Rate Control
in
Random Access Networks

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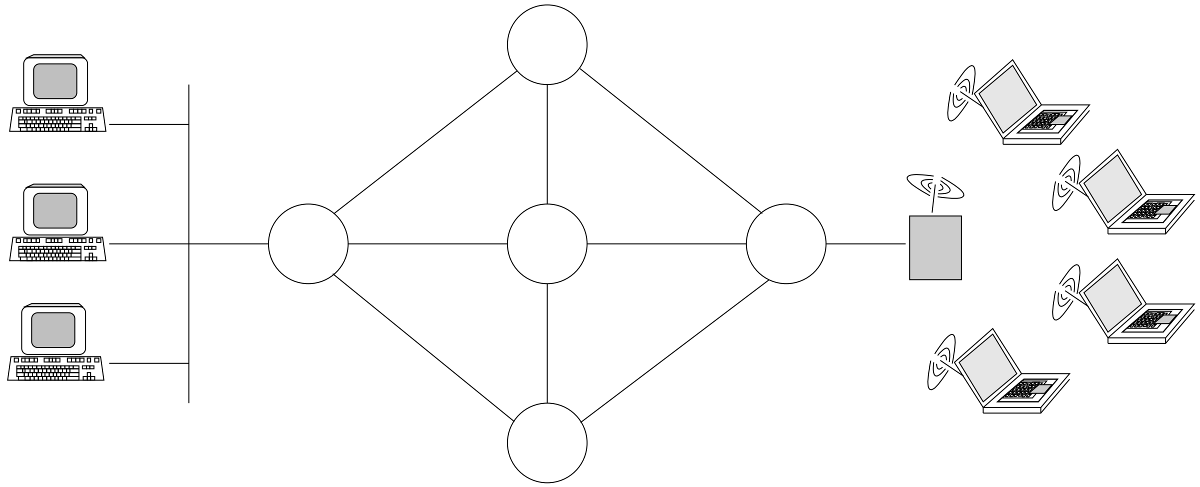
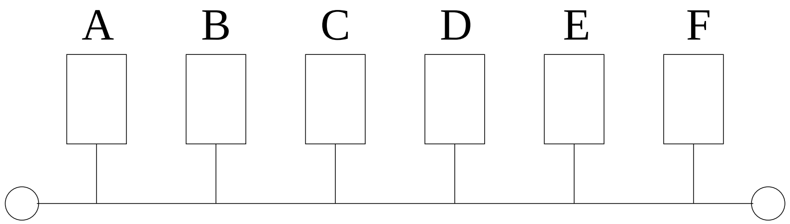
Rate Control



- Point-to-Point Wide Area Networks
  - TCP
  - Price-Based Control
- Contention-Based Local Area Networks

Contention-Based Local Area Networks



- Collisions - Backlog - Stability
- Retransmission Strategies for Backlogged Packets
- Rate Control
  - Collision: Reduce Rate
  - Idle: Increase Rate
- Questions
  - Stable?
  - Operating Point?

Approach

- Channel Model: Slotted Aloha
  - CSMA, CSMA/CD
- Use Channel Feedback to Modulate Rate
  - Idle, Successful, Collision Slot
- Markov Chain Formulation
- Stability: Pakes Mean-Drift Criteria
- Operating Point

Slotted Aloha

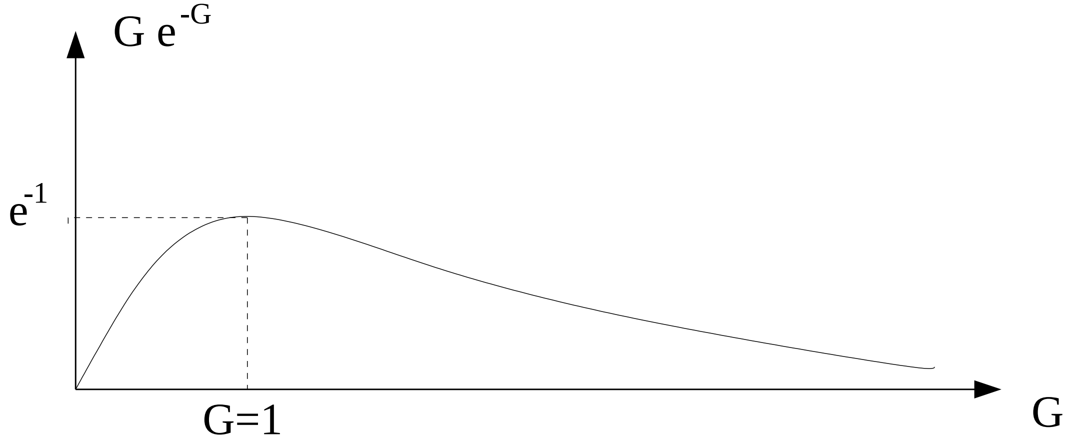
- Unit Time = Packet Transmission Delay
- Poisson Arrivals with (Aggregated) Rate of λ Packets/Unit-Time
- Collision or Perfect Reception
- Immediate Feedback: $0, 1, e$
- Retransmission Probability: q
- Infinite Set of Hosts

