Pricing in Wireless Networks

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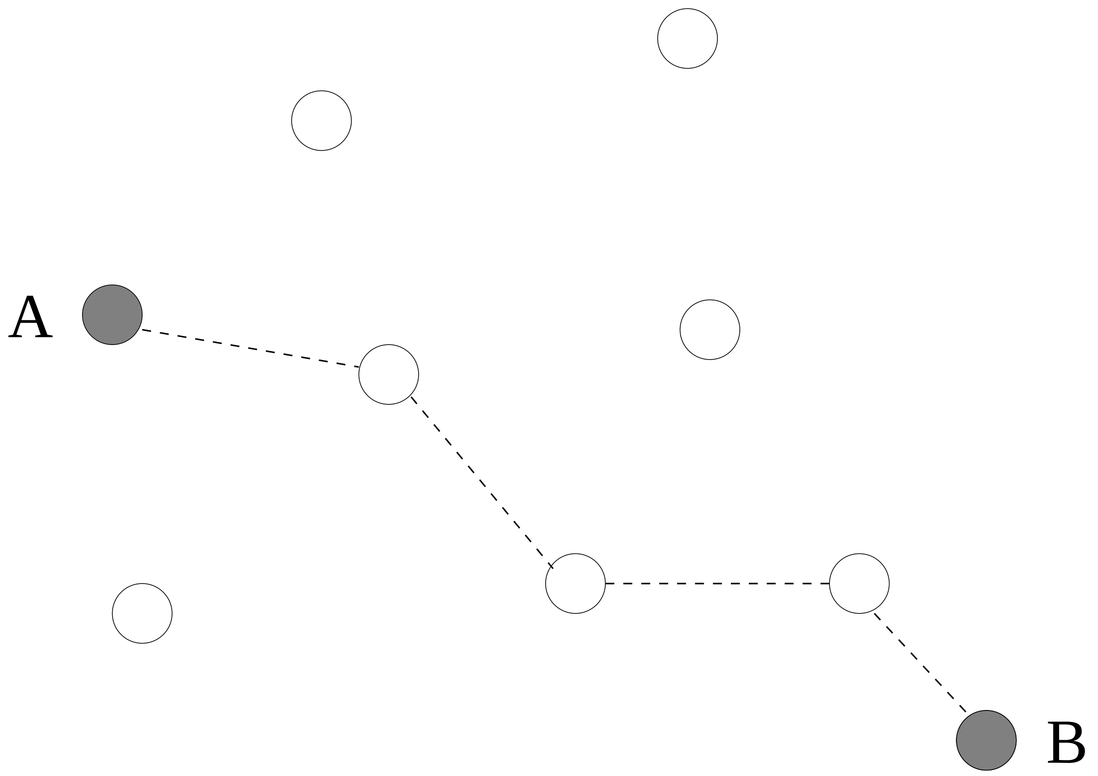
Joint work with Ying Qiu

Ad-Hoc Networks



Pricing to Stimulate Co-operation

Nodes are

- rewarded for forwarding packets (providing resources)
- charged for sending packets (using resources)

Nodes decide on

- how many packets to send (Transmission Rate)
- how many packets to forward
- how much to charge for forwarding packets

Issues

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