The Cartesian product (or cross product) of A and B, denoted by $A \times B$, is the set

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$$

1. the elements (a, b) of $A \times B$ are ordered pairs 2. for pairs (a, b), (c, d) we have

$$(a,b)=(c,d)\iff a=c$$
 and $b=d$

Definition 2

The *n*-fold product of sets A_1, A_2, \ldots, A_n is the set of *n*-tuples

 $A_1 \times A_2 \times \ldots \times A_n = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A_i \text{ for all } 1 \le i \le n\}$

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$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$



$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$$



$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$$



October 30, 2007 2 / 12

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 $B = \{4, 5\}$

a) $B^2 = B \times B = \{(4, 4), (4, 5), (5, 4), (5, 5)\}$ b) $B^3 = B \times B \times B = \{(4, 4, 4), (4, 4, 5), (4, 5, 4), (4, 5, 5), (5, 4, 4), (5, 4, 5), (5, 5, 4), (5, 5, 5)\}$



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 $B = \{4,5\}$ a) $B^2 = B \times B = \{(4,4), (4,5), (5,4), (5,5)\}$ b) $B^3 = B \times B \times B = \{(4,4,4), (4,4,5), (4,5,4), (4,5,5), (5,4,4), (5,4,5), (5,5,4), (5,5,5)\}$



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A (binary) relation from A to B is a subset of $A \times B$. A (binary) relation on A is a subset of $A \times A$.

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A (binary) relation from A to B is a subset of $A \times B$. A (binary) relation on A is a subset of $A \times A$.

$$A = \{2, 3, 4\} \text{ and } B = \{4, 5\}$$

$$a) R_1 = \{(2, 4), (3, 5)\}$$

$$b) R_2 = \{(2, 4), (3, 4), (4, 4)\}$$

$$c) R_3 = \{(2, 4), (2, 5), (4, 4), (4, 5)\}$$

$$d) R_4 = \emptyset$$

$$A = 0$$

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A (binary) relation from A to B is a subset of $A \times B$. A (binary) relation on A is a subset of $A \times A$.

$$A = \{2, 3, 4\} \text{ and } B = \{4, 5\}$$
a) $R_1 = \{(2, 4), (3, 5)\}$
b) $R_2 = \{(2, 4), (3, 4), (4, 4)\}$
c) $R_3 = \{(2, 4), (2, 5), (4, 4), (4, 5)\}$
d) $R_4 = \emptyset$

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A (binary) relation from A to B is a subset of $A \times B$. A (binary) relation on A is a subset of $A \times A$.

$$A = \{2, 3, 4\} \text{ and } B = \{4, 5\}$$

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A (binary) relation from A to B is a subset of $A \times B$. A (binary) relation on A is a subset of $A \times A$.



Relation $\mathcal{R} = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid 1 \le x \le y \le 4\} =$ = $\{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)\}$



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Relation $\mathcal{R} = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid 1 \le x \le y \le 4\} =$ = $\{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)\}$



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{Justin, Joey, Kevin, Nick} × {Britney, Christina, Jessica, Kelly, Sarah}



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Who dated whom? $\{(Ju, Br), (Ju, Je), (Jo, Ke), (Jo, Sa), (Ke, Br), (Ke, Ch), (Ni, Ch), (Ni, Ke), (Ni, Sa)\}$



Who is dating whom? $\{(Ju, Je), (Ke, Br), (Ni, Ch), (Ni, Ke)\}$



Theorem 4

For any set A, we have
$$A imes \emptyset = \emptyset$$
 (and $\emptyset imes A = \emptyset$)

Proof. If $(a, b) \in A \times \emptyset$, then $a \in A$ and $b \in \emptyset$, impossible.

Theorem 5

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For any sets A, B, C

a) A \times (B \cap C) = (A \times B) \cap (A \times C)

b) A \times (B \cup C) = (A \times B) \cup (A \times C)

c) (A \cap B) \times C = (A \times C) \cap (B \times C)

d) (A \cup B) \times C = (A \times C) \cup (B \times C)
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Proof. a) $(a, b) \in A \times (B \cap C) \iff a \in A \text{ and } b \in B \cap C \iff a \in A \text{ and } b \in B \text{ and } b \in C \iff (a, b) \in A \times B \text{ and}$ $(a, b) \in A \times C \iff (a, b) \in (A \times B) \cap (A \times C)$

Theorem 4

For any set A, we have
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b) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

c)
$$(A \cap B) \times C = (A \times C) \cap (B \times C)$$

d) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

Proof. a) $(a, b) \in A \times (B \cap C) \iff a \in A \text{ and } b \in B \cap C \iff$ $a \in A \text{ and } b \in B \text{ and } b \in C \iff (a, b) \in A \times B \text{ and}$ $(a, b) \in A \times C \iff (a, b) \in (A \times B) \cap (A \times C)$

Observation 6

For any two sets A, B, the number of elements in $A \times B$ is

 $|A \times B| = |A| \cdot |B|$

Hence there are exactly $|\mathcal{P}(A \times B)| = 2^{|A \times B|} = 2^{|A| \cdot |B|}$ different relations from A to B.

