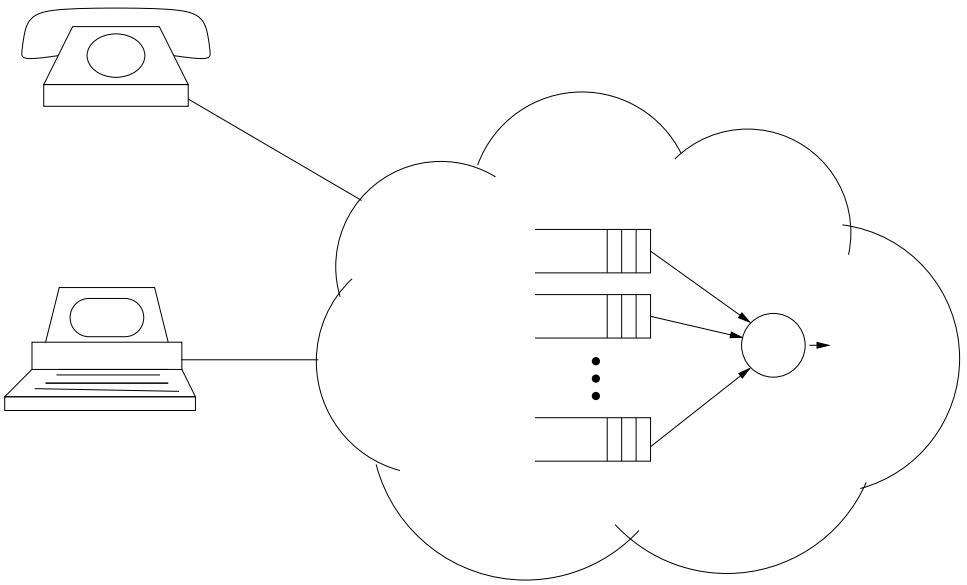
Differentiated Services and Max-Min Fairness

Peter Marbach
Department of Computer Science
University of Toronto

Overview

- Problem Formulation
- Problem Motivation
- Results
- Related Work
- Mathematical Model

Problem Formulation



- Priority Levels: $1, ..., N$
- Price: u_i per Submitted Packet

- **Questions**
  - Stable Operation?
  - Pricing - Network Performance?
  - Bandwidth Allocation?

- **Approach**
  - Non-Cooperative Game

    "Utility - Cost"
  - Equilibrium?
  - Pricing?
  - Bandwidth Allocation?

Problem Motivation: DiffServ

- Services Classes with Different QoS
  - Example: Premium Service and Best Effort Service
- Pricing
- Network Control vs. Economic Mechanism
- Dumb Networks and Intelligent Hosts

Results

- Single Link Case
  - Equilibrium Exists
  - Pricing a Tool to Control Congestion
    - Smooth Traffic:
      Absolute QoS: $P_{tr}(i) \geq q_i$
    - Bursty Traffic:
      Relative QoS: $\frac{P_{tr}(i)}{P_{tr}(i+1)} \geq q_i$

- Network Case
  - Equilibrium Exists
  - Max-Min Fair Bandwidth Allocation

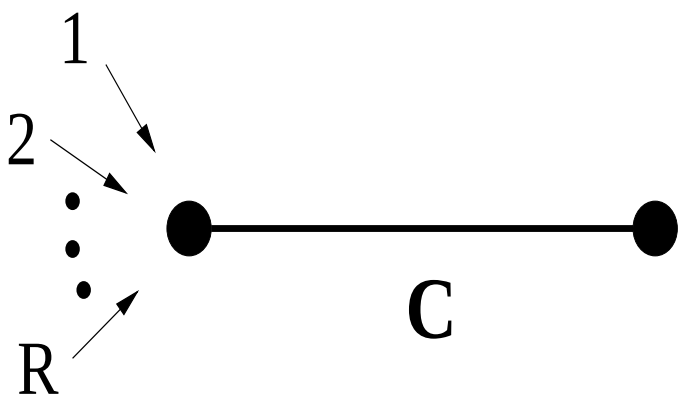
Related Work

- A. Odlyzko:
  Paris Metro Pricing

- R. Cocchi/S. Shenker/D. Estrin/L. Zhang:
  Experimental Results

- HP Lab:
  Internet Service Provider (Index Project)

