STA 247 — Practice problem set #3 (non-credit, not for handing in)

Question 1: Suppose that a technical support person takes 6 minutes on average to deal with one customer's problem, that the standard deviation of the time required to deal with one customer is 2 minutes, and that it never takes more than 15 minutes to deal with one customer. Suppose that each customer's problem is independent of the problems of other customers. Find a good approximation to the probability that the technical support person will take more than 175 minutes to deal with the problems of 25 customers.

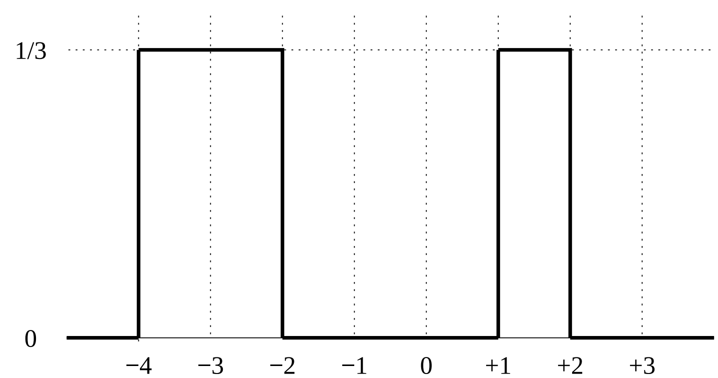
Question 2: A cosmic ray detector installed on a mountain in Alberta records the arrival time and energy of every cosmic ray that it detects, and sends the data on these events via satellite to a computer in Toronto. To reduce the number of satellite transmissions needed, the detector waits until it has data on 400 new cosmic ray events before sending the data for these events to Toronto as a single packet. The detector then forgets these events, and waits until 400 more events have occurred before sending the next packet.

Suppose that the time in seconds, T, from when one cosmic ray is detected to when the next cosmic ray is detected has exponential distribution with rate 2, so the probability density function for T is $2e^{-2t}$. Suppose also that times between detections are independent.

a) What are the mean and standard deviation of the distribution for the time between when one packet is sent and the next packet is sent?

b) Find a good approximation to the probability that the time between one packet and the next packet is greater than 215 seconds. Explain why the approximation you use should be good.

Question 3: Here is a graph of the probability density function for a random variable X:

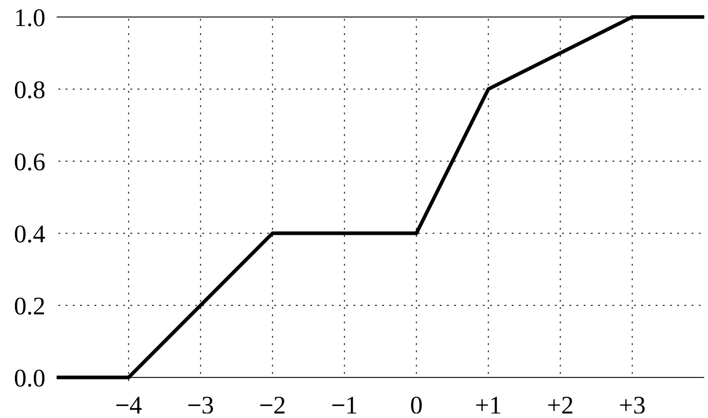


a) Draw a graph of the cumulative distribution function for this random variable.

b) Compute $P(X \leq 0)$.

c) Compute $E(X)$.

Question 4: Here is a graph of the cumulative distribution function for a random variable X:



a) Draw a graph of the probability density function for this random variable.

b) Find $P(-3 \leq X \leq 1)$.

c) Compute $E(X)$.

Question 5: Let X and Y be independent random variables. Suppose that X has the geometric distribution with parameter p_X and Y has the geometric distribution with parameter p_Y. Let Z be the minimum of X and Y. Prove that Z has a geometric distribution, and find the parameter, p_Z, of this distribution.

Question 6: Suppose that the life time of a light bulb has an exponential distribution with mean 50 hours. You wish to study for 5 hours in a room light by a lamp holding such a light bulb.

a) Suppose you put a new light bulb in the lamp when you start studying. What is the probability that the light bulb will last at least as long as you are studying (5 hours)?

b) Suppose you know that a new light bulb was put in the lamp 12 hours before you start studying, and that the lamp has been on since then. What is the probability that the light bulb will last at least as long as you are studying (5 more hours)?