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Observation 6

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Hence there are exactly $|\mathcal{P}(A \times B)| = 2^{|A \times B|} = 2^{|A| \cdot |B|}$ different relations from A to B.

5.1.7 - If $A = \{1, 2, 3, 4, 5\}$ and $B = \{w, x, y, z\}$, how many elements are there in $\mathcal{P}(A \times B)$.

5.1.3 - For $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$

a) $|A \times B| =$? Answer: 9

- b) # of relations from A to B $\frac{1}{2}$
- c) # of relations on A?
- d) # of relations from A to B that contain (1,2) and (1,5)?
- e) # of relations from A to B that contain exactly five ordered pairs ?
- f) # of relations on A that contain at least seven elements ? Answer: $\binom{9}{7} + \binom{9}{8} + \binom{9}{8} = 123$

5.1.7 - If $A = \{1, 2, 3, 4, 5\}$ and $B = \{w, x, y, z\}$, how many elements are there in $\mathcal{P}(A \times B)$. Answer: $2^{20} = 1,048,576$

5.1.7 - If $A = \{1, 2, 3, 4, 5\}$ and $B = \{w, x, y, z\}$, how many elements are there in $\mathcal{P}(A \times B)$. Answer: $2^{20} = 1,048,576$ 5.1.3 - For $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$ a) $|A \times B| = ?$

5.1.7 - If $A = \{1, 2, 3, 4, 5\}$ and $B = \{w, x, y, z\}$, how many elements are there in $\mathcal{P}(A \times B)$. Answer: $2^{20} = 1,048,576$	
5.1.3 - For $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$	
a) $ A \times B = ?$	Answer: 9
b) $\#$ of relations from A to B ?	
c) # of relations on A ?	
d) # of relations from A to B that contain (1,2) and (1,5) ?	
e) # of relations from A to B that contain exactly five ordered pairs ?	
f) # of relations on A that contain at least seven elements ? Answer: $\binom{9}{7}$	
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5.1.7 - If $A = \{1, 2, 3, 4, 5\}$ and $B = \{w, x, y \}$ elements are there in $\mathcal{P}(A \times B)$.	$\{x, z\}$, how many Answer: $2^{20} = 1,048,576$
5.1.3 - For $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$	
a) $ A \times B = ?$	Answer: 9
b) $\#$ of relations from A to B ?	
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d) # of relations from A to B that contain $(1,2)$ and $(1,5)$?	
e) # of relations from A to B that contain exactly five ordered pairs ?	
f) # of relations on A that contain at least seven elements ? Answe	

5.1.7 - If $A = \{1, 2, 3, 4, 5\}$ and $B = \{w, x, y \}$ elements are there in $\mathcal{P}(A \times B)$.	, z }, how many nswer: 2 ²⁰ = 1,048,576
5.1.3 - For $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$	
a) $ A \times B = ?$	Answer: 9
b) $\#$ of relations from A to B ?	Answer: $2^9 = 512$
c) $\#$ of relations on A ?	
d) # of relations from A to B that contain $(1,2)$ and $(1,5)$?	
e) # of relations from A to B that contain exactly five ordered pairs ?	
f) # of relations on A that contain at least seven elements ? Answer	

5.1.7 - If $A = \{1, 2, 3, 4, 5\}$ and $B = \{w, x, y, z\}$, how many elements are there in $\mathcal{P}(A \times B)$. Answer: $2^{20} = 1,048,576$ 5.1.3 - For $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$ a) $|A \times B| = ?$ Answer: 9 Answer: $2^9 = 512$ b) # of relations from A to B? c) # of relations on A?

