Random Access Networks: Transmission Costs, Selfish Nodes, and Protocol Design

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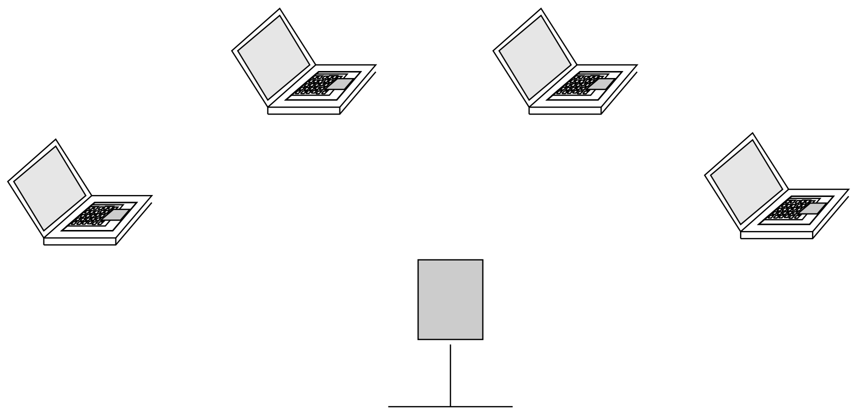
Random Access Networks with Transmission Cost



- Transmission cost

Random Access Networks with Transmission Cost

Questions

- Is transmission cost sufficient to guarantee stability?
- If not, what additional mechanisms are needed?

Answers

- Transmission cost does not guarantee stability
- Pricing mechanism: stability and system performance

Model

- Non-Cooperative Game
- Slotted Aloha (CSMA/CD)

Non-Cooperative Game - Slotted Aloha Model

- Poisson Arrivals
- Collision - Retransmission
- Probabilistic Retransmissions
- Transmission Cost γ
- Infinite set of hosts

Non-Cooperative Game

- Poisson Arrivals with (Aggregated) Rate $\lambda(u)$, $u \geq 0$
  - Packets have different values
  - $\lim_{u\to\infty} \lambda(u) = 0$
- State n: number of backlogged packets
  - whether to accept a new packet
  - retransmission probability for backlogged packet
- Strategy $\pi = (u, q)$
  - $u = (u(0), u(1), u(2), ...)$
  - $q = (q(1), q(2), ...)$
- Strategy $\pi = (\lambda, q)$
  - $\lambda(n) = \lambda\big(u(n)\big), \qquad n = 0, 1, 2, ...$

Markov Chain Formulation

- Nodes are indistinguishable (symmetric strategies)
- Strategy π: Markov chain $\{n_k; k \geq 0\}$
- Successful transmission of a backlogged packet for given node:

$$e^{-\lambda(n)}(1 - q(n))^{n-1} \approx e^{-\lambda(n)-(n-1)q(n)}$$

- Offered load: $G(n) = \lambda(n) + nq(n)$
- Instantaneous throughput

$$G(n)e^{-G(n)}$$

Markov Chain Formulation

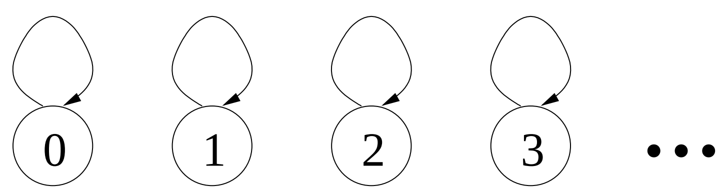
- Cost for successfully transmitting a packet
  - new packet: $R(\pi, n)$
  - backlogged packet: $Q(\pi, n)$
- Retransmission Probabilities $\hat{q}$
  - new packet: $R(\pi, n, \hat{q})$
  - backlogged packet: $Q(\pi, n, \hat{q})$