Slotted Aloha

- λ: Aggregated Arrival Rate
- n: Number of Backlogged Packets

$$n_{t+1} = n_t + Y_t - I[Z_t = 1]$$

- $G(n) = \lambda + nq$: Offered Load
- Throughput: $G(n)e^{-G(n)}$



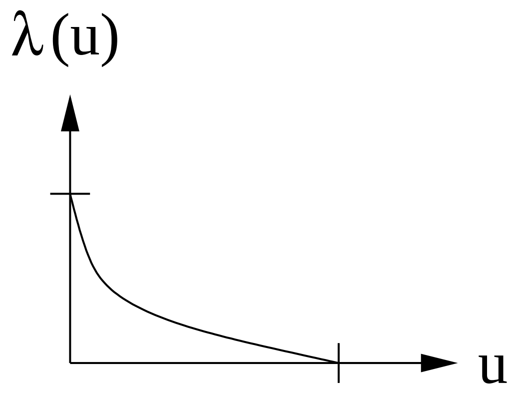
Rate Control

- Price Signal u
- Aggregated Transmission Rate $\lambda(u)$
- Collision: Increase Price
- Idle Slot: Decrease Price
- Price Adaptation: $\alpha < 0$, $\gamma > 0$

$$u_{t+1} = \Big[u_t + \alpha I[Z_t = 0] + \beta I[Z_t = 1] + \gamma I[Z_t \geq 2]\Big]^+$$

Stability - Markov Chain (n_t, u_t)

Assumption 1 *There exist positive constants* λ_{max} *and* u_{max} *such that* $\lambda : \mathbb{R}_+ \to [0,\lambda_{max}]$ *is strictly decreasing, with* $\lambda(u) = 0$ *when* $u \geq u_{max}$.



Stability: Is the Markov chain positive recurrent?

Operating Point

- Mean Drift of Backlog n

$$d_n(n,u) \triangleq \mathbf{E}\big(n_{t+1} - n_t \,\big|\, n_t = n, u_t = u\big)$$

- Mean Drift of Price u

$$d_u(n,u) \triangleq \mathbf{E}\big(u_{t+1} - u_t \,\big|\, n_t = n, u_t = u\big)$$

- Operating Point (n^*, u^*)

$$d_n(n^*, u^*) = d_u(n^*, u^*) = 0$$

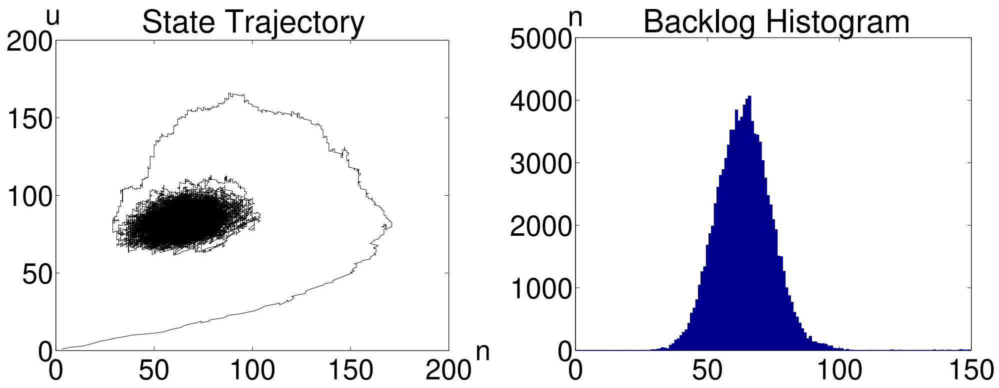
- Questions
  - Does operating point exist?
  - Is there a unique operating point?

Results

- System is stable.
- (Under suitable conditions) There exists a unique operating point (n^*, u^*)
- $G^* = \lambda^* + n^* q$
- We can set G^* by choosing α,β,γ.
  - Throughput $\lambda^* = G^* e^{-G^*}$
  - Backlog n^*
  - Average Delay $D^* = n^*/\lambda^*$
  - $\beta = \frac{\gamma}{G^*}\left(G^* + 1 - e^{G^*}\right) - \frac{\alpha}{G^*}$

Numerical Results

- $G^* = 1$ and $S^* = e^{-1} = 0.368$, $D^* = 171.82$
- $\alpha = -1$, $\gamma = 1$, and $\beta = 0.2817$
- $\lambda(u) = \left[4(1 - u/150)^3\right]^+$
- $q = 0.01$



- $S = 0.367$ and $D = 170.28$

Extensions

- Delay Differentiation: q_c, $c = 1, ..., C$
- Dynamic Retransmission Probabilities: $q(u)$
  - $\lim_{u\to\infty} \lambda(u) = 0$
  - $q(u) = e^{-bu}$, $b > 0$
  - $q(u) = (1 + bu)^{-r}, r > 1$ and $b > 0$.
- Finite Number of Nodes: Fairness, Throughput Differentiation

$$\lambda(u) = \sum_{m=1}^{M} \lambda_m(u).$$

- Integration with Price-Based Rate Control for Point-to-Point Networks
  - Marking Scheme by by Athuraliya and Low.