## The solution to Q3 (midterm)

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Q3.

Let G denote the graph in the question and f is a max flow for G.

For any edge e of G, we just need to increase the capacity if f(e) > c(e) - 1.

Identify a max flow f. Create another graph G' similar to G (the same nodes and edges) with new edges' weights. For any edge e in G: if f(e) > c(e) - 1, set the weight of the corresponding edge in G' to p(e); otherwise, set the weight to 0. We want to find a path from s to t with the smallest weight in this graph (the shortest path from s to t) that can be done by Dijkstra algorithm. The run time of the algorithm is  $\mathcal{O}(m \log n)$ .