Pricing Priority Services

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"A Business Model for Internet Services"
Alcatel Bell, Belgium

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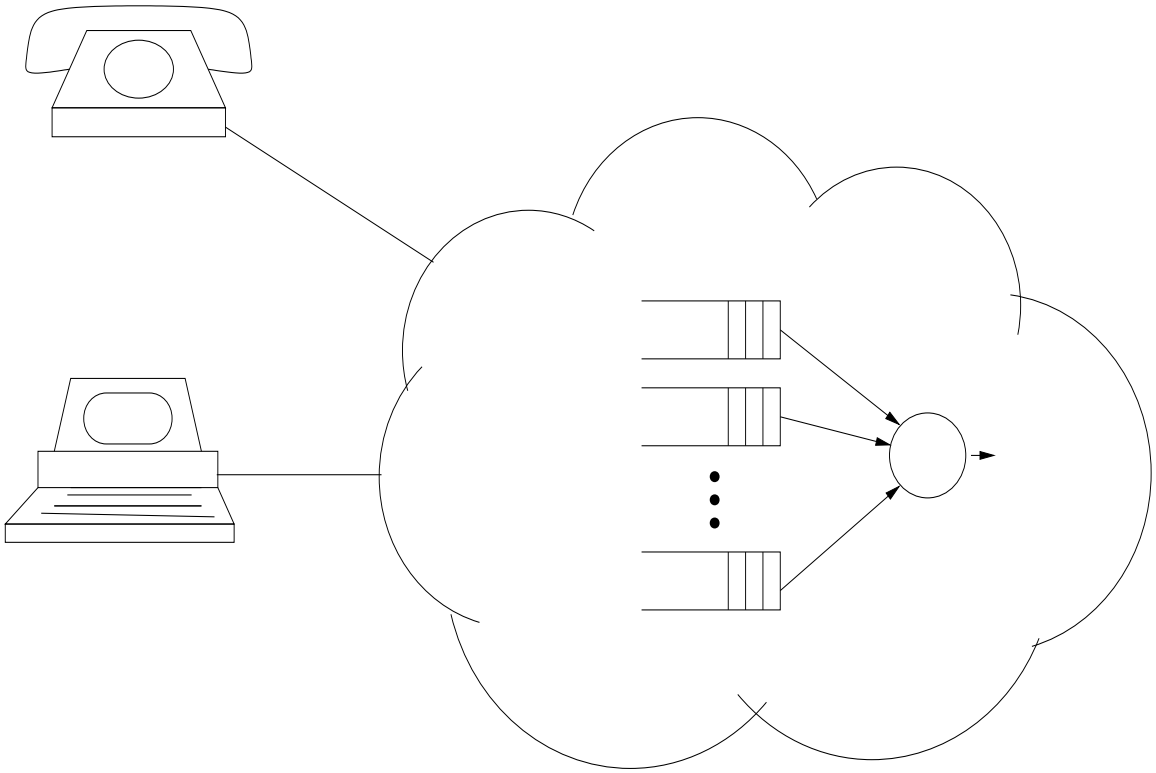
QoS Through Priority Service



- Priority Levels: $1, ..., N$
- Price: u_i per Submitted Packet with Priority i
- Non-Cooperative Game

"Utility - Cost"

Questions

- Stability
  - Equilibrium?
  - Pricing?
- "Fairness"

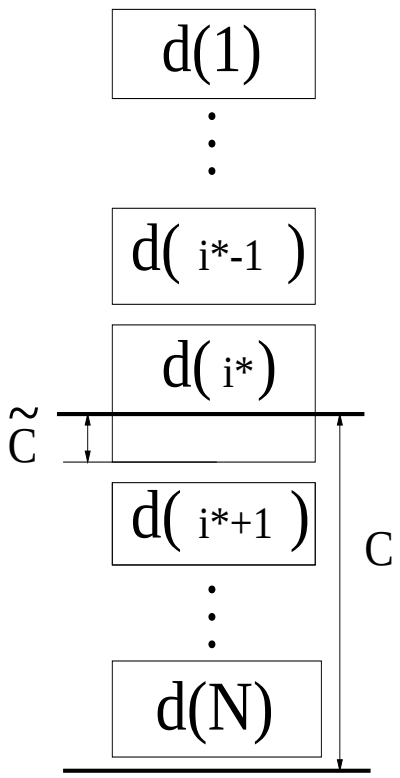
Stability: Single Link Case

- Equilibrium Exists

- Pricing as a Tool to Control Congestion

Link Model

- Slotted Time, Capacity C, No Buffer
- $d = (d(1), d(2), ..., d(N))$



$$P_{tr}(i,d) = 0, \qquad i < i^*$$

$$P_{tr}(i^*,d) = \frac{\tilde{C}}{d(i^*)}, \qquad i = i^*$$

$$P_{tr}(i,d) = 1, \qquad i > i^*$$

Elastic Traffic

- Care only about Throughput
- Customers $1, ..., R$
- Allocation $d_r = (d_r(1), ..., d_r(N))$
- Aggregate Allocation $d = \sum_{r=1}^{R} d_r$
- Utility:

$$U_r(d_r, P_{tr}(d)) = \bar{U}_r(x_r),$$

$$x_r = \sum_{r=1}^{N} P_{tr}(i, d) d_r(i)$$

- Demand $D_r(u)$:

$$\max_{x \geq 0} \{\bar{U}(x) - xu\}, \qquad u \geq 0$$

Non-Cooperative Game

- Customer's Problem:

$$\max_{y_r \in \Re_+^N} \left\{ U_r(y_r, P_{tr}(d)) - \sum_i y_r(i) u_i \right\}$$

