, y, 1), (d, x, 2)\}$

Tripartite matching

$A = \{a, b, c, d\}$
$B = \{w, x, y, z\}$
$C = \{1, 2, 3, 4\}$

$T = \{(a, x, 1), (a, z, 4),$
$(b, x, 2), (b, x, 3),$
$(c, w, 3), (c, x, 3),$
$(d, y, 1), (d, x, 2)\}$

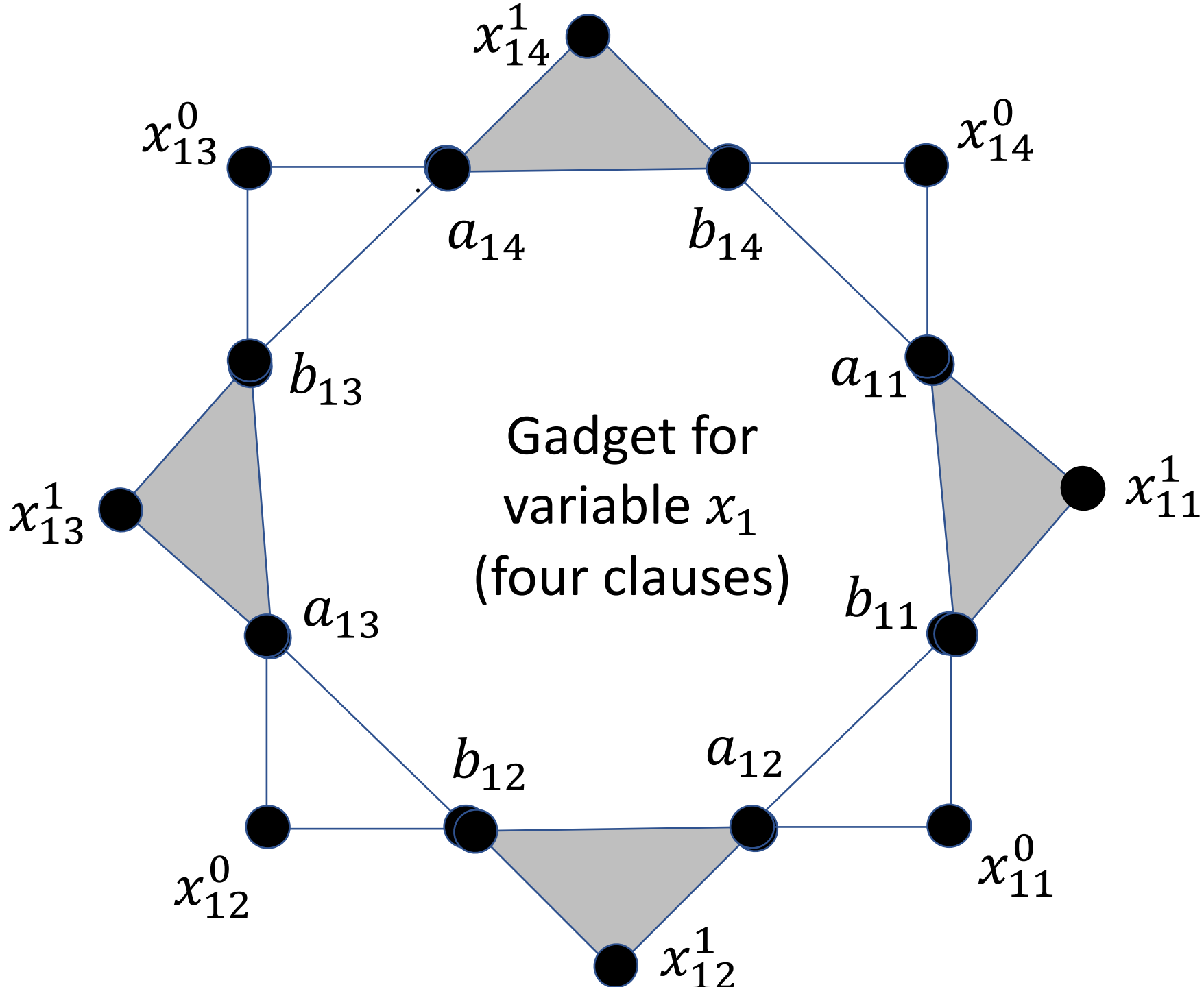
Matching M

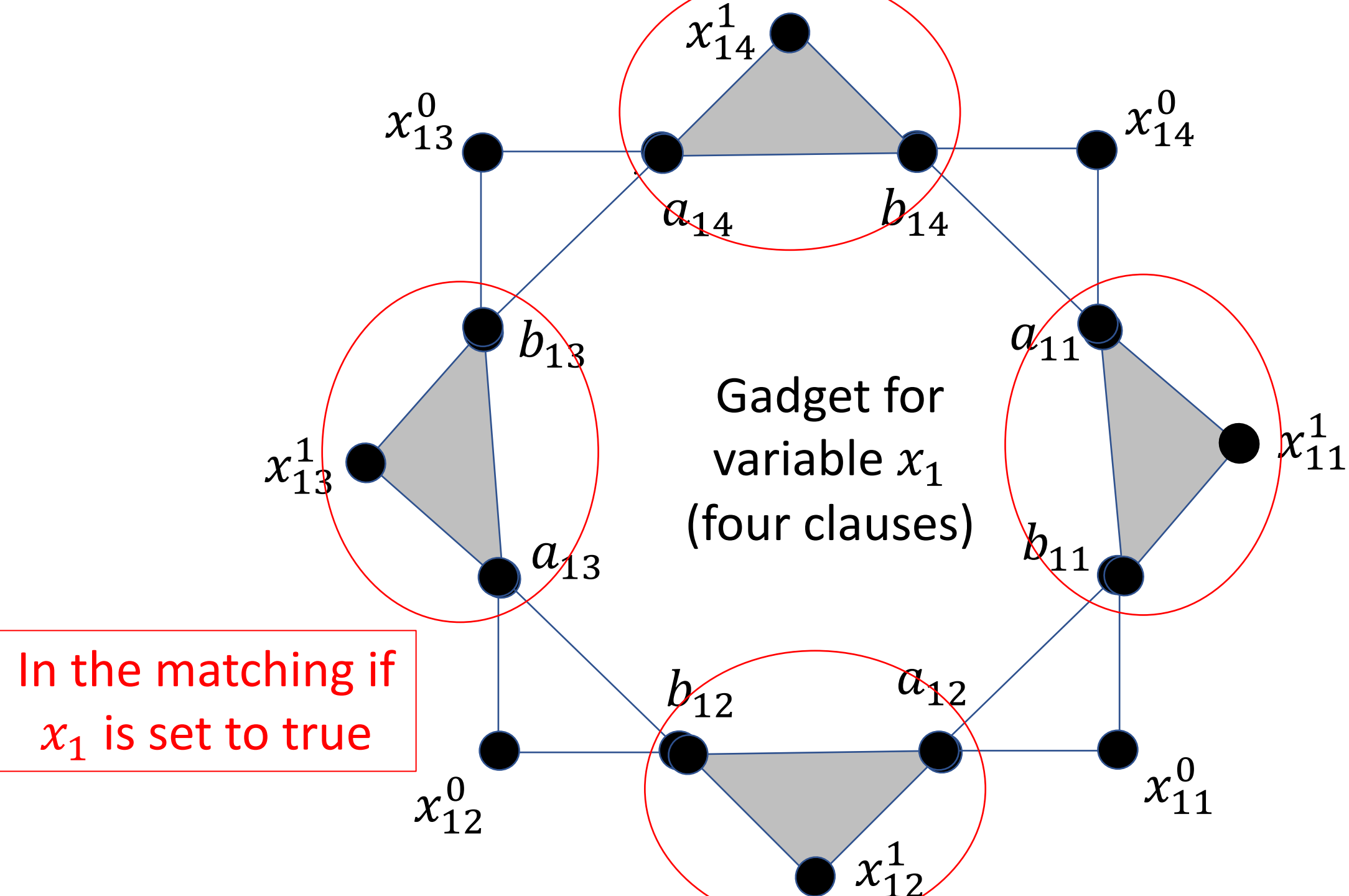
