CSC363 - Summer 2005
Assignment 1

due on Tuesday, June 7th, at 6pm

Problem 1 [10p]
Give a regular Turing Machine deciding the language $L = \{ww \mid w \in \{0,1\}^*\}$. Begin with a high-level description of how the machine works, followed by a formal description. To specify the transition function, you may use a table or a diagram. Also include informal descriptions of the states of your machine.

Problem 2 [10p]
A 2-head Turing Machine has a single two-way infinite tape, and 2 heads which can operate on the tape independently of each other. The transition function of a 2-head TM is of the form

$$\delta(q, c_1, c_2) = (q', c_1', D_1, c_2', D_2)$$

where q, q' are states, c_1, c_2, c_1', c_2' are characters of the tape alphabet, and D_1, D_2 are directions Left or Right. This transition means that, when the TM is in state q, head number 1 is reading c_1 and head number 2 is reading c_2, the machine moves to state q', head number 1 writes c_1' and moves in the direction D_1, and head number 2 writes c_2' and moves in the direction D_2.

We adopt the following convention: if the two heads are reading the same cell prior to a transition, and they attempt to write distinct characters to the same cell during that transition, the character written by head number 1 will end up on the tape.

Show that 2-head TMs recognize the same class of languages as regular TMs. In your proof, you may give implementation-level descriptions of TMs (descriptions in English of how the machine moves its head and how it stores data on its tape).

Problem 3 [10p]
This is problem 4.18 from p.170 in Sipser. Let A and B be two disjoint co-Turing-recognizable languages (meaning that their complements are Turing-recognizable, see p.167). Show that there exists a decidable language C, such that $A \subseteq C$ and $B \subseteq \overline{C}$.

Problem 4 [15p]
Let $A_1, A_2, \ldots, A_i, \ldots$ be a list of some languages such that for every i, A_i is decidable (i.e. for every i, there exists a decider M_i such that $A_i = L(M_i)$). Let

$$A = \bigcup_{i \geq 1} A_i$$

(a) [5p] Prove that A is recognizable.

(b) [5p] Can A be recognizable but not decidable? Prove your answer.

(c) [5p] Let k be some fixed integer. Consider the language

$$L_k = \{w \in \{0,1,\ldots,9\}^* \mid |w| \leq k \text{ and } w \text{ appears in the decimal expansion of } \pi\}$$

In plain English, L_k contains those strings w of length at most k such that w appears in the decimal expansion of π. We know $\pi = 3.14159265..$, so, for example, $41 \in L_2$ and $926 \in L_4$. Is L_k recognizable? Is L_k decidable?

(d) [Bonus 2p] Do you think that the following language is decidable? Explain.

$$L = \{w \in \{0,1,\ldots,9\}^* \mid w \text{ appears in the decimal expansion of } \pi\}$$