Family name: *Given names:* *Student ID:*

1	
2	
3	
T	

CSC 363 — Test #1 — 2010-02-03

No books, notes, or other information storage systems are allowed.

1) Consider the Nondeterministic Finite Automaton (NFA) with alphabet $\Sigma = \{0, 1\}$, state space $Q = \{q_0, q_1, q_2, q_3, q_4\}$, start state q_0, set of accepting states $F = \{q_2, q_4\}$, and transition function δ defined as follows:

$$\begin{array}{lll} \delta(q_0,0)=\{q_0\} & \delta(q_0,1)=\{q_1,q_3\} & \delta(q_0,\epsilon)=\emptyset \\ \delta(q_1,0)=\emptyset & \delta(q_1,1)=\{q_2\} & \delta(q_1,\epsilon)=\emptyset \\ \delta(q_2,0)=\{q_2\} & \delta(q_2,1)=\emptyset & \delta(q_2,\epsilon)=\emptyset \\ \delta(q_3,0)=\{q_3\} & \delta(q_3,1)=\{q_4\} & \delta(q_3,\epsilon)=\emptyset \\ \delta(q_4,0)=\emptyset & \delta(q_4,1)=\emptyset & \delta(q_4,\epsilon)=\emptyset \end{array}$$

1a) [20 marks] For each of the following strings, say whether or not this NFA accepts the string, and if it does accept, give the sequence of states gone though for each of the accepting branches of the computation.

- 11
- 1010
- 0011
- 11000

1b) [10 marks] Write a regular expression that describes the language recognized by this NFA.

2) Define a *2-tape One-Way-Read-Only-Input Turing Machine* (a 2-OWROI Turing Machine) as follows. It has two tapes, with the input string being stored on tape 1 (followed by a blank), and tape 2 initialized to all blanks (infinitely far to the right). Each tape has a head that is initially at the leftmost square. There is a finite-state control unit as for regular Turing Machines. Transitions are determined by the current state and the two symbols on the squares seen by the heads on the two tapes. In each transition, the square under the head on tape 2 is replaced by a new symbol (which may be the same as the old). The head for tape 1 either stays on the same square (S), or moves to the right (R), except a move right does nothing when the head is at the blank symbol after the input. Note that the head for tape 1 *cannot* move left. The head for tape 2 either moves right (R) or left (L), with a move left when the tape is on the leftmost square doing nothing. The transition function is therefore of the following form:

$$\delta : \quad Q \times \Gamma^2 \rightarrow Q \times \Gamma \times \{ R, S \} \times \{ L, R \}$$

Note that the symbols on tape 1 are never changed from the initial string. A 2-OWROI Turing Machine has accept and reject states like a regular Turing Machine, and accepts an input string if and only if it ever enters the accept state, as for a regular Turing Machine.

2a) [20 Marks] Prove that every language that is recognizable by a regular Turing Machine is recognizable by a 2-OWROI Turing. You should be explicit about the logic of the proof. You should give implementation-level details of any Turing Machine construction you use (but needn't specify every transition in complete detail if not necessary for understanding), and explicitly state what state space and tape alphabet are used.

2b) [15 Marks] Prove that every language that is recognizable by a 2-OWROI Turing Machine is recognizable by a regular Turing Machine. For this proof, you may *not* use the fact (discussed in the text and in class) that k-tape Turing Machines have the same power as regular Turing Machines, though your proof might use ideas similar to those used in the proof of that. You should be explicit about the logic of the proof. You may describe the operation of any Turing Machine you construct at a high level (similar to that used in the proofs in Sipser's book), but you should explicitly state what tape alphabet is used.

Note: This question involves more details than other questions on the test, so you may want to leave it to the end, or write down just your basic idea (for part marks) and fill in the details if you have time.

3) Define the language $A_{TM\times 2}$ as follows:

$$A_{TM\times 2} \;=\; \{\, \langle M_1, M_2, w\rangle \mid M_1 \text{ and } M_2 \text{ are Turing Machines that both accept the string } w \,\}$$

3a) [20 marks] Prove that $A_{TM\times 2}$ is recognizable.

3b) [15 marks] Prove that $A_{TM\times 2}$ is not decidable.