STA 247 — Practice problem set #4 (non-credit, not for handing in)

Question 1: Suppose you buy a disk drive from a manufacturer who makes them in two factories. The two factories produce equal numbers of disk drives, and one cannot tell which factory a drive was made in from looking at it. However, drives from factory A are more reliable than drives from factory B — the time (in years) from purchase to failure for a drive from factory A has the exp(1/3) distribution (with mean 3), whereas the time from purchase to failure for a drive from factory B has the exp(1/2) distribution (with mean 2).

a) Suppose your drive fails after 4 years. What is the probability that it was made in factory A?

b) Suppose that after 4 years of use, your drive still hasn't failed. What is the probablity that it was made in factory A?

Question 2: Bits sent through a communications channel are sometimes received with the wrong value. For some channels, such errors often occur in bursts, with several errors occurring in a row. We can model such error behaviour using a Markov chain. Let E_i be the random variable having the value 1 if an error occurred in bit i, and 0 otherwise. Suppose that these errors have the Markov property, so that

$$P(E_i = e_i \,|\, E_{i-1} = e_{i-1}) \;=\; P(E_i = e_i \,|\, E_{i-1} = e_{i-1},\, E_{i-2} = e_{i-2}, \ldots, E_0 = e_0)$$

We can then specify the error behaviour of the channel by the one-step transition probabilities for this Markov chain. Suppose that these transition probabilities are the same at all times (ie, the Markov chain is homogeneous). The one-step transition probabilities will then be determined by just two numbers, $P^{(1)}(0 \to 1) \;=\; P(E_i = 1 \,|\, E_{i-1} = 0)$ and $P^{(1)}(1 \to 1) \;=\; P(E_i = 1 \,|\, E_{i-1} = 1)$.

a) Find $P(E_{i+3} = 1 \,|\, E_i = 1)$ exactly, assuming that $P^{(1)}(0 \to 1) = 0.01$ and $P^{(1)}(1 \to 1) = 0.4$.

b) Find the steady-state probabilities for this Markov chain. In other words, find the limit of $P(E_i = 1)$ as i becomes very large. Express this probability as a function of $P^{(1)}(0 \to 1)$ and $P^{(1)}(1 \to 1)$ and also find its numerical value for the specific values $P^{(1)}(0 \to 1) = 0.01$ and $P^{(1)}(1 \to 1) = 0.4$.

Question 3: Suppose that the number of cases of Bubonic Plague in Canada in a year has the Poisson(1.2) distribution. Suppose also that 3/4 of the people in Canada who get Bubonic Plague die, and that the death of one such person is independent of the death of another. Find the distribution of the number of people who die of Bubonic Plauge in Canada in a year. State and prove a theorem that generalizes this result.