Tripartite matching

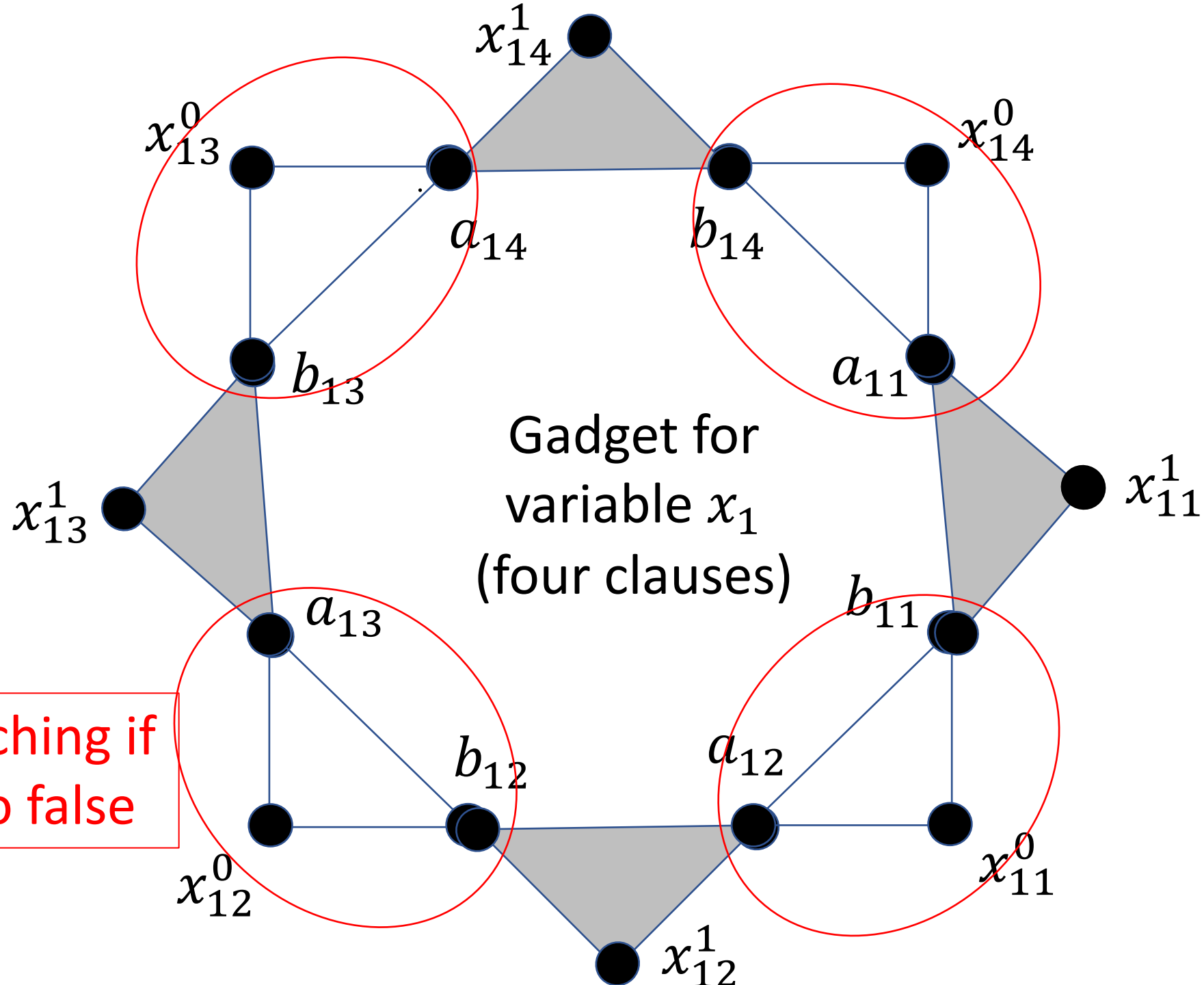
$A = \{a, b, c, d\}$
$B = \{w, x, y, z\}$
$C = \{1, 2, 3, 4\}$

$T' = \{(a, x, 1), (a, z, 4),$
$(b, x, 2), (b, x, 3),$
$(c, w, 1), (c, x, 3),$
$(d, y, 1), (d, x, 2)\}$

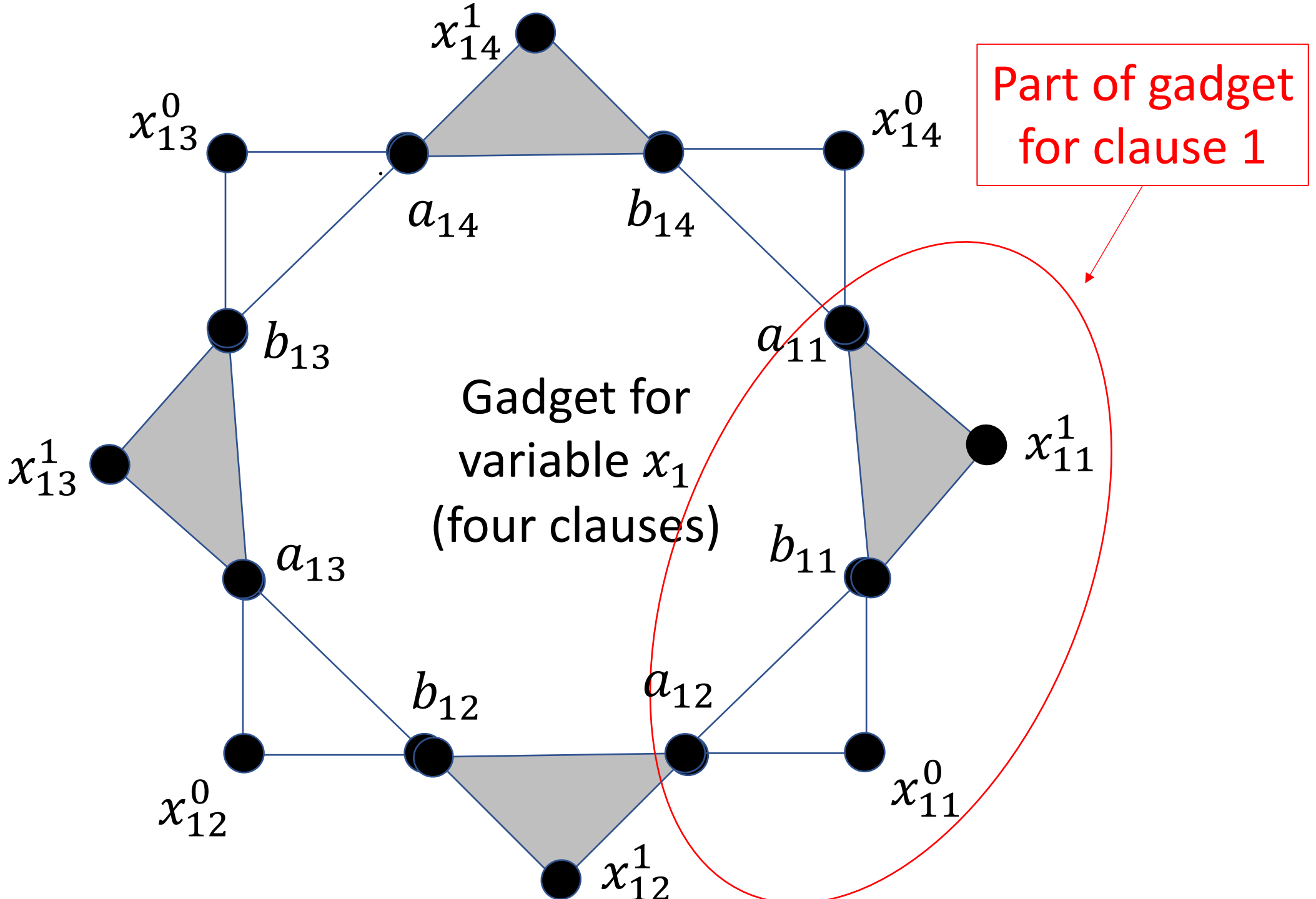
No matching!

