Finding a Set of Basis Functions

These linear constraints on β define a subspace of the space of functions. There should be a set of basis functions that span this subspace. Here's one possible set:

$$h_1(X) = 1, \quad h_2(X) = X, \quad h_3(X) = X^2,$$

$$h_4(X) = (X - \xi_1)_+, \quad h_5(X) = (X - \xi_1)_+^2$$

$$h_6(X) = (X - \xi_2)_+, \quad h_7(X) = (X - \xi_2)_+^2$$

Here, $(X - a)_+$ means $\max(X - a, 0)$, and $(X - a)_+^2$ means $[(X - a)_+]^2$. The numbering of h_m above is unrelated to the numbering of earlier basis functions.

We can see that this is a correct set of basis functions by noting that

- The basis functions are all piecewise polynomials (with the boundaries of the pieces given by ξ_1 and ξ_2.
- The basis functions are all continuous.
- There are 7 basis functions, and they are linearly independent.

So linear combinations of these basis functions are continuous piecewise polynomials, as we want. We get all we want because the dimensionality is right.