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UNIVERSITY OF TORONTO
Faculty of Arts and Science

JULY 2013 EXAMINATIONS

CSC 373 H
Instructor: Milad Eftekhar

Duration — 2 hours

Examination Aids: *One* 8.5" × 11" sheet of paper, *hand*written on both sides.

Student Number: |_|_|_|_|_|_|_|_|_|

Last (Family) Name(s): _____

First (Given) Name(s): _____

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*Do **not** turn this page until you have received the signal to start.*
In the meantime, please read the instructions below *carefully*.

This midterm exam paper consists of 6 questions on 10 pages (including this one), printed on one side of each sheet. *When you receive the signal to start, please make sure that your copy of the midterm exam is complete, fill in the identification section above, and write your student number where indicated at the bottom of every page (except page 1).*

Answer each question directly on this paper, in the space provided, and use the reverse side of the pages for rough work. If you need more space for one of your solutions, use the reverse side of a page and *indicate clearly the part of your work that should be marked.*

In your answers, you may use without proof any result or theorem covered in lectures, tutorials, or the textbook, as long as you give a clear statement of the result(s)/theorem(s) you are using. You must justify all other facts required for your solutions.

Write up your solutions carefully! In particular, use notation and terminology correctly and explain what you are trying to do — part marks *will* be given for showing that you know the general structure of an answer, even if your solution is incomplete.

If you are unable to answer a question, you will get 20% of the marks for that question if you write "I don't know" and nothing else — you will *not* get those marks if your answer contains contradictory statements (such as "I don't know" followed or preceded by parts of a solution that have not been crossed off).

MARKING GUIDE

# 1:	_____/10
# 2:	_____/10
# 3:	_____/10
# 4:	_____/15
# 5:	_____/10
# 6:	_____/10
TOTAL:	_____/65

Good Luck!

CONT'D...

Question 1. [10 MARKS]

Let $G = (V, E)$ be a connected undirected weighted graph with weight $w(e)$ for each edge $e \in E$. A MAXIMUM spanning tree in this graph is a spanning tree T such that sum of the weights of edges in T is maximum. Prove that if the weights of all edges in G are distinct, then G has a unique MAXIMUM spanning tree.

Question 2. [10 MARKS]

Show that if all characters occur with a frequency less than $\frac{1}{3}$ (that is $\frac{1}{3}$ of the sum of the frequencies), then in the Huffman encoding scheme there is guaranteed to be no codeword of length 1.

Question 3. [10 MARKS]

Consider a flow network $G = (V, E)$ with a source node s, a sink node t and capacities $c(e)$ for each edge $e \in E$. Assume there is a cost function $p : E \to \mathbb{N}$ associated with the edges. The interpretation of the cost function p is that $p(e)$ is the (non-negative) cost to increase the capacity of edge e by one unit.

Suppose that we have already computed an optimal flow f for this network. Now suppose that you want to increase this max flow f by one unit by increasing the capacity of some edges. Our goal is to do this at the least cost. Describe a method for optimally determining which edge capacities to increase. Your method should run in time that is asymptotically better than $\mathcal{O}(mn)$.

Question 4. [15 MARKS]

You are consulting for an investment company. The members of this company are doing a simulation in which they look at n consecutive days of a given stock in the past. For each day $i = 1, 2, ..., n$ they have the price $p(i)$ per share for the stock on that day (assume the price is fixed during each day). They want to buy 100 shares on some day and sell all these shares on some (later) day. They want to know when should they have bought and when should they have sold in order to have made as much money as possible (If there is no way to make money during these n days, you should report this instead). Show how to find the correct numbers k (the day they should have bought) and l (the day they should have sold) in time $\mathcal{O}(n)$.

Question 5. [10 MARKS]

The company Elgoog! has two offices in New York and Toronto. The employees in the New York office collaborate with some teams in the Toronto office and they need to have regular face-to-face meetings. Thus, every week several employees from the New York office travel to Toronto. These employees have a choice of two air lines: Air Canada, and United Airlines:

- Air Canada charges a fixed rate r per person; thus if k people travel by Air Canada, they should pay $k \times r$ dollars.
- United Airlines makes contracts for a fixed amount c per week, independent of the number of travelers. However, contracts with United Airlines must be made in blocks of four consecutive weeks at a time.

Given a sequence of the number of employees traveling to Toronto every week, Elgoog! should decide which airline to choose for each of the n weeks. The cost of traveling for the whole n weeks is the total amount paid to both airlines. Give a polynomial-time algorithm that returns the minimum cost. You are NOT required to output the chosen airline for each week.

Example: If $r = 1$, $c = 10$, and the sequence of the number of employees traveling is

$$10, 8, 8, 11, 11, 11, 11, 8, 8, 10$$

then the minimum cost is 92 (this happens when we choose Air Canada for the first three weeks, then United Airlines for a block of four consecutive weeks, and then Air Canada for the final three weeks).

Question 6. [10 MARKS]

Consider the following variation of the maximum flow problem:
Consider a flow network $G = (V, E)$ with a source node s, a sink node t and capacities $c(e)$ for each edge $e \in E$. Moreover, assume there is a capacity $c(v)$ for each vertex $v \in V$ that determines the maximum flow that can enter node v (this is a new constraint that we did not see in the original network flow problem). Explain how we can efficiently identify the maximum flow in this variation of the problem.
(**Hint:** reduce this variation to the original maximum flow problem.)

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Total Marks = 65