CSC 373H (2007): Assignment 3

Worth 5%. Due July 12 at 6pm in lecture.

The work you submit must be your own. You may discuss problems with each others; however, you should prepare written solutions alone. Copying assignments is a serious academic offence, and will be dealt with accordingly.

Question 1 [Flow Network]

Some of you friends have recently graduated and started a small company. They're in the process of porting all their software from an old system to a new, revved-up system; and they're facing the following problem.

They have a collection of n software applications, $\{1, 2, \ldots, n\}$, running on their old system; and they'd like to port some of these to the new system. If they move application i to the new system, they expect a net monetary benefit of $b_i \geq 0$. The different software applications interact with one another; if applications i and j have extensive interaction, then the company will incur an expense if they move one of i or j to the new system but not both; let's denote this expense by $x_{i,j} \geq 0$.

So, if the situation were really this simple, your friends would just port all n applications, achieving a total benefit of $\sum_i b_i$. Unfortunately, there is a problem....

Due to small but fundamental incompatibilities between the two systems, there's no way to port application 1 to the new system; it will have to remain on the old system. Nevertheless, it might still pay off to port some of the other applications, accruing the associated benefit and incurring the expense of the interaction betwwen applications on different systems.

So this is the question they pose to you: Which of the remaining applications, if any, should be moved? Give a polynomial-time algorithm using flow network to find a set $S \subseteq \{1, 2, \ldots, n\}$ for which the sum of the benefits minus the expenses of moving the applications in S to the new system is maximized. **Assume that all $b_i, x_{i,j}$ are nonnegative integers.**

Question 2 [Flow Network]

Consider the following definition. We are given a set of n countries that are engaged in trade with one another. For each country i, we have the value s_i of its budget surplus; this number may be positive or negative, with a negative number indicating a deficit. For each pair of countries i, j, we have the total value $e_{i,j}$ of all exports from i to j; this number is always nonnegative. We say that a subset S of countries is *free-standing* if the sum of the budget surpluses of the countries in S, minus the total value of all exports from countries in S to countries not in S, is nonnegative. (~~A trivial free-standing set is the set of all n countries.~~)

Give a polynomial-time algorithm that takes this data for a set of n countries and decides whether it contains a nonempty free-standing subset that is not equal to the full set. **Assume that all s_i are integers, and $e_{i,j}$ are nonnegative integers.**

Question 3 [Linear Programming]

A film producer wants to make a motion picture. For this, she needs to choose among n available actors. Actor i demands a payment of s_i dollars to participate in the picture.

The funding for the picture will come from m investors. The kth investor will pay the producer p_k dollars, but only under the following condition. The investor has a list of actors $L_k \subseteq \{1, 2, \ldots, n\}$, and he will only invest if *all* the actors on his list appear in the picture.

The profit of the producer is the sum of the payments from the investors that she agrees to take funding from, minus the sum of payments she makes to the actors that appear in the picture. The goal is to maximize the producer's profit.

Formulate this problem as a 0-1 Integer Programming problem so that the optimal value of the objective function is the maximum profit of the producer. Precisely specify the variables, the objective function, and the constraints. (You are *not* required to solve the 0-1 Integer Programming problem.)