5.1 ele	7 - If $A = \{1, 2, 3, 4, 5\}$ and $B = \{w, x, y, z\}$ ments are there in $\mathcal{P}(A \times B)$. Answ	, how many ver: $2^{20}=1,048,576$
5.1	3 - For $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$	
a)	$ A \times B = ?$	Answer: 9
b)	# of relations from A to B ?	Answer: $2^9 = 512$
c)	# of relations on A ?	Answer: $2^9 = 512$
	# of relations from A to B that contain $(1,2)$ and $(1,5)$?	
	# of relations from A to B that contain exactly five ordered pairs ?	
f)	# of relations on A that contain at least seven elements ? Answer: (
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5.1.7 - If $A = \{1, 2, 3, 4, 5\}$ and $B = \{w, x, y \}$ elements are there in $\mathcal{P}(A \times B)$.	$,z\}$, how many nswer: 2 ²⁰ = 1,048,576
5.1.3 - For $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$	
a) $ A \times B = ?$	Answer: 9
b) $\#$ of relations from A to B ?	Answer: $2^9 = 512$
c) $\#$ of relations on A?	Answer: $2^9 = 512$
d) # of relations from A to B that contain $(1,2)$ and $(1,5)$?	
e) $\#$ of relations from A to B that contain exactly five ordered pairs ?	
f) # of relations on A that contain at least seven elements ? Answer	

5.1.7 - If $A = \{1, 2, 3, 4, 5\}$ and $B = \{w, x, elements are there in \mathcal{P}(A \times B).$	y, z, how many Answer: $2^{20} = 1.048.576$
5.1.3 - For $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$	1,010,010
a) $ A \times B = ?$	Answer: 9
b) $\#$ of relations from A to B?	Answer: $2^9 = 512$
c) $\#$ of relations on A ?	Answer: $2^9 = 512$
d) # of relations from A to B that contain (1,2) and (1,5) ?	Answer: $2^7 = 128$
e) # of relations from A to B that contain exactly five ordered pairs ?	
f) # of relations on A that contain at least seven elements ? Answ	

5.1.7 - If $A=\{1,2,3,4,5\}$ and $B=\{w,x,$	y, z, how many
elements are there in $\mathcal{P}(A \times B)$.	Answer: $2^{20} = 1,048,576$
5.1.3 - For $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$	
a) $ A \times B = ?$	Answer: 9
b) $\#$ of relations from A to B ?	Answer: $2^9 = 512$
c) $\#$ of relations on A ?	Answer: $2^9 = 512$
d) # of relations from A to B	A 07 100
that contain $(1,2)$ and $(1,5)$?	Answer: $2' = 128$
e) $\#$ of relations from A to B	
that contain exactly five ordered pairs ?	
f) # of relations on A that contain at least seven elements ? Answ	

5.1.7 - If $A=\{1,2,3,4,5\}$ and $B=\{w,x,$	y, z, how many
elements are there in $\mathcal{P}(A imes B)$.	Answer: $2^{20} = 1,048,576$
5.1.3 - For $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$	
a) $ A \times B = ?$	Answer: 9
b) $\#$ of relations from A to B ?	Answer: $2^9 = 512$
c) $\#$ of relations on A ?	Answer: $2^9 = 512$
d) $\#$ of relations from A to B	
that contain $(1,2)$ and $(1,5)$?	Answer: $2^7 = 128$
e) $\#$ of relations from A to B	
that contain exactly five ordered pairs ?	Answer: $\binom{9}{5} = 126$
f) $\#$ of relations on A that	
contain at least seven elements ? Answ	

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5.1.7 - If $A = \{1, 2, 3, 4, 5\}$ and $B = \{w, x, y, elements are there in \mathcal{P}(A \times B).$	z}, how many swer: $2^{20} = 1,048,576$
5.1.3 - For $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$	
a) $ A \times B = ?$	Answer: 9
b) $\#$ of relations from A to B ?	Answer: $2^9 = 512$
c) $\#$ of relations on A ?	Answer: $2^9 = 512$
d) # of relations from A to B that contain $(1, 2)$ and $(1, 5)$?	Answer: $2^7 = 128$
e) # of relations from A to B that contain exactly five ordered pairs ?	Answer: $\binom{9}{5}=126$
f) # of relations on A that contain at least seven elements ? Answer	

5.1.7 - If $A = \{1, 2, 3, 4, 5\}$ and $B = \{w, x, y\}$ elements are there in $\mathcal{P}(A \times B)$.	y, z}, how many Answer: $2^{20} = 1,048,576$
5.1.3 - For $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$	
a) $ A \times B = ?$	Answer: 9
b) $\#$ of relations from A to B?	Answer: $2^9 = 512$
c) $\#$ of relations on A ?	Answer: $2^9 = 512$
d) # of relations from A to B that contain $(1, 2)$ and $(1, 5)$?	Answer: $2^7 = 128$
e) # of relations from A to B that contain exactly five ordered pairs ?	Answer: $\binom{9}{5} = 126$
f) # of relations on A that contain at least seven elements ? Answe	er: $\binom{9}{7} + \binom{9}{8} + \binom{9}{9} = 121$
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October 30, 2007 12 / 12