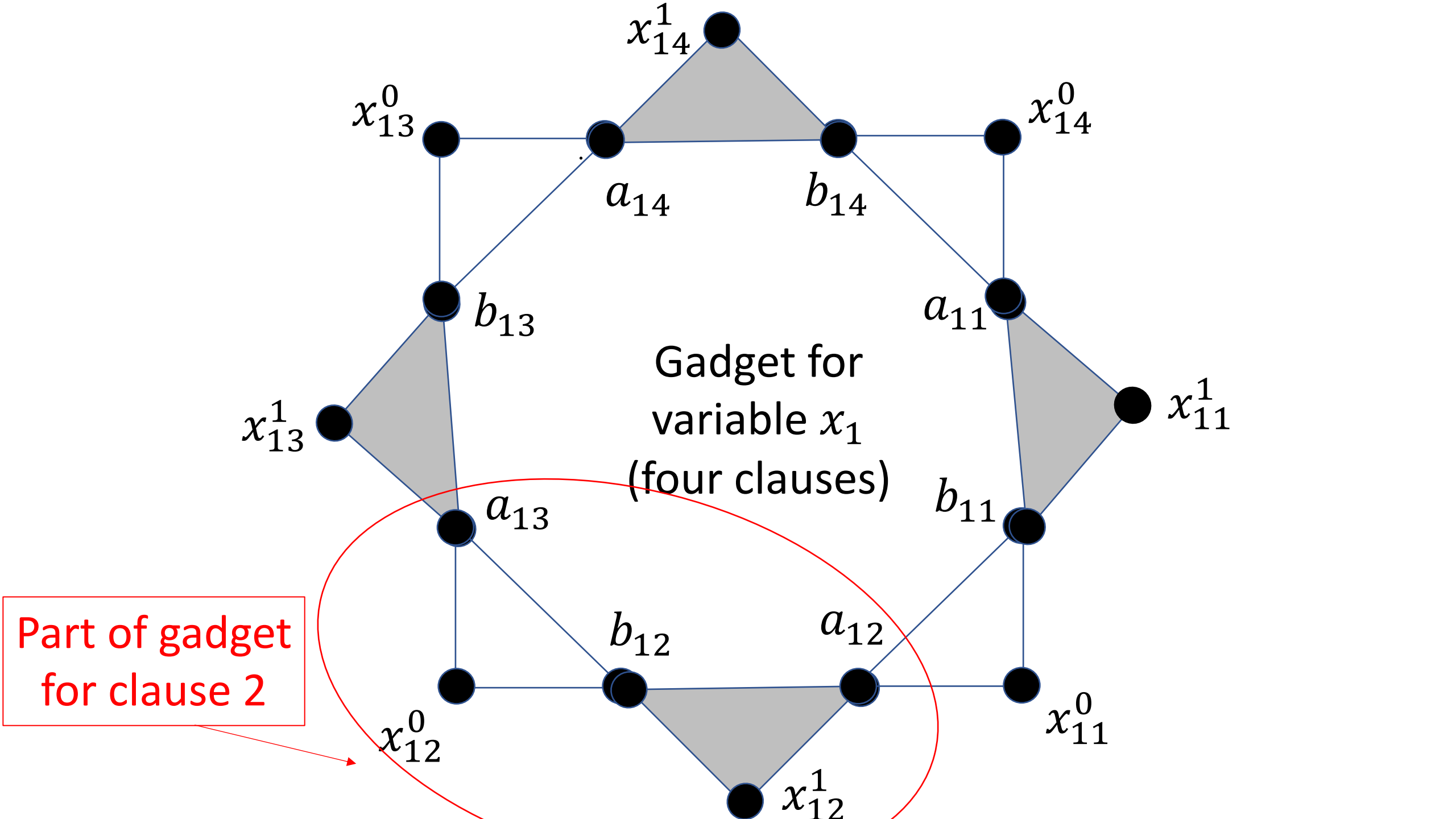

x_{14}^1
x_{13}^0
x_{14}^0
a_{14}
b_{14}
b_{13}
a_{11}
Gadget for
variable x_1
(four clauses)
x_{13}^1
x_{11}^1
a_{13}
b_{11}
b_{12}
a_{12}
x_{12}^0
x_{11}^0
x_{12}^1

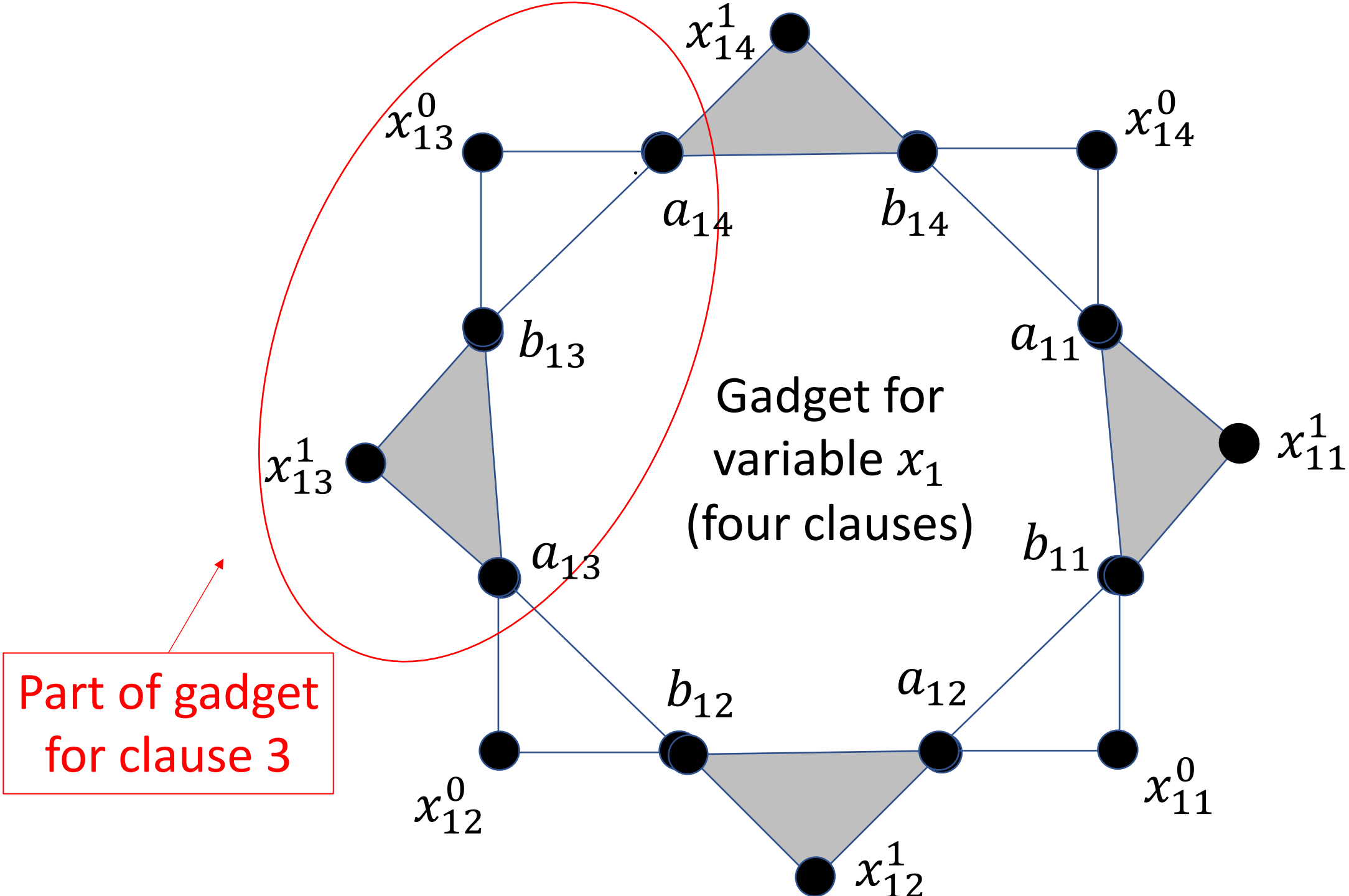

x_{14}^1
x_{13}^0
x_{14}^0
a_{14}
b_{14}
b_{13}
a_{11}
x_{13}^1
x_{11}^1
Gadget for variable x_1 (four clauses)
a_{13}
b_{11}
b_{12}
a_{12}
x_{12}^0
x_{11}^0
x_{12}^1
In the matching if x_1 is set to true

