STA 4503, Spring 2013 — Theory Assignment #3

Due at the start of class on March 22. Please hand it in on 8 1/2 by 11 inch paper, stapled in the upper left, with no other packaging.

This assignment is to be done by each student individually. You may discuss it in general terms with other students, but the work you hand in should be your own. In particular, you should not leave any discussion with someone else with any written notes (either on paper or in electronic form).

This assignment is concerned with the correct method of accepting or rejecting a Metropolis-Hastings proposal when the proposal is obtained by randomly choosing from a finite set of deterministic transformations of the current state. You will find the correct method for a simple situation, by looking at the limit of stochastic proposals as the proposal distribution becomes more and more concentrated at two points.

For this assignment, the distribution of interest is for a positive real variable, x, with probability density function $\pi(x)$ (which is zero for $x \leq 0$). We consider sampling from this distribution using a Metropolis-Hastings algorithm in which the proposal distribution, $g(x^*|x)$, is an equal mixture of two uniform distributions, as follows:

$$g(\,\cdot\,|x) \;=\; \frac{1}{2}\Big(\,U((a-\epsilon)\,x,\;(a+\epsilon)\,x) \;+\; U(x/(a+\epsilon)),\;x/(a-\epsilon))\,\Big)$$

Here, a is a constant greater than one, and ϵ is positive constant less than $a-1$.

a) Derive an expression for the Metropolis-Hastings acceptance probability for a proposal, x^*, drawn according to the above proposal distribution. This expression should be suitably simplified, and appropriate for direct implementation in a computer program.

b) Consider the limit of this Metropolis-Hastings transition as ϵ goes to zero. Describe in as simple a form as possible the manner in which a proposal is generated in this limit, as would be suitable for implementation in a computer program. Find the limiting form of the acceptance probability as ϵ goes to zero, simplifying it as much as possible.

(Note that in the limit as ϵ goes to zero, the above Metropolis-Hastings update is not ergodic. You can suppose that it is used in combination with other updates, which together produce an ergodic chain.)