CS 2429
Propositional Proof Complexity
ASSIGNMENT # 2
Due: November 21, 2002

1. Prove the following theorem. Let $\mathcal{P}$ be a tree-like Resolution proof of $A(\overline{p},\overline{q}) \wedge B(\overline{p},\overline{r})$ and let the variables in $\overline{p}$ occur only positively in A and only negatively in B. Then there is a monotone boolean formula that computes an interpolant for $A(\overline{p},\overline{q}) \wedge B(\overline{p},\overline{q})$ of size polynomial in the size of $\mathcal{P}$.

2. Let $\mathcal{P}$ be a Cutting Planes refutation of some CNF formula $f(x_1,\ldots,x_n)$ with $poly(n)$ lines. Show that there exists a polynomial, $p(n)$ such that from $\mathcal{P}$, we can obtain another Cutting Planes refutation, $\mathcal{Q}$, such that all coefficients in Q have absolute value of size at most $2^{O(p(n))}$.

3. Let $\mathcal{P}$ be a Cutting Planes refutation of some CNF formula $f(x_1,\ldots,x_n)$. Show that from $\mathcal{P}$ we can obtain another Cutting Planes refutation, $\mathcal{Q}$ such that the size of $\mathcal{Q}$ is polynomial in the size of $\mathcal{P}$, and such that the multiplication rule is never used in $\mathcal{Q}$. (Hint: Use the previous question.)

4. Let CP^* be Cutting Planes but where all coefficients have absolute value of size at most $poly(n)$. Show that CP^* is complete for unsatisfiable CNF formulas. That is, let $f_1, f_2, \cdots$ be an an unsatisfiable family of CNF formulas, where f_i has i underlying variables. Show that for all i, $L(f_i)$, the inequalities associated with f_i, have Cutting Planes refutations where the coefficients have absolute value at most $poly(i)$.

5. (*This one is challenging and is for extra credit.*) Let CP^* denote the Cutting Planes system where all coefficients have absolute value of size at most $poly(n)$. Does CP^* polynomially-simulate CP for CNF formulas? If your answer is yes, then show how to take any CP refutation $\mathcal{P}$ and from it, obtain a CP^* refutation $\mathcal{Q}$ such that the size of $\mathcal{Q}$ is polynomial in the size of $\mathcal{P}$. If your answer is no, then give an example of a family of CNF formulas that have short CP refutations, but that require superpolynomial-size CP^* refutations. I will accept partial solutions. Try to come up with something. Even a result conditional on a plausible complexity assumption would be interesting.