Pricing, Games, and DiffServ

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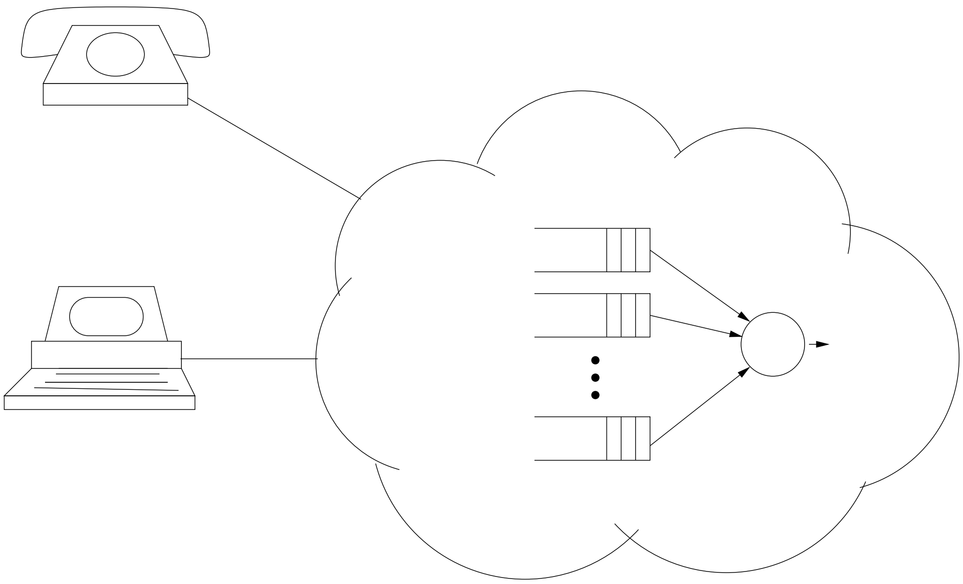
Problem Formulation



- Priority Levels: $1, ..., N$
- Price: u_i per Submitted Packet

- **Questions**
  - Stable Operation?
  - Pricing - Network Performance?
  - Bandwidth Allocation?

- **Approach**
  - Non-Cooperative Game

    "Utility - Cost"
  - Equilibrium?
  - Pricing?
  - Bandwidth Allocation?

Problem Motivation: DiffServ

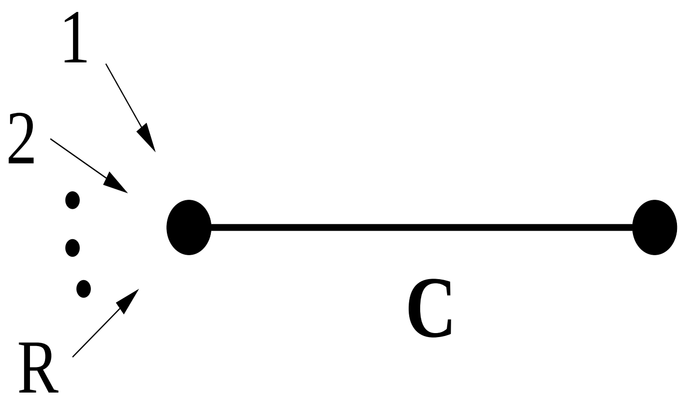
- Services Classes with Different QoS
  - Example: Premium Service and Best Effort Service
- Pricing
- Network Control vs. Economic Mechanism
- Dumb Networks and Intelligent Hosts

Results

- Single Link Case
  - Equilibrium Exists
  - Pricing a Tool to Control Congestion
    * Smooth Traffic:
      Absolute QoS: $P_{tr}(i) \geq q_i$
    * Bursty Traffic:
      Relative QoS: $\frac{P_{tr}(i)}{P_{tr}(i+1)} \geq q_i$