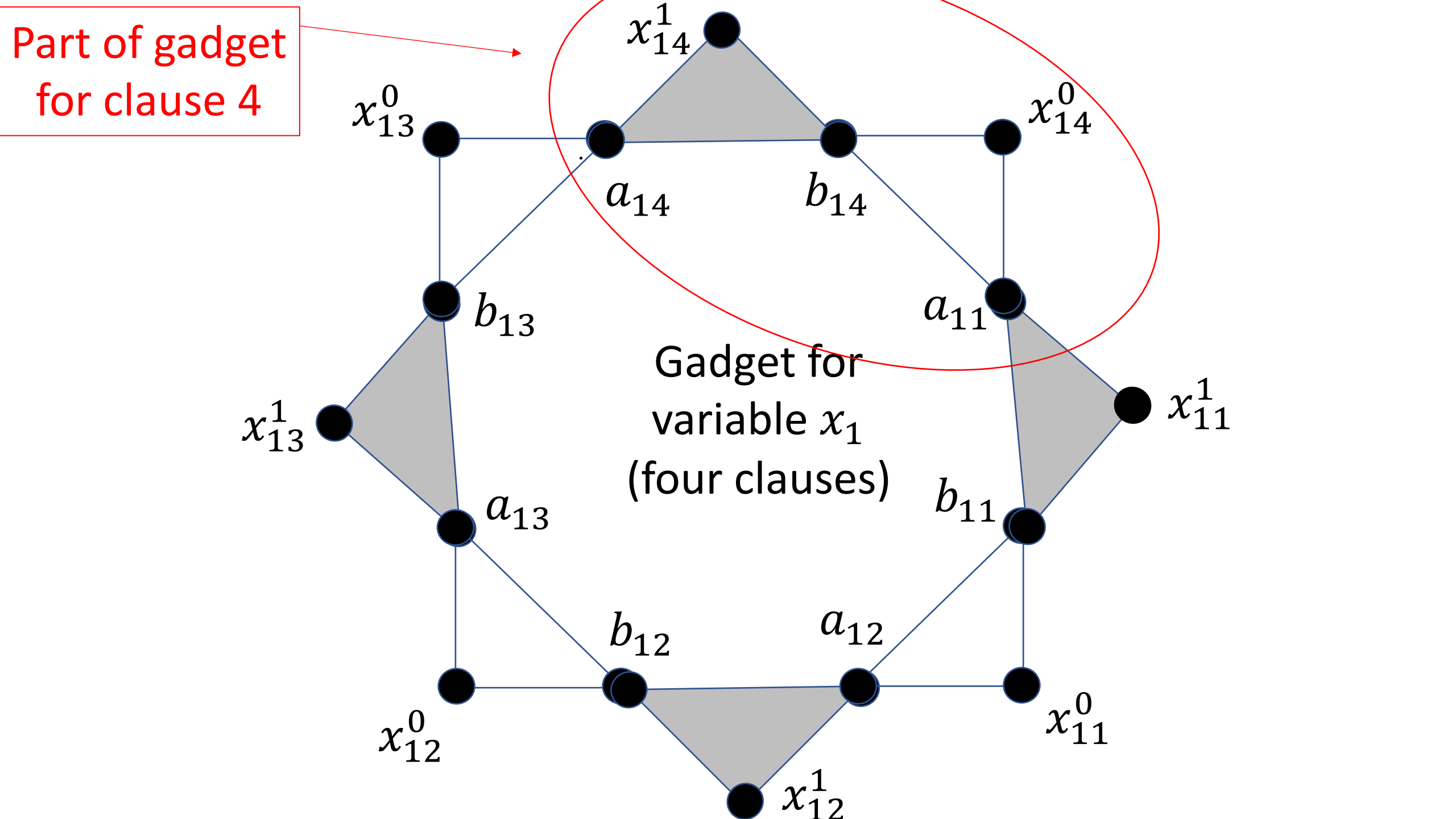


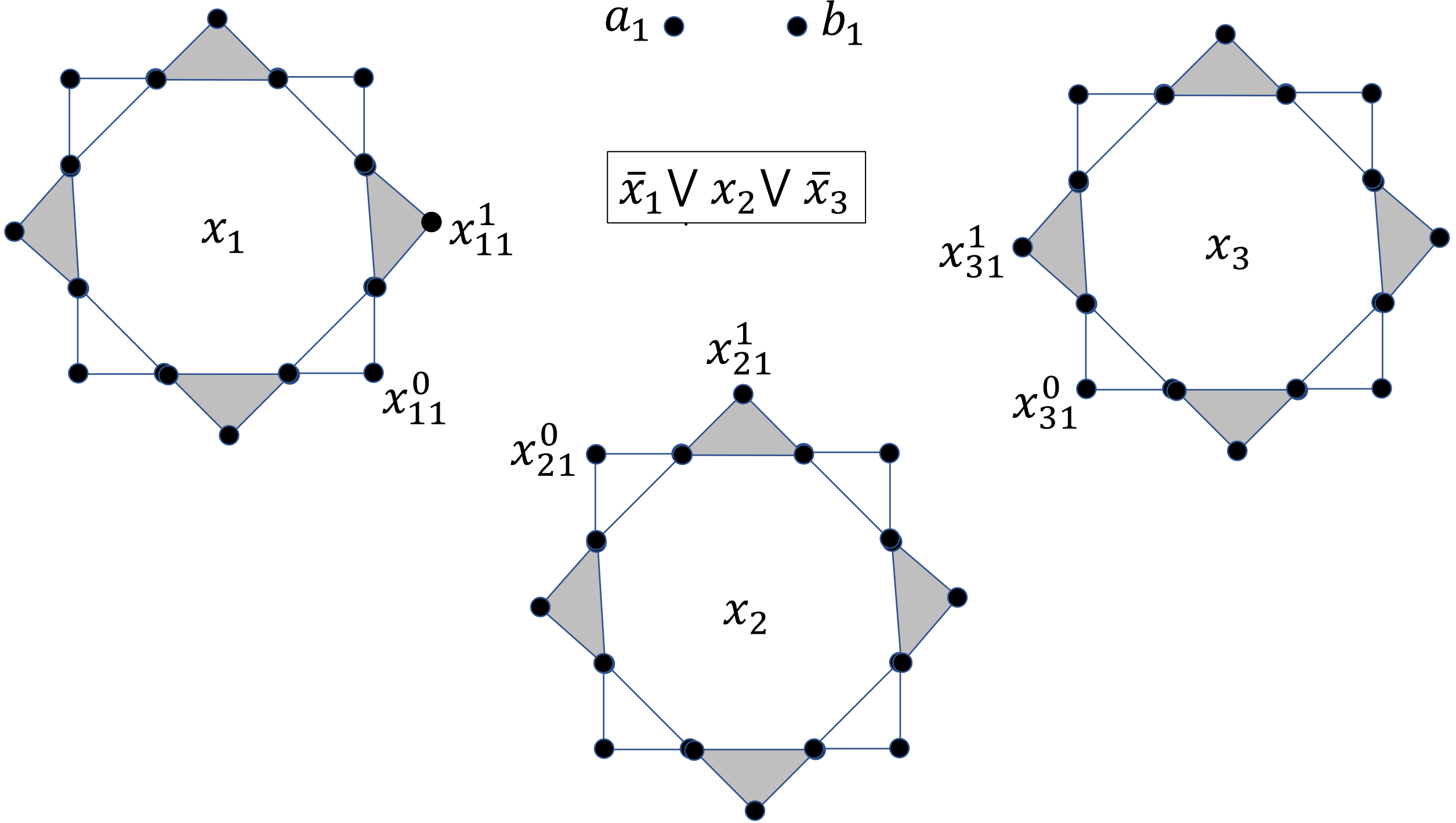
In the matching if x_1 is set to false

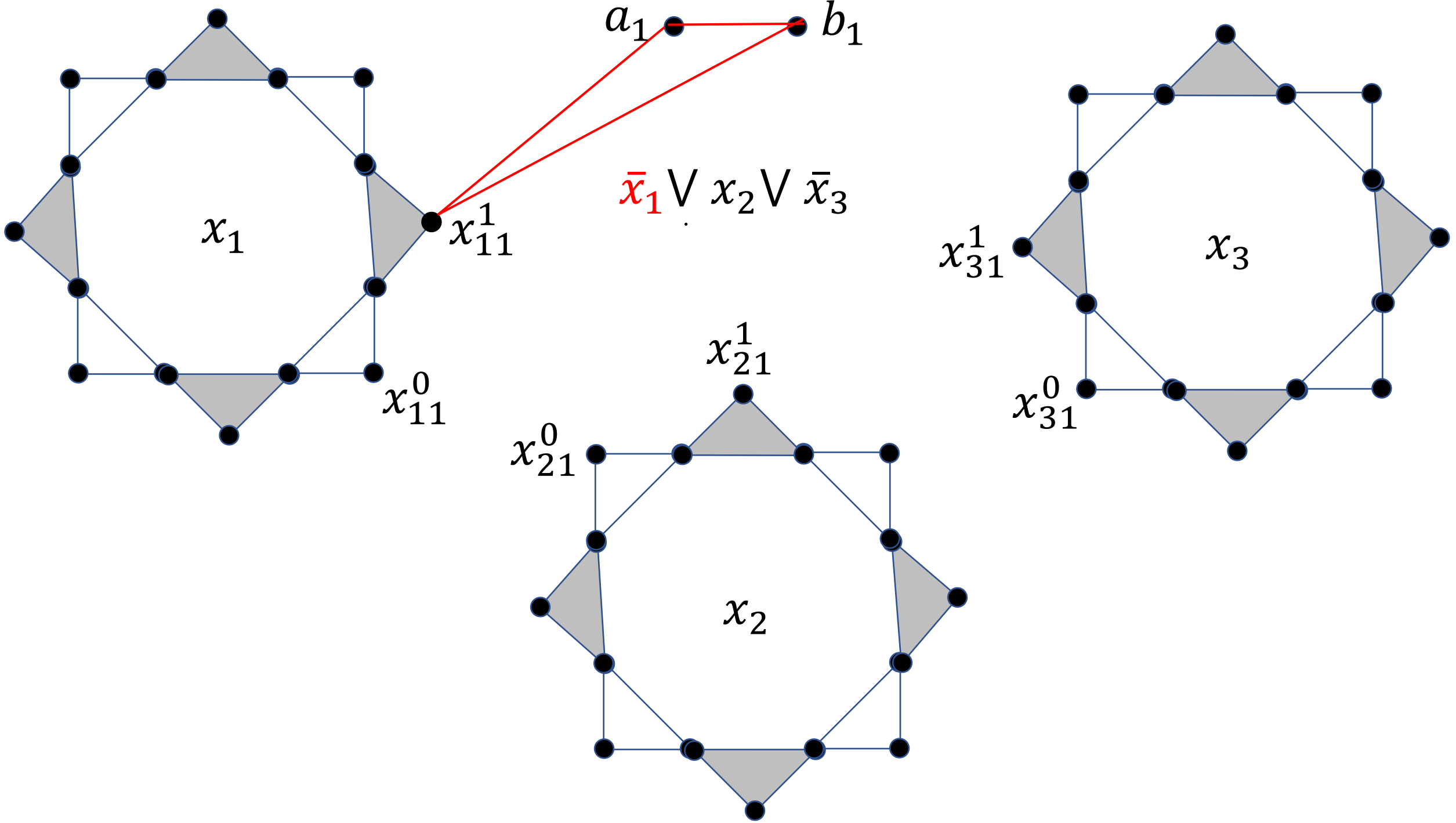

x_{14}^1
x_{13}^0
x_{14}^0
a_{14}
b_{14}
b_{13}
a_{11}
x_{13}^1
x_{11}^1
Gadget for
variable x_1
(four clauses)
a_{13}
b_{11}
b_{12}
a_{12}
x_{12}^0
x_{11}^0
x_{12}^1
Part of gadget
for clause 1

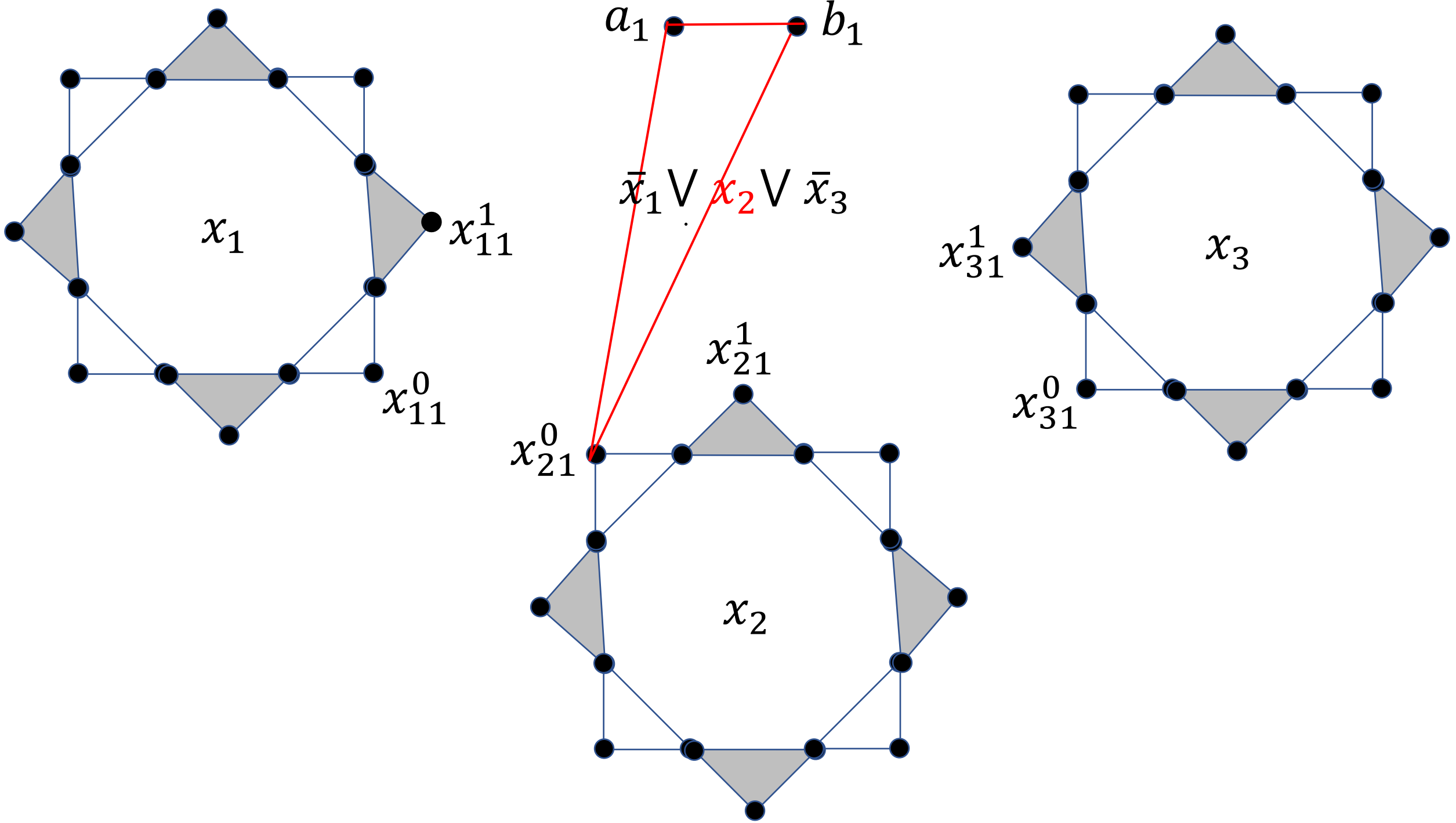

x_{14}^1
x_{13}^0