Equilibrium Strategy

- Admissible retransmission vector $\hat{q}$
  - $T_n(\pi, \hat{q})$ is a random variable, $n \geq 0$
  - set of all admissible retransmission vectors: $\mathcal{Q}(\pi)$
- Admissible strategy π
  - $\lambda(0) > 0$
  - $q \in \mathcal{Q}(\pi)$
- Equilibrium strategy
  - $q = \arg\min_{\hat{q} \in \mathcal{Q}(\pi)} Q(\pi, n, \hat{q}), \qquad n \geq 0$
  - $u(n) = R(\pi, n)$
- Symmetric Nash equilibrium

Stable Strategy

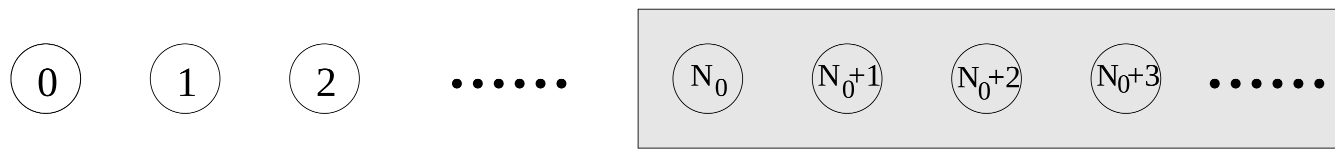
- Stable strategy
  - "Expected number of backlogged nodes stays bounded"
- Stable equilibrium strategy
  - Single positive recurrent class, and possibly some transient states



- Questions
  - Does a stable equilibrium strategy exist?
  - Does a unique stable equilibrium strategy exist?
  - What is the performance at a stable equilibrium strategy?

A particular class of Strategies

- Set $\mathcal{F}_\kappa$ of admissible strategies
  - class $\mathcal{N}_c = \{n; n \geq N_0\}$
  - $\lambda(n) + (n-1)q(n) = \kappa, \qquad n \in \mathcal{N}_c$



- Transmitting backlogged packet
  - $e^{-\lambda(n)-(n-1)q(n)} = e^{-\kappa}, \qquad n \in \mathcal{N}_c$
- Cost $Q(\pi, n)$
  - $Q(\pi, n) = \gamma e^{\kappa}, \qquad n \in \mathcal{N}_c$

Existence of a Equilibrium Strategy $\pi \in \mathcal{F}_\kappa$

Proposition 1 *There exists a stable equilibrium strategy* $\pi \in \mathcal{F}_\kappa$ *if and only if the following conditions hold*

(a) $\max_{r \geq 0}(f_\infty(r) - r) \geq 0$,

(b) $\lambda(r_\infty) < \kappa e^{-\kappa}$, *and*

(c) $\lambda(r_0) \geq \kappa$.

Idea: The transmission cost γ needs to be large enough in order to have a equilibrium strategy $\pi \in \mathcal{F}_\kappa$.

Existence of a Stable Equilibrium Strategy

Proposition 2 *If* π *is a stable equilibrium strategy then there exists a* $\kappa > 0$ *such that* $\pi \in \mathcal{F}_\kappa$*.*

Interpretation

- If transmission cost γ is too small then there does not exist a stable equilibrium allocation
- If there exists a stable equilibrium allocation, then there is typically a continuum of stable equilibria (in κ).
- Different values of κ lead to different throughput and delay.

Protocol Design

- Need additional mechanism to guarantee stability
- Would like mechanism for choosing κ
- **Idea:** cost c for successfully transmitted packets
- Can choose c and κ to
  - set throughput/delay (trade-off)
- MAC protocol: choosing κ
  - Pick κ in advance ($\kappa = 1$)
  - Choose c for throughput/delay
  - Determine $q(n)$ (probability of successful retransmission is the same at all states)
  - No node has incentive to deviate (no cheating)
  - MAC standard

Conclusions

- Random Access Networks with Transmission Costs
- Insight into Protocol Design
- Price-Based Rate Control
  - C. Yuen and P. Marbach, "Price-Based Rate Control in Random Access Networks," to appear in IEEE/ACM Transactions on Networking

Related Work

- Selfish Users - Retransmission Probabilities
  - MacKenzie and Wicker
- Selfish Users - Experimental
  - Altman, El Azouzi, Jimenez
- Cheating in WiFi Hotspots
  - Raja, Hubaux, Aad