Mathematical Model

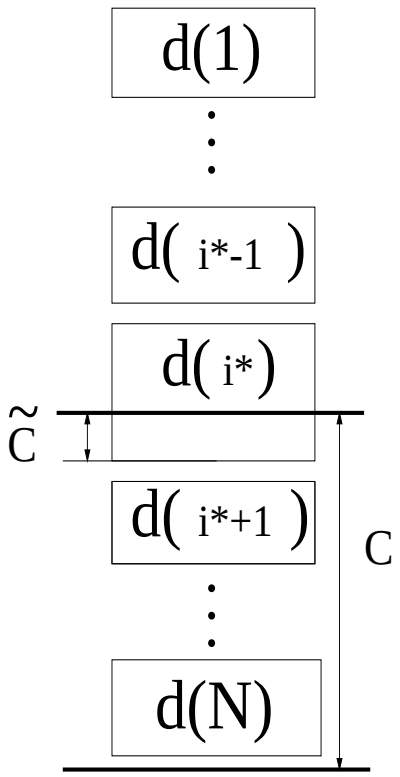


- Slotted Time, Capacity C, No Buffer

$$d_r = (d_r(1), ..., d_r(N))$$

$$d = \sum_{r=1}^{R} d_r$$

- $d = (d(1), d(2), \ldots, d(N))$



$$P_{tr}(i,d) = 0, \qquad i < i^*$$

$$P_{tr}(i^*,d) = \frac{\tilde{C}}{d(i^*)}, \qquad i = i^*$$

$$P_{tr}(i,d) = 1, \qquad i > i^*$$

Elastic Traffic

- Users care only about Throughput

- User's Problem:

$$\max_{d_r \in \Re_+^N} \left\{ U_r(x_r) - \sum_i d_r(i) u_i \right\},$$

$$x_r = \sum_{i=1}^{N} d_r(i) P_{tr}(i,d)$$

- Demand $D_r(u)$:

$$D_r(u) = \arg\max_{x \geq 0} \{U_r(x) - xu\}, \qquad u \geq 0$$

- Competitive Price-Taking Assumption
  - Users do not know how they affect $P_{tr}(i)$
  - Many Small users Share the Link

- Equilibrium $(d_1^*, ..., d_R^*)$
  - For every user $r = 1, ..., R$, we have

$$d_r^* = \arg \max_{d_r \in \Re_+^N} \left\{ U_r(x_r) - \sum_{i=1}^{N} d_r(i) u_i \right\},$$

where

$$x_r = \sum_{r=1}^{R} d_r(i) P_{tr}(i, d^*).$$

Results: Single Link Case

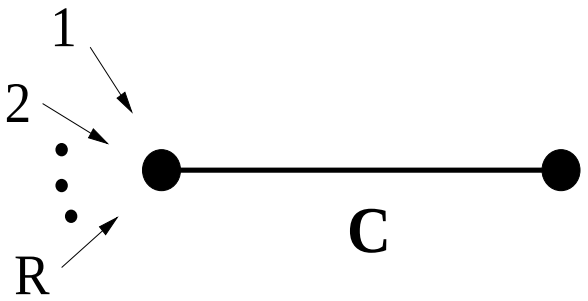
- At most two priorities i^* and i^*+1 are used
- $P_{tr}(i^*, d) \geq \frac{u_{i^*}}{u_{i^*+1}}$
- $x_r = D_r(u^*)$

$$u^* = \frac{u_{i^*}}{P_{tr}(i, d^*)}$$

- $\sum_{r=1}^{R} x_r = C$

Max-Min Fairness

- Single Link:

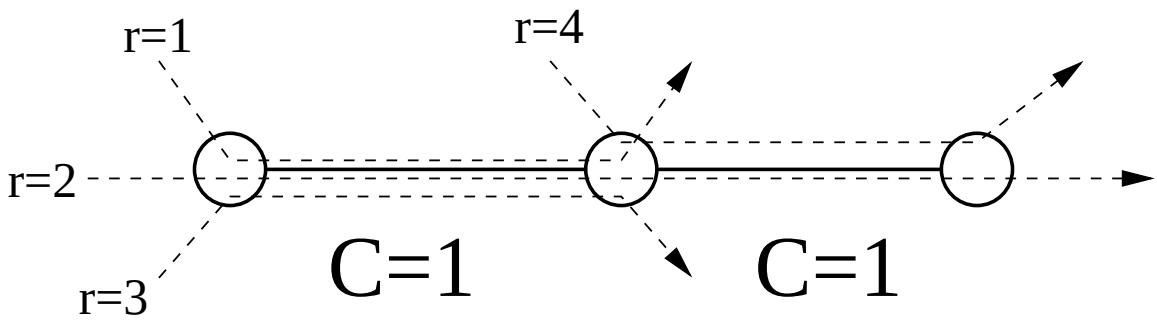


$$x^* = x_r = C/R, \quad \sum_r x^* = R$$

- Priority Service

$$x_r = D_r(u^*), \quad \sum_r x_r = C$$

- Network:



$$x_1 = x_2 = x_3 = \frac{1}{3}, \quad x_4 = \frac{2}{3}$$

"Idealized" Priority Service

- Priorities: $\mathcal{I} = [u_{\min}, \infty)$
- Price: u per Submitted Packet with Priority u
- Non-Cooperative Game

