STA 247 — Practice problem set #1 (non-credit, not for handing in)

Question 1: The random variable X has a range of $\{0, 1, 2\}$ and the random variable Y has a range of $\{1, 2\}$. The joint distribution of X and Y is given by the following table:

x	y	$P(X = x, Y = y)$
0	1	0.2
0	2	0.1
1	1	0.0
1	2	0.2
2	1	0.3
2	2	0.2

1. Write down tables for the marginal distributions of X and of Y, i.e. give the values of $P(X = x)$ for all x, and of $P(Y = y)$ for all y.
2. Write down a table for the conditional distribution of X given that $Y = 2$, i.e. give the values of $P(X = x \,|\, Y = 2)$ for all x.
3. Compute $E(X)$ and $E(Y)$.
4. Compute $E(XY)$.
5. Are X and Y independent? Explain why or why not.

Question 2: You roll one red die and one green die. Define the random variables X and Y as follows:

$$\begin{aligned} X &= \text{The number showing on the red die} \\ Y &= \text{The number of dice that show the number six} \end{aligned}$$

For example, if the red and green dice show the numbers 6 and 4, then $X = 6$ and $Y = 1$.

Write down a table showing the joint probability mass function for X and Y, find the marginal distribution for Y, and compute $E(Y)$.

Question 3. Suppose you roll two fair, six-sided dice, one of which is red and the other of which is green. Define the following random variables:

$$X = \text{The number shown on the red die}$$

$$Y = \begin{cases} 0 & \text{if the two dice show the same number} \\ 1 & \text{if the number on the green die is bigger than the number on the red die} \\ 2 & \text{if the number on the red die is bigger than the number on the green die} \end{cases}$$

a) Write down a table showing the joint probability mass function for X and Y.

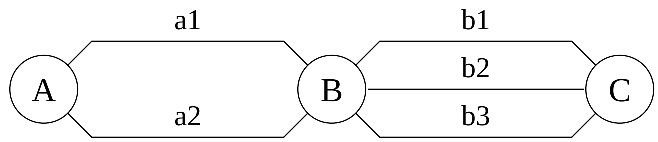
b) Find the marginal probability mass function for Y, and compute its expected value.

c) Find the conditional probability mass function for X given $Y = 1$.

Question 4. You flip a fair coin. If the coin lands heads, you roll a fair six-sided die 100 times. If the coin lands tails, you roll the die 101 times. Let X be 1 if the coin lands heads and 0 if the coin lands tails. Let Y be the total number of times that you roll a 6. Find $P(X = 1 \,|\, Y = 15)$.

Question 5. Suppose you randomly pick an integer from 1 to 3, with the three possibilities being equally likely. Call this integer N. You then randomly pick an integer from N to 3, with the $4 - N$ possibilities being equally likely. Call this second integer M. What is the probability that M will be 3?

Question 6. Three computers, A, B, and C, are linked by network connections as shown below:



Two network connections, a1 and a2, link computer A and computer B. Three network connections, b1, b2, and b3, link computer B and computer C. Since there is no direct connection from computer A to computer C, messages sent from computer A to computer C must pass through computer B.

When computer A sends a message to computer B, it randomly chooses whether to use connection a1 or connection a2, with probabilities of 2/3 for a1 and 1/3 for a2. When computer B sends a message to computer C, it randomly chooses whether to use connection b1, connection b2, or connection b3, with probabilities of 1/2 for b1, 1/4 for b2, and 1/4 for b3.

The time to send a message through each connection is 1ms for a1, 2ms for a2, 1ms for b1, 2ms for b2, and 3ms for b3. When a message is sent from computer A to computer C through computer B, it takes no time for computer B to take the message received from connection a1 or a2 and send it on connection b1, b2, or b3.

Let X be the random variable whose value is the total time (in milliseconds) taken for a message sent from computer A to computer C to arrive.

a) Find $P(X = 4)$.

b) Find $E(X)$.

c) Suppose that a message sent from computer A to computer C takes 4ms or more to arrive (ie, $X \geq 4$). How likely is it that this message was sent from computer B to computer C using connection b3?