x_{14}^0
a_{14}
b_{14}
b_{13}
a_{11}
Gadget for
variable x_1
(four clauses)
x_{13}^1
x_{11}^1
a_{13}
b_{11}
Part of gadget
for clause 2
b_{12}
a_{12}
x_{12}^0
x_{11}^0
x_{12}^1

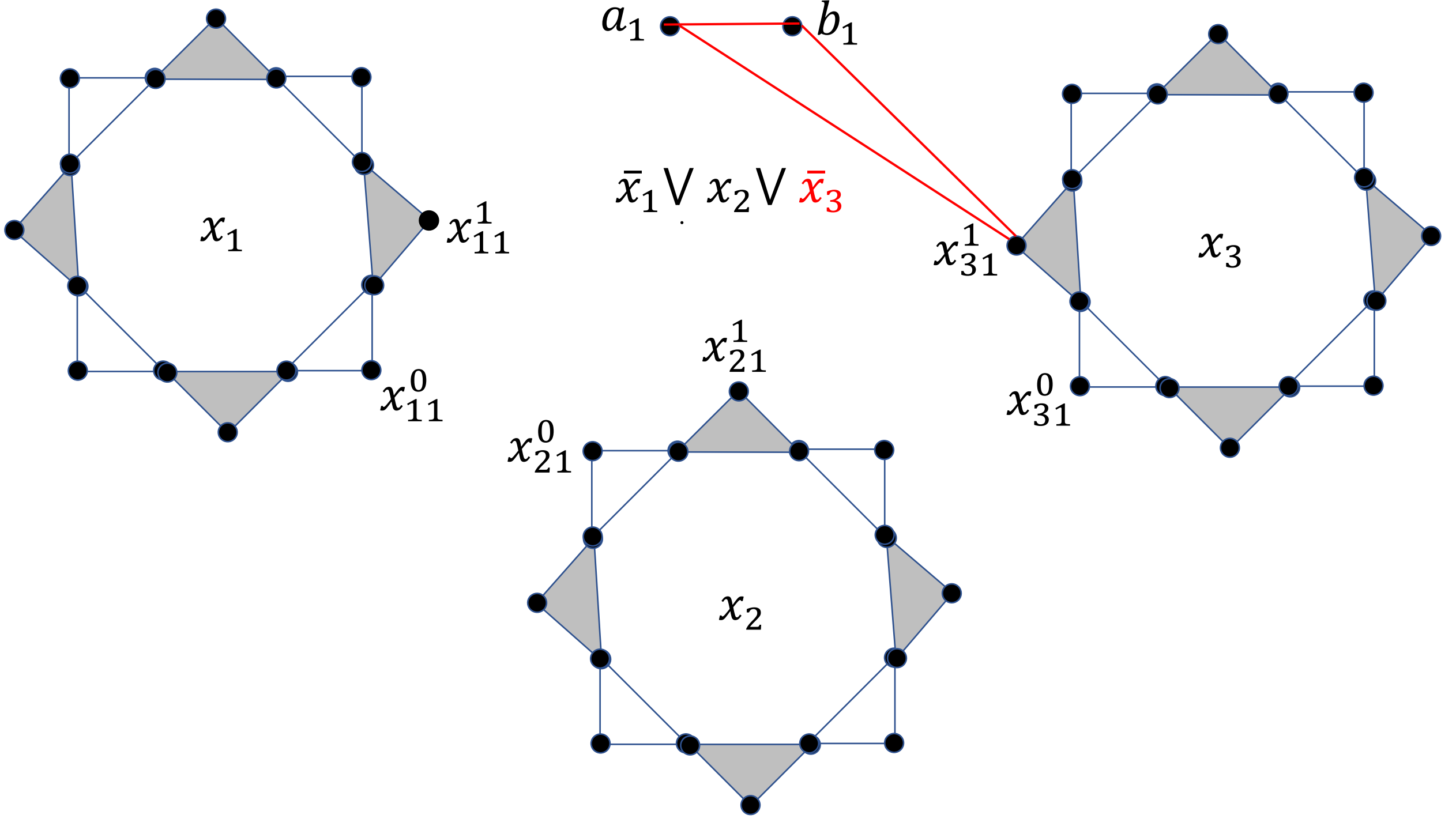

x_{14}^1
x_{13}^0
x_{14}^0
a_{14}
b_{14}
b_{13}
a_{11}
x_{13}^1
x_{11}^1
Gadget for variable x_1 (four clauses)
a_{13}
b_{11}
b_{12}
a_{12}
x_{12}^0
x_{11}^0
x_{12}^1
Part of gadget for clause 3

