CS 2429
Propositional Proof Complexity
ASSIGNMENT # 3
Due: December 16, 2002

1. Give the shortest tree-like Resolution refutations of $\neg PHP_n^{n+1}$ that you can come up with. Analyze the complexity of your refutations.

2. Give the shortest dag-like Resolution refutations of $\neg PHP_n^{n+1}$ that you can come up with. Analyze the complexity of your refutations.

3. Using Hastad's Switching Lemma, prove the following Switching Lemma. Let $X = \{x_1, \ldots, x_n\}$. Let f be an r-disjunction over the variables X. Let $\mathcal{Q}_n^l$ be the set of all restrictions that leave exactly l unset variables, and of the set variables, exactly half are set to zero, and exactly half are set to 1. Prove for $s \geq 0$, $l = pn$, $p \leq 1/7$,
$$\frac{|\{\rho \in \mathcal{Q}_n^l \ : \ |T(f\lceil_\rho)| \geq s\}|}{|Q_n^l|} \leq (7pr)^s \cdot n$$

4. Suppose that we wanted to prove a Switching Lemma over $\mathcal{S}_n^l$, which are the set of all restrictions that leave exactly l unset variables, and of the set variables, either they are all set to 0, or all set to 1. Is such a Switching Lemma possible? If so, give a proof, and if not, provide a counterexample.