CSC 310, Spring 2002 — Assignment #3

Due at 3:10pm on March 22. Worth 5% of the course grade.

Note that this assignment is to be done by each student individually. You may discuss it in general terms with other students, but the work you hand in should be your own.

These questions all concern a channel whose input and output alphabets are both $\{\,-1,\,0,\,+1\}$. Transmission through this channel behaves according to the following forward probabilities:

$$P_{-1,-1} = 1, \quad P_{-1,0} = P_{-1,+1} = 0$$
$$P_{+1,+1} = 1, \quad P_{+1,0} = P_{+1,-1} = 0$$
$$P_{0,0} = P_{0,-1} = P_{0,+1} = 1/3$$

The channel input probabilities will be denoted by p_{-1}, p_0, p_{+1}. The input of the channel is denoted by $\mathcal{A}$, and the output by $\mathcal{B}$.

For all questions, show how you derived your answers.

Question 1 (15 marks): Find the conditional distributions for the input symbol given each of the three possible output symbols, in terms of the input probabilities. In other words, in the notation of the slides and text, find the backward probabilities, Q_{ij} for $i, j \in \{\,-1,\,0,\,+1\}$.

Question 2 (15 marks): Find a simple expression for $H(\mathcal{B}\,|\,\mathcal{A})$ in terms of the input probabilities.

Question 3 (15 marks): Find an expression for $H(\mathcal{B})$ in terms of the input probabilities.

Question 4 (20 marks): Prove that the input probabilities that maximize $I(\mathcal{A},\,\mathcal{B})$ are such that $p_{-1} = p_{+1}$.

Question 5 (35 marks): Find the capacity of this channel, and the input distribution that achieves this capacity. You should give this input distribution in terms of simple exact expressions, and also give the numerical values of the input probabilities and of the capacity to four decimal places.

In answering this question, you may assume the statement in Question 3, even if you haven't been able to prove it yourself.