## Definition 1

The Cartesian product (or cross product) of $A$ and $B$, denoted by $A \times B$, is the set

$$
A \times B=\{(a, b) \mid a \in A \text { and } b \in B\}
$$

the elements $(a, b)$ of $A \times B$ are ordered pairs for pairs $(a, b),(c, d)$ we have


## Definition 2

The n-fold product of sets $A_{1}, A_{2}, \ldots, A_{n}$ is the set of $n$-tuples $A_{1} \times A_{2} \times \ldots \times A_{n}=\left\{\left(a_{1}, a_{2}, \ldots, a_{n}\right) \mid a_{i} \in A_{i}\right.$ for all $\left.1 \leq i \leq n\right\}$

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1. the elements $(a, b)$ of $A \times B$ are ordered pairs
2. for pairs $(a, b),(c, d)$ we have

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(a, b)=(c, d) \Longleftrightarrow a=c \text { and } b=d
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$\longrightarrow \begin{array}{llll}\mid & \mid & \mid & \\ 2 & 3 & 4 & A\end{array}$

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$$
\begin{array}{ll}
A=\{2,3,4\} & \text { a) } A \times B=\{(2,4),(2,5),(3,4),(3,5),(4,4),(4,5)\} \\
B=\{4,5\} & \text { b) } B \times A=\{(4,2),(4,3),(4,4),(5,2),(5,3),(5,4)\}
\end{array}
$$




$$
B=\{4,5\}
$$

а) $B^{2}=B \times B=\{(4,4),(4,5),(5,4),(5,5)\}$
b) $B^{3}=B \times B \times B=\{(4,4,4),(4,4,5),(4,5,4),(4,5,5)$,

$$
(5,4,4),(5,4,5),(5,5,4),(5,5,5)\}
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$\{\Delta, \square\} \times\{x, y\} \times\{\varrho, \oplus, \boldsymbol{\infty}\}$

## Tree Diagram



## Definition 3

A (binary) relation from $A$ to $B$ is a subset of $A \times B$.
A (binary) relation on $A$ is a subset of $A \times A$.


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$A=\{2,3,4\}$ and $B=\{4,5\}$
a) $R_{1}=\{(2,4),(3,5)\}$
b) $R_{2}=\{(2,4),(3,4),(4,4)\}$
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d) $R_{4}=\emptyset$


Relation $\mathcal{R}=\{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid 1 \leq x \leq y \leq 4\}=$
$=\{(1,1),(1,2),(1,3),(1,4),(2,2),(2,3),(2,4),(3,3),(3,4),(4,4)\}$

## Notation

Relation $\mathcal{R}=\{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid 1 \leq x \leq y \leq 4\}=$
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## Notation

$$
\begin{gathered}
(x, y) \in \mathcal{R} \\
\hat{\mathbb{1}} \\
x \mathcal{R} y
\end{gathered}
$$

(think of $\mathcal{R}$ as $\leq$ )
\{Justin, Joey, Kevin, Nick \} $\times$ \{Britney, Christina, Jessica, Kelly, Sarah\}


Who dated whom? \{(Ju, Br), (Ju, Je), (Jo, Ke), (Jo, Sa), (Ke, Br), (Ke, Ch), (Ni, Ch), (Ni, Ke), (Ni, Sa) \}


Who is dating whom? $\{(J u, J e),(K e, B r),(N i, C h),(N i, K e)\}$
Justin_Sritney

## Theorem 4

For any set $A$, we have $A \times \emptyset=\emptyset \quad($ and $\emptyset \times A=\emptyset)$
Proof. If $(a, b) \in A \times \emptyset$, then $a \in A$ and $b \in \emptyset$, impossible.

## Theorem 5

For any sets $A, B, C$

Proof. a) $(a, b) \in A \times(B \cap C) \Longleftrightarrow a \in A$ and $b \in B \cap C \Longleftrightarrow$
$\square$ $(a, b) \in A \times C \Longleftrightarrow(a, b) \in(A \times B) \cap(A \times C)$

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## Theorem 5

For any sets $A, B, C$
a) $A \times(B \cap C)=(A \times B) \cap(A \times C)$
b) $A \times(B \cup C)=(A \times B) \cup(A \times C)$
c) $(A \cap B) \times C=(A \times C) \cap(B \times C)$
d) $(A \cup B) \times C=(A \times C) \cup(B \times C)$

Proof. a) $(a, b) \in A \times(B \cap C) \Longleftrightarrow a \in A$ and $b \in B \cap C \Longleftrightarrow$ $a \in A$ and $b \in B$ and $b \in C \Longleftrightarrow(a, b) \in A \times B$ and $(a, b) \in A \times C \Longleftrightarrow(a, b) \in(A \times B) \cap(A \times C)$

## Observation 6

For any two sets $A, B$, the number of elements in $A \times B$ is

$$
|A \times B|=|A| \cdot|B|
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Hence there are exactly $|\mathcal{P}(A \times B)|=2^{|A \times B|}=2^{|A| \cdot|B|}$ different relations from $A$ to $B$.

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## Exercises:

5.1.7 - If $A=\{1,2,3,4,5\}$ and $B=\{w, x, y, z\}$, how many elements are there in $\mathcal{P}(A \times B)$.
5.1.3 - For $A=\{1,2,3\}$ and $B=\{2,4,5\}$
a) $|A \times B|=$ ?
b) \# of relations from $A$ to $B$ ?
c) \# of relations on $A$ ?
d) \# of relations from $A$ to $B$
that contain $(1,2)$ and $(1,5)$ ?
e) \# of relations from $A$ to $B$
that contain exactly five ordered pairs ?
f) \# of relations on $A$ that
contain at least seven elements ?

## Exercises:

5.1.7 - If $A=\{1,2,3,4,5\}$ and $B=\{w, x, y, z\}$, how many elements are there in $\mathcal{P}(A \times B)$.

Answer: $2^{20}=1,048,576$
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5.1.3 - For $A=\{1,2,3\}$ and $B=\{2,4,5\}$
a) $|A \times B|=$ ?

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b) \# of relations from $A$ to $B$ ?

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Answer: 9
Answer: $2^{9}=512$

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Answer: $2^{7}=128$
e) \# of relations from $A$ to $B$ that contain exactly five ordered pairs ?

Answer: $\binom{9}{5}=126$

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that contain exactly five ordered pairs ? Answer: $\binom{9}{5}=126$
f) \# of relations on $A$ that
contain at least seven elements ? Answer: $\binom{9}{7}+\binom{9}{8}+\binom{9}{9}=121$