- Network Case
  - Equilibrium Exists
  - Max-Min Fair Bandwidth Allocation

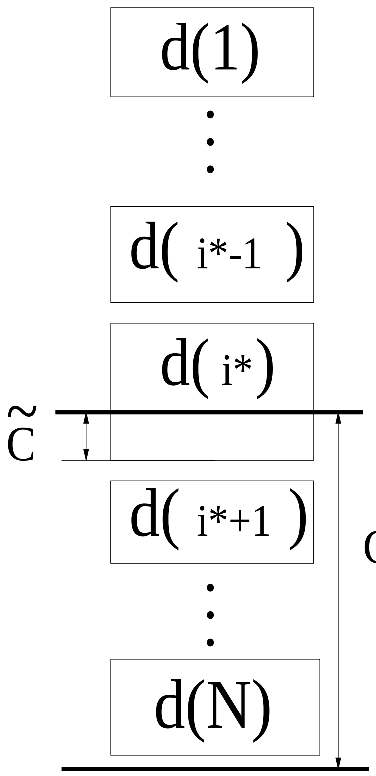
Mathematical Model



- Slotted Time, Capacity C, No Buffer

$$d_r = (d_r(1), ..., d_r(N))$$
$$d = \sum_{r=1}^{R} d_r$$

- $d = (d(1), d(2), ..., d(N))$



$$P_{tr}(i,d) = 0, \qquad i < i^*$$

$$P_{tr}(i^*,d) = \frac{\tilde{C}}{d(i^*)}, \qquad i = i^*$$

$$P_{tr}(i,d) = 1, \qquad i > i^*$$

Elastic Traffic

- Users care only about Throughput

- User's Problem:

$$\max_{d_r \in \Re_+^N} \left\{ U_r(x_r) - \sum_i d_r(i) u_i \right\},$$

$$x_r = \sum_{i=1}^{N} d_r(i) P_{tr}(i, d)$$

- Demand $D_r(u)$:

$$D_r(u) = \arg\max_{x \geq 0} \{U_r(x) - xu\}, \qquad u \geq 0$$

- Competitive Price-Taking Assumption
  - Users do not know how they affect $P_{tr}(i)$
  - Many Small users Share the Link
- Equilibrium $(d_1^*, ..., d_R^*)$
  - For every user $r = 1, ..., R$, we have

$$d_r^* = \arg \max_{d_r \in \Re_+^N} \left\{ U_r(x_r) - \sum_{i=1}^{N} d_r(i) u_i \right\},$$

where

$$x_r = \sum_{r=1}^{R} d_r(i) P_{tr}(i, d^*).$$

Results: Single Link Case

- At most two priorities i^* and i^*+1 are used

- $P_{tr}(i^*, d) \geq \frac{u_{i^*}}{u_{i^*+1}}$, or

$$\frac{u^{i^*}}{P_{tr}(i^*, d)} \leq u_{i^*+1}$$

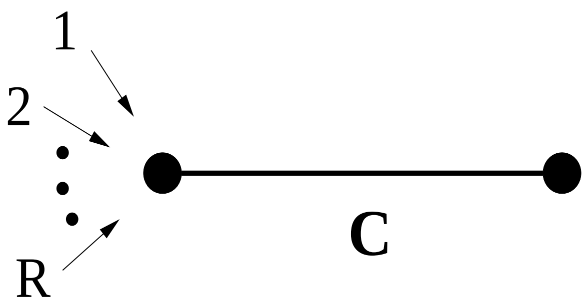
- $x_r = D_r(u^*)$

$$u^* = \frac{u_{i^*}}{P_{tr}(i, d^*)}$$

- $\sum_{r=1}^{R} x_r = C$

Max-Min Fairness

- Single Link:



$$x^* = x_r = C/R, \quad \sum_r x^* = R$$

- Priority Service

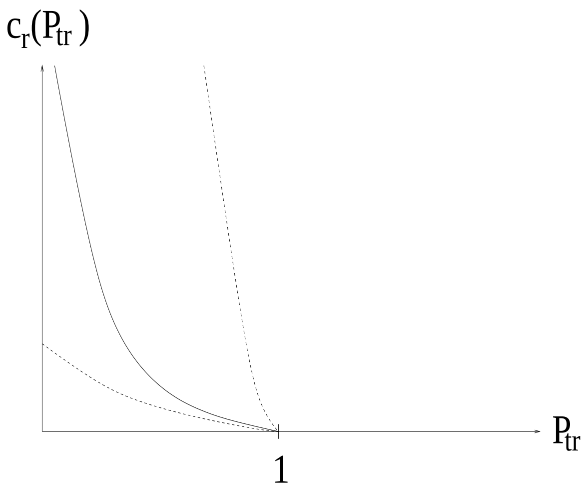
$$x_r = D_r(u^*), \quad \sum_r x_r = C$$

