CS 2429

Communication Complexity, Information Complexity and Privacy

ASSIGNMENT # 1

Due: October 31, 2012

1. Let f be a boolean function on $X \times Y$. prove that if all of the rows of M_f are distinct, then $D(f) \geq \log\log|X|$. Prove that $D(f) \leq rank(f) + 1$.

2. $MED(x, y)$ is defined to be the median of the multiset $x \cup y$. (Here we are viewing x and y as n-bit binary strings each representing subsets of $[n]$.) Using binary search, one can show that $D(MED) = O(\log^2 n)$. Give an $O(\log n)$-bit protocol for MED.

3. $GT(x, y)$, the greater-than function, is 1 if and only if $x > y$ (viewing x and y as numbers expressed in binary, each as n-bit numbers). What is the communication comlexity of the greater-than function?

   (a) Prove a lower bound of n on the deterministic communication complexity of GT.

   (b) Given an upper bound of $O(\log^2 n)$ on the randomized complexity.

4. For $x, y \in \{0, 1\}^n$, let $d(x, y)$ denote the Hamming distance between x and y, that is, the number of indices i such that $x_i \neq y_i$. Let R be a relation consisting of all triples (x, y, m) such that $|m - d(x, y)| \leq n/3$. That is, computing R is the problem of approximating the Hamming distance between x and y. Prove that $D(R) = \Omega(n)$. (Observe that computing the Hamming distance exactly is as hard as computing the equality function.)