"Utility - Cost"

Results

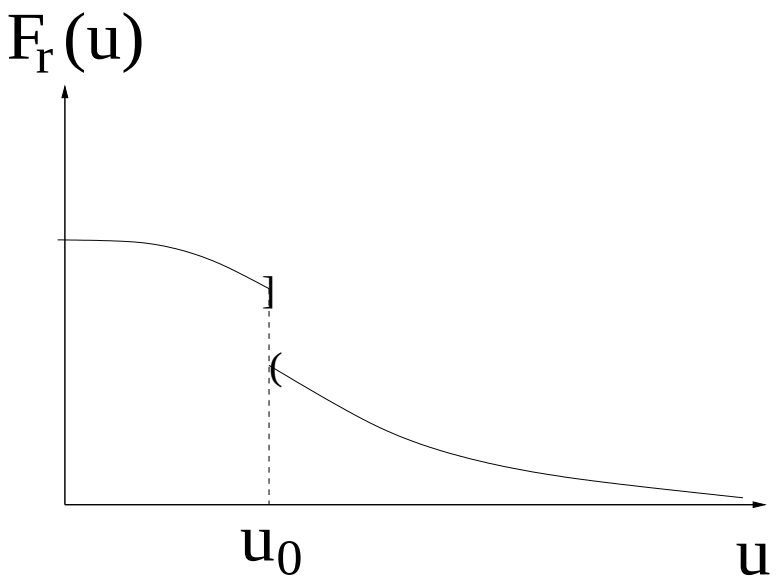
- Equilibrium Exists
- User r sends all packets in priority u_r^*
- $x_r = D_r(u_r^*)$
- Max-Min Fair

Proof:

- "Water Filling" Principle

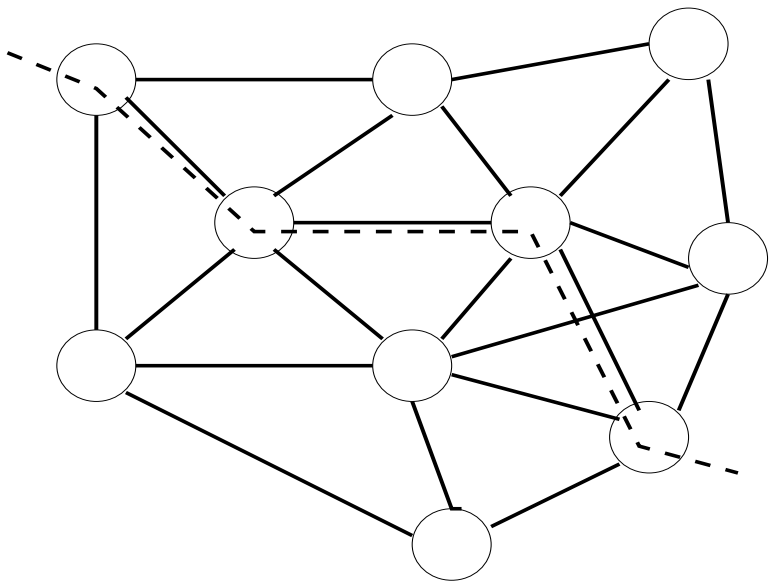
Mathematical Model

- Allocation (Distribution): $F_r(u)$



- Network

  - Network: $l \in \mathcal{L}$, C_l
  - Route: $(l_r^1, ..., l_r^{K_r})$
  - Allocation: $\underline{F} = (F_1, ..., F_R)$
  - Link Trans. Prob.: $P_{tr,l}(u, \underline{F})$



- System Of Equations

$$
\begin{aligned}
F_{r,l_r^1}(u,\underline{F}) &= F_r(u) \\
F_{r,l_r^k}(u,\underline{F}) &= \int_u^\infty P_{tr,l_r^{k-1}}(v,\underline{F})(-dF_{r,l_r^{k-1}}(v)) \\
P_{tr,l}(u,\underline{F}) &= P_{tr}\left(u,\sum_{r\in H_l} F_{r,l}(\underline{F})\right)
\end{aligned}
$$

Summary

- Priority Service with Pricing
  - Pricing to Control Congestion
  - Min-Max Fair Pricing
- Link Model
- Algorithm for Choosing d_r