- Nash Equilibrium: $(d_1, ..., d_R)$
  - Equilibrium Exists
  - $P_{tr}(i^*, d) \geq \frac{u_{i^*}}{u_{i^*+1}}$
  - $x_r = D_r(u^*), \qquad u_{i^*} \leq u^* \leq u_{i^*+1}$
  - $\sum_r x_r = C$

- "Fairness"

Quality of Service

- Utility:

$$\bar{U}_r(x_r) - \sum_{i=1}^{N} c_r\left(P_{tr}(i,d)\right) d_r(i)$$

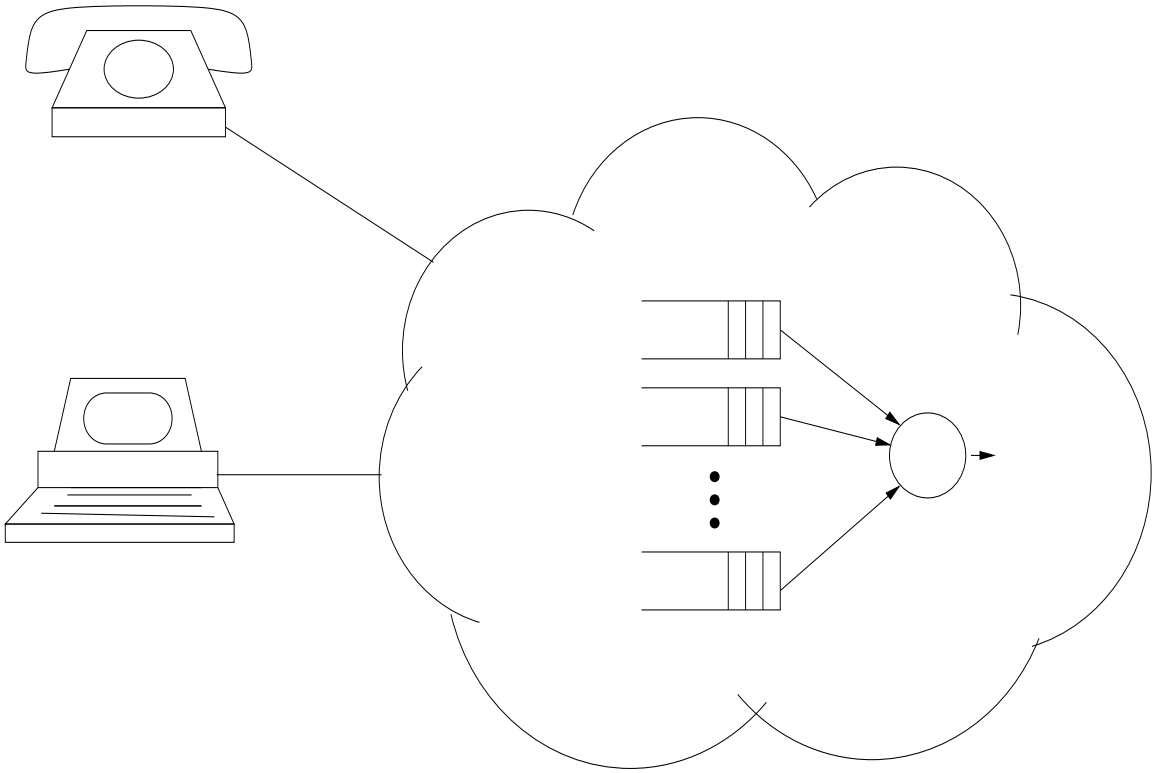
- Competitive Price-Taking Assumption
  - Nash Equilibrium Exists
  - $P_{tr}(i^*) \geq \frac{u_{i^*}}{u_{i^*+1}}$
  - $D_r(u_{i^*+1}) \leq x_r \leq D_r(u_{i^*})$
  - $\sum_r x_r = C$

- Differentiated QoS

Fairness: "Idealized" Service

- Unique Equilibrium
- Pricing is "Min-Max" Fair

"Idealized" Priority Service

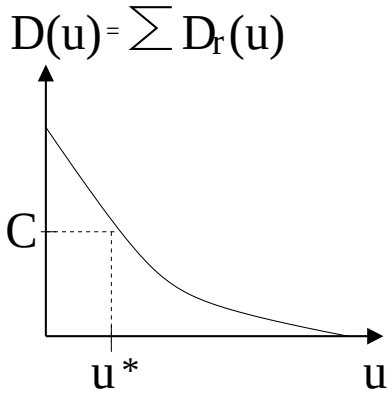


- Priorities: $\mathcal{I} = [u_{\min}, \infty)$
- Price: u per Submitted Packet with Priority u
- Non-Cooperative Game

"Utility - Cost"

"Min-Max" Fair Pricing

- Single Link



$$x_r = D_r(u_r) = D_r(u^*)$$

- Network:
  Minimize the price of the customer with the **max**imal price, subject to the capacity constraints.

Summary

- Priority Service with Pricing
  - Pricing to Control Congestion
  - Differentiated QoS
  - Min-Max Fair Pricing
- Link Model
- Algorithm for Choosing d_r

Related Work

- A. Odlyzko:
  Paris Metro Pricing

- A. Gupta/D. Stahl/A. Whinston:
  Dynamic Pricing

- R. Cocchi/S. Shenker/D. Estrin/L. Zhang:
  Experimental Results

- S. Chen and K. Park:
  Cost only Performance Related

- J. MacKie-Mason and H. Varian:
  Smart Market

- F. Kelly, A. Maulloo, D. Tan:
  Proportional Fairness