Users with QoS Requirements

- Utility Function:

$$U_r(x_r) - \sum_{i=1}^{N} c_r\left(P_{tr}(i,d)\right) d_r(i)$$

- Penalty Term $c_r(P_{tr})$



Results

- Equilibrium Exists

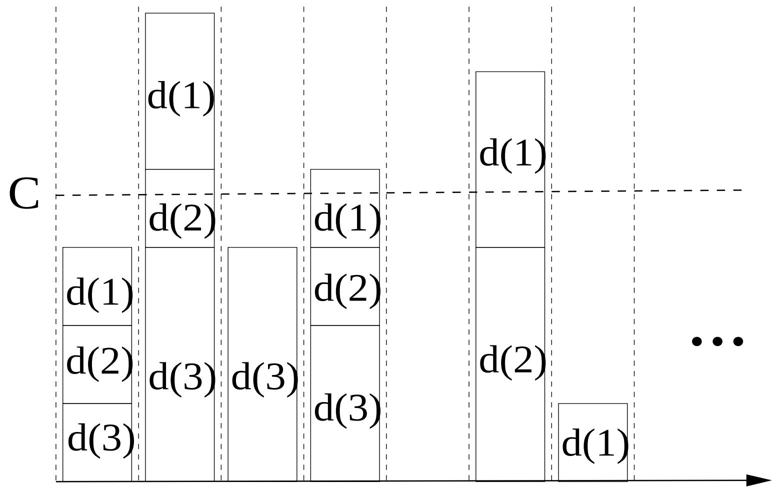
- $P_{tr}(i^*) \geq \frac{u_{i^*}}{u_{i^*+1}}$

- $x_r = D_r(u_r^*), \qquad u_{i^*} \leq u_r^* \leq u_{i^*+1}$

- $\sum_r x_r^* = C$

Bursty Traffic

- User r submits a burst of size B in priority i with probability $\lambda_r(i)$



- User's Problem ($B = 1$):

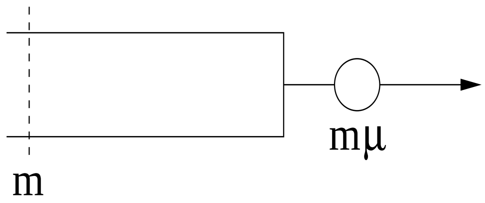
$$\max_{d_r \in \Re_+^N} \left\{ U_r(\kappa_r) - \sum_i \lambda_r(i) u_i \right\},$$

$$\kappa_r = \sum_{i=1}^{N} \lambda_r(i) P_{tr}(i, d)$$

- Non-Cooperative Game
  - all priorities $i \leq i^*$ get used
  - $\frac{P_{tr}(i^*, d)}{P_{tr}(i^*+1, d)} \geq \frac{u_{i^*}}{u_{i^*+1}}$

Case Study

- Priority Queue with Poisson Arrival and Exponential Service Time



- $R = 5$, $N = 8$

- $\frac{u_i}{u_{i+1}} = \frac{1}{2}$, i=1,...,7

- Equilibrium Allocation

m	5	10	20	50	100
$\lambda^*(1)$	2.13	0.466	2.13	2.10	0
$\lambda^*(2)$	0.88	1.81	0.26	0	0
$\lambda^*(3)$	1.70	3.02	1.85	0	0
$\lambda^*(4)$	1.01	0.27	0.76	0	0
$\lambda^*(5)$	1.77	0	0	0	0
$\lambda^*(6)$	3.94	9.84	21.26	54.06	106.02
$\lambda^*(7)$	0	0	0	0	0
$\lambda^*(8)$	0	0	0	0	0

Summary

- Priority Service with Pricing
  - Pricing to Control Congestion
  - Min-Max Fair Pricing
- Link Model
- Algorithm for Choosing d_r