Family name: *Given name:* *Student ID:*

STA 247 — Quiz #2, 2001-10-21, 3:10pm – 35 minutes long

No books, no notes, and no calculators may be used.

All numerical answers must be actual numbers (decimals such as 0.15 or simple fractions such as 3/13), not just a formula. If this requires arithmetic on numbers bigger than 1000, you've either made a mistake, or you should think of an easier way to solve the problem.

Q1 (40 marks): An urn contains three balls, which are labelled with the numbers 1, 2, and 3. You drawn two balls from this urn, *not* replacing the first before drawing the second. Let X be the number on the first ball you draw, and let Y be the number on the second ball you draw. Also, define random variables U and V as $U = X + Y$ and $V = X - Y$.

a) Write down a table of the joint probability mass function for X and Y — that is, a table giving $P(X = x, Y = y)$ for all values of x and y in $\{1, 2, 3\}$. No explanations are required.

b) Write down a table of the joint probability mass function for U and V. No explanations are required.

c) Find the marginal distributions for U and V (that is, find $P(U = u)$ for all possible u and $P(V = v)$ for all possible V). Show your work.

d) Find $E(V^2)$. Show your work.

e) Are U and V independent? Show why or why not.

Q2 (35 marks): You roll a six-sided die. Let X be the number showing on this die (from 1 to 6). You then flip a coin once if $X = 1$, twice if $X = 2$, or three times if $X \geq 3$. Let Y be the number of times the coin lands heads in these flips.

a) Write down a table of the joint probability mass function for X and Y — that is, a table giving $P(X = x, Y = y)$ for all possible values of x and y. Show your work.

b) Write down the probability mass function for the conditional distribution of X given $Y = 1$. Show your work.

Q3 (25 marks): Let X and Y be random variables, both with range $\{1, 2, 3\}$. Prove that if X and Y are independent, then the event $X > 1$ is independent of the event $Y = 2$. Use only definitions, properties of numbers and sets, and the basic axioms of probability in your proof.