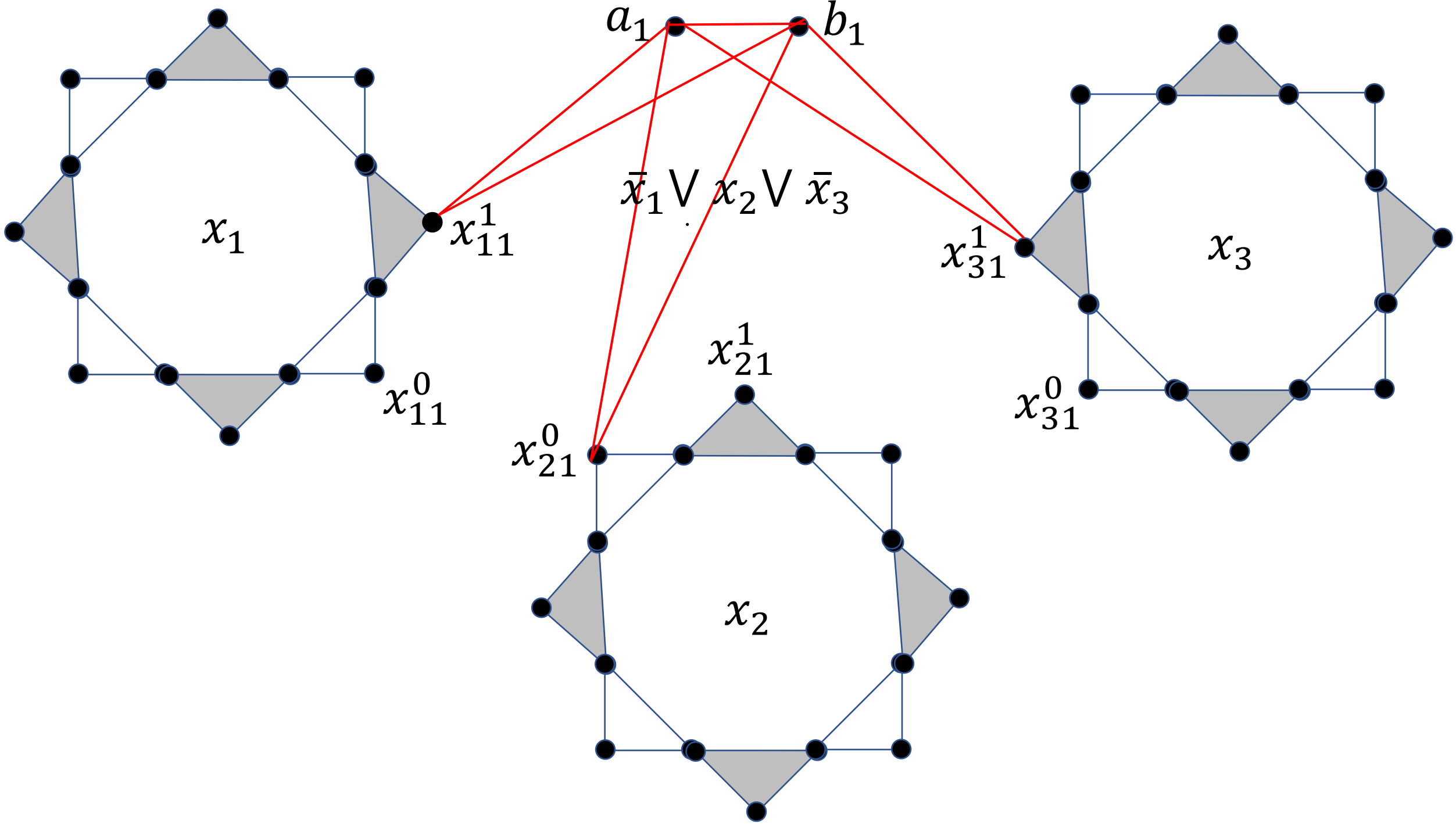

Part of gadget for clause 4
x_{14}^1
x_{13}^0
x_{14}^0
a_{14}
b_{14}
b_{13}
a_{11}
Gadget for variable x_1 (four clauses)
x_{13}^1
x_{11}^1
a_{13}
b_{11}
b_{12}
a_{12}
x_{12}^0
x_{11}^0
x_{12}^1


a_1
b_1
$\bar{x}_1 \vee x_2 \vee \bar{x}_3$
x_1
x_{11}^1
x_{11}^0
x_{21}^1
x_{21}^0
x_2
x_{31}^1
x_3
x_{31}^0

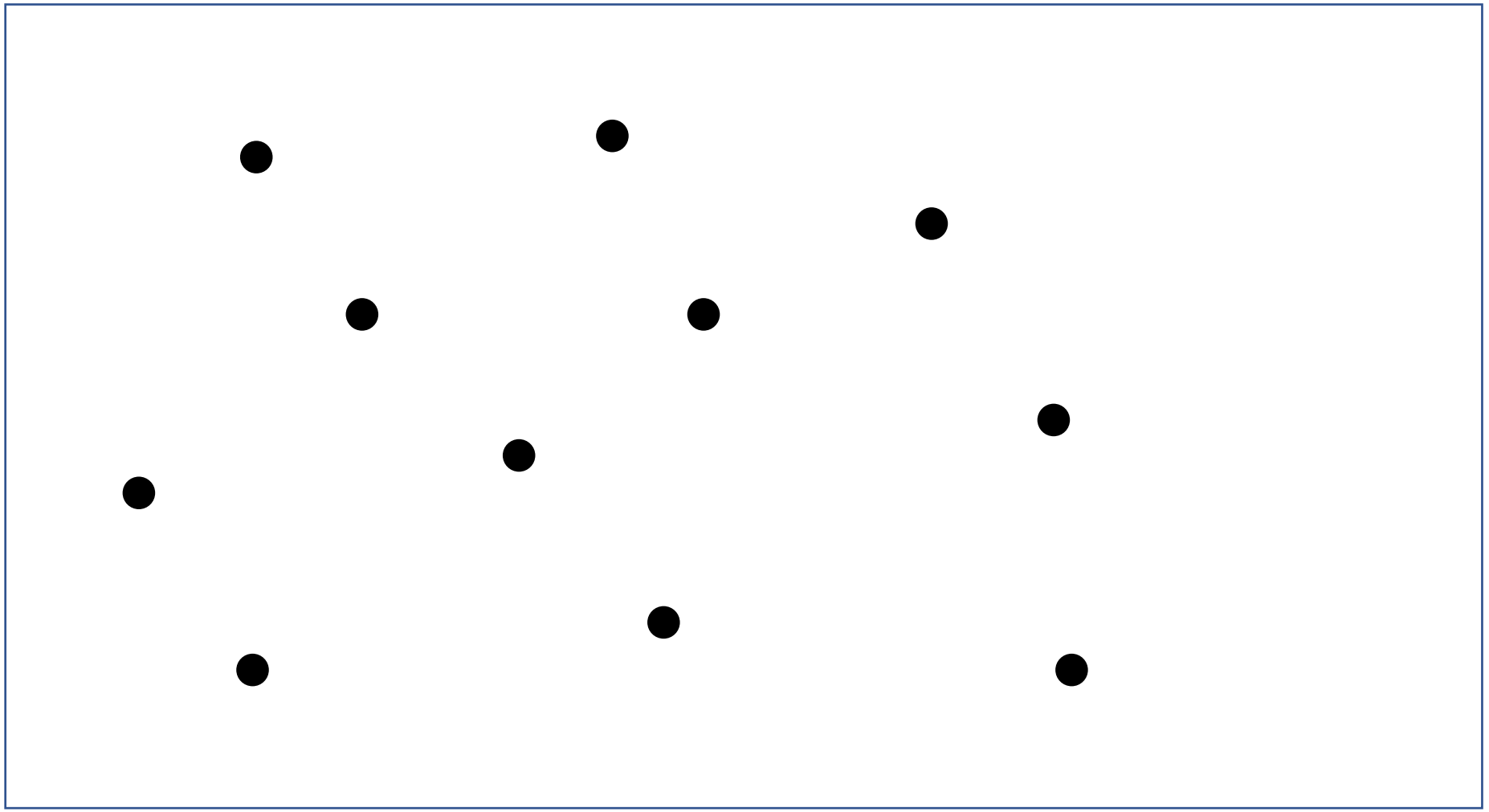

a_1
b_1
$\bar{x}_1 \vee x_2 \vee \bar{x}_3$
x_1
x_{11}^1
x_{11}^0
x_{21}^1
x_{21}^0
x_2
x_{31}^1
x_3
x_{31}^0


a_1
b_1
$\bar{x}_1 \vee x_2 \vee \bar{x}_3$
x_1
x_{11}^1
x_{11}^0
x_{21}^1
x_{21}^0
x_2
x_{31}^1
x_3
x_{31}^0

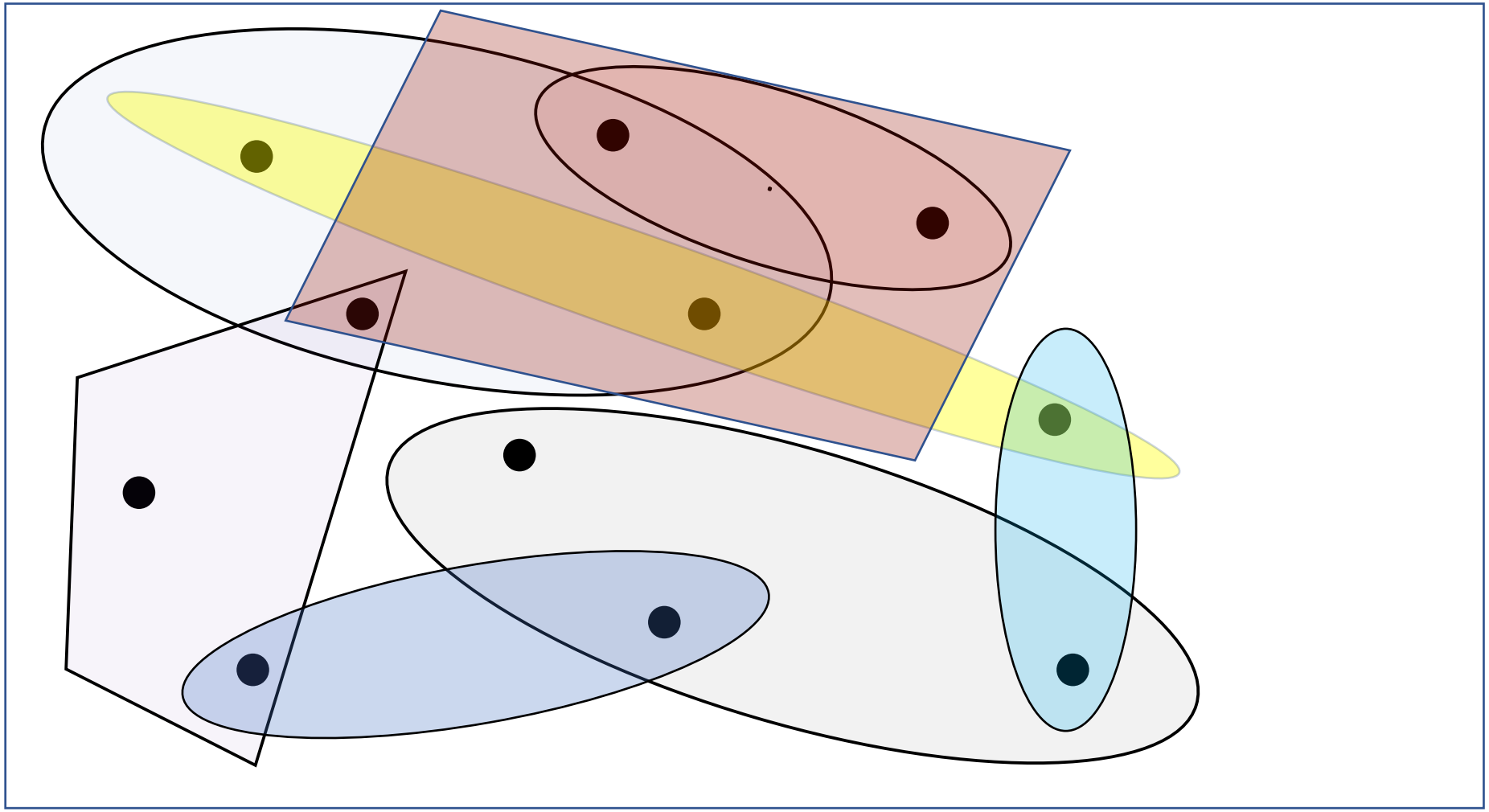

a_1
b_1
$\bar{x}_1 \vee x_2 \vee \bar{x}_3$
x_1
x_{11}^1
x_{11}^0
x_{21}^1
x_{21}^0
x_2
x_{31}^1
x_{31}^0
x_3


a_1
b_1
$\bar{x}_1 \vee x_2 \vee \bar{x}_3$
x_1
x_{11}^1
x_{11}^0
x_{21}^1
x_{21}^0
x_2
x_{31}^1
x_{31}^0
x_3

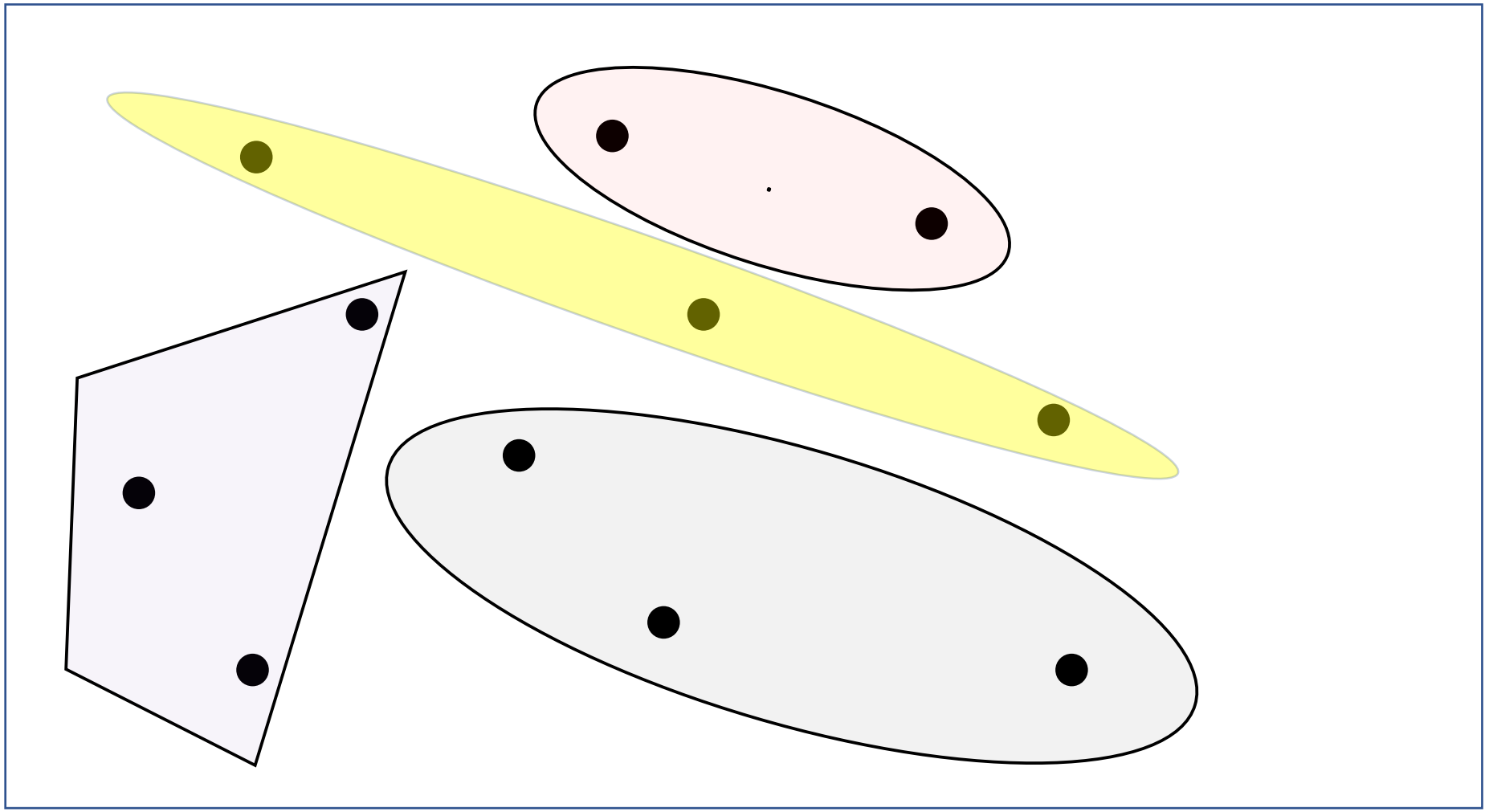
The Universe of elements U



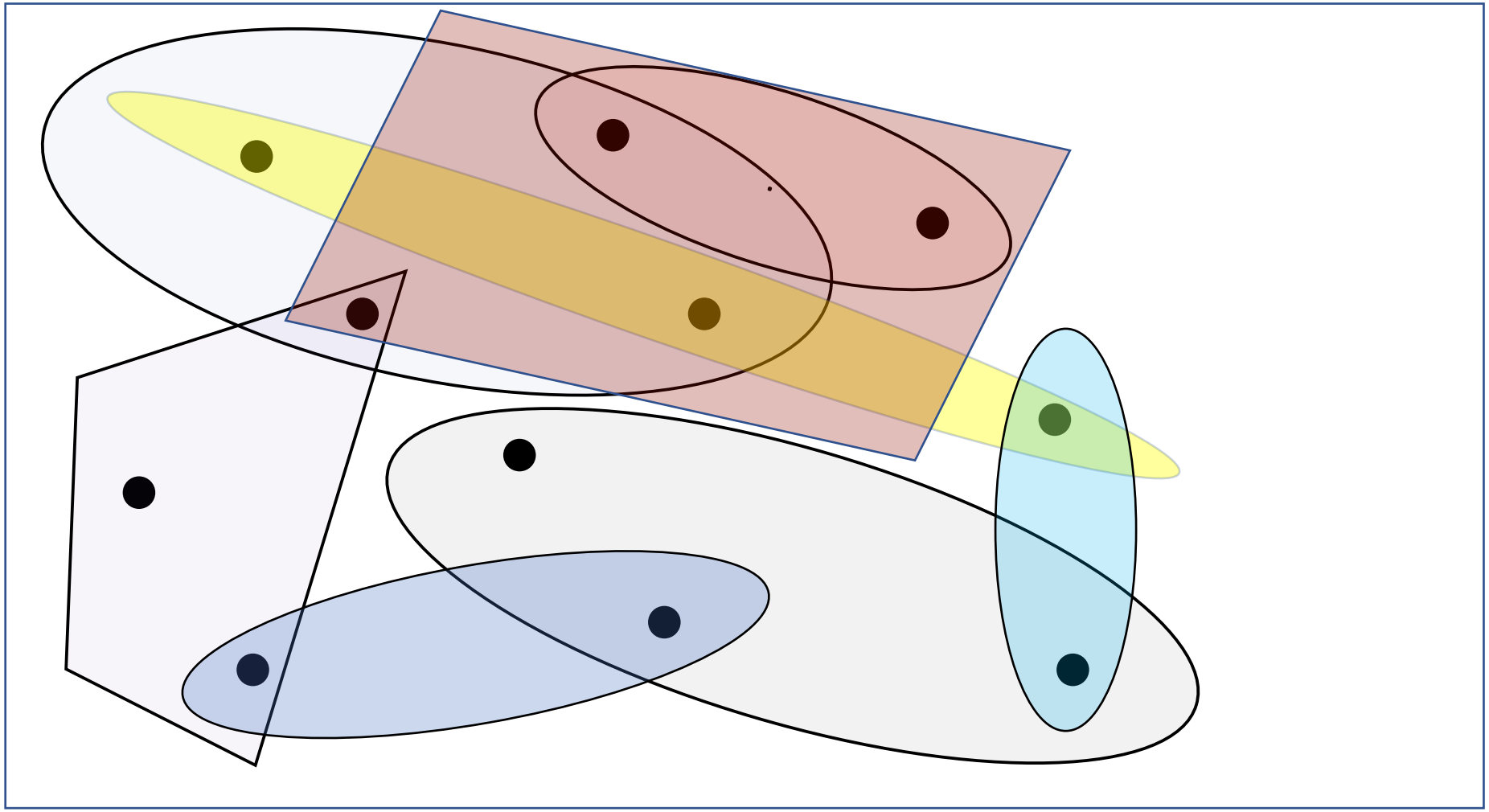
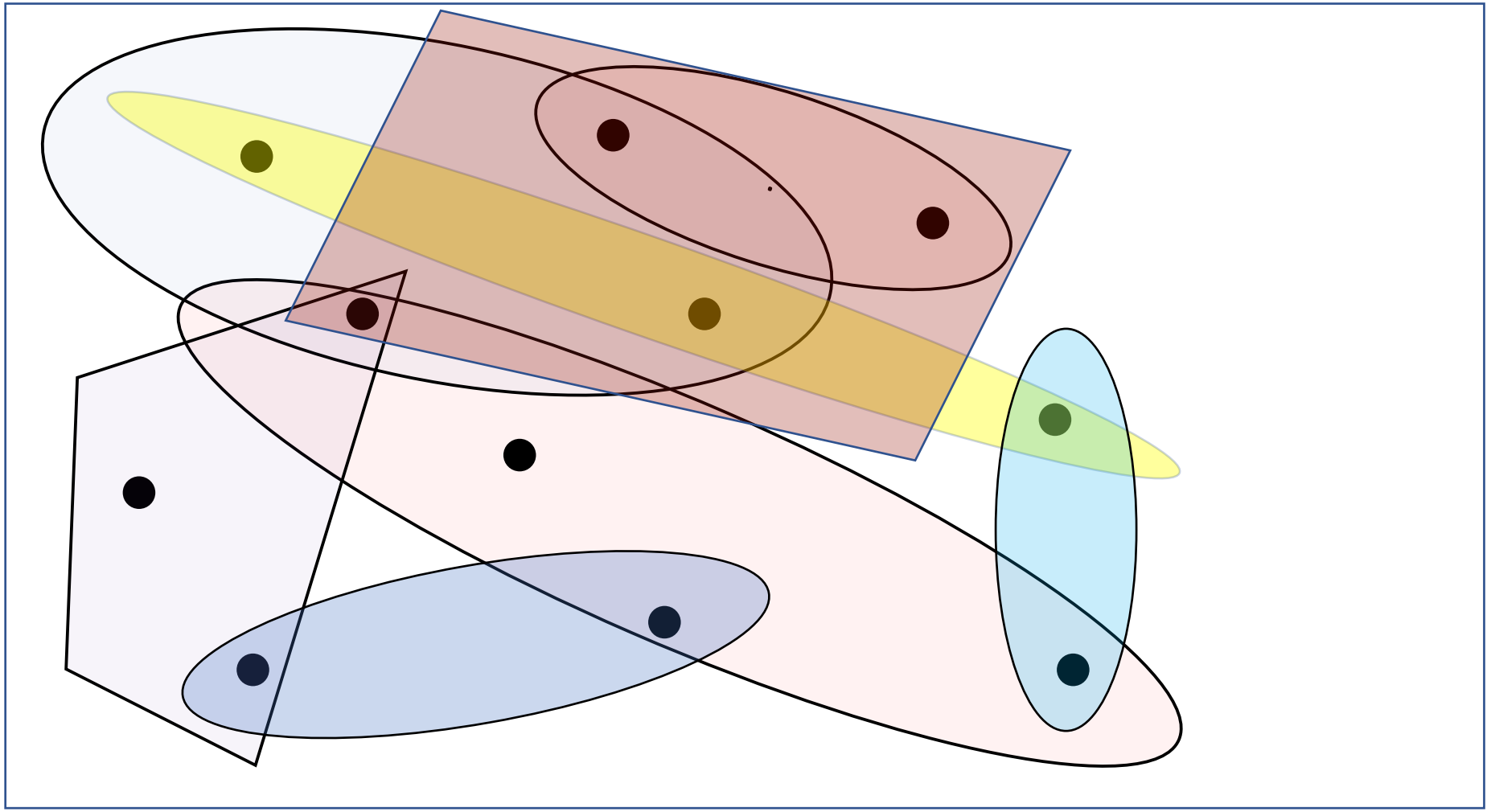
A set $\mathcal{S}$ of subsets of U



An exact cover of $\mathcal{S}$



A set $\mathcal{S}$ of subsets of U



A set $\mathcal{S}'$ of subsets of U with no exact cover