Turing Machine for $\{w\#w^R : w \in \{0,1\}^*\}$

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A machine to recognise this language is the following machine M, with input alphabet $\sigma = \{0,1\}$, tape alphabet $\Gamma = \{0,1,\#,b\}$ (b is the blank syjmbol), accept state q_y, reject state q_n, start state q_0 and δ described by the table below. Here is an informal description of how M operates: In passes it remembers and erases the leftmost symbol, scans the string string to the extreme right end and erases the rightmost symbol, ensuring it is the same as the stored symbol(i.e., rejecting if not). During the scan it also verifies that the divider '#' is present. Then, the head returns to the left end to start again. When the LHS is empty, the machine checks that the RHS is also empty, and accepts.

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State | Symbol  ||  New State | New Symbol | Direction | Comment
================++================================================
q_0   | b       ||  q_n       |            |           | Look for leftmost symbol or #
q_0   | #       ||  q_1       | #          | R         |  and remember it
q_0   | 0       ||  q_2       | b          | R         |  Should not see b
q_0   | 1       ||  q_3       | b          | R         |
-----------------------------------------------------------------
q_1   | b       ||  q_y       |            |           | Saw # in q_0, so RHS
q_1   | 0,1,#   ||  q_n       |            |           |  should be empty
-----------------------------------------------------------------
q_2   | b       ||  q_n       |            |           | Remembering 0,
q_2   | #       ||  q_4       | #          | R         |  and look for #
q_2   | 0,1     ||  q_2       | 0,1        | R         |
-----------------------------------------------------------------
q_3   | b       ||  q_n       |            |           | Remembering 1,
q_3   | #       ||  q_5       | #          | R         |  and look for #
q_3   | 0,1     ||  q_3       | 0,1        | R         |
-----------------------------------------------------------------
q_4   | #       ||  q_n       |            |           | Remembering 0,
q_4   | b       ||  q_6       | b          | L         |  look for end
q_4   | 0,1     ||  q_4       | 0,1        | R         |
-----------------------------------------------------------------
q_5   | #       ||  q_n       |            |           | Remembering 1,
q_5   | b       ||  q_7       | b          | L         |  look for end
q_5   | 0,1     ||  q_5       | 0,1        | R         |
-----------------------------------------------------------------
q_6   | #,b,1   ||  q_n       |            |           | Check end char is 0
q_6   | 0       ||  q_8       | b          | L         |
-----------------------------------------------------------------
q_7   | #,b,0   ||  q_n       |            |           | Check end char is 1
q_7   | 1       ||  q_8       | b          | L         |
-----------------------------------------------------------------
q_8   | 0,1,#   ||  q_8       | 0,1,#      | L         | Scan to left end
q_8   | b       ||  q_0       | b          | R         |  and start again
-----------------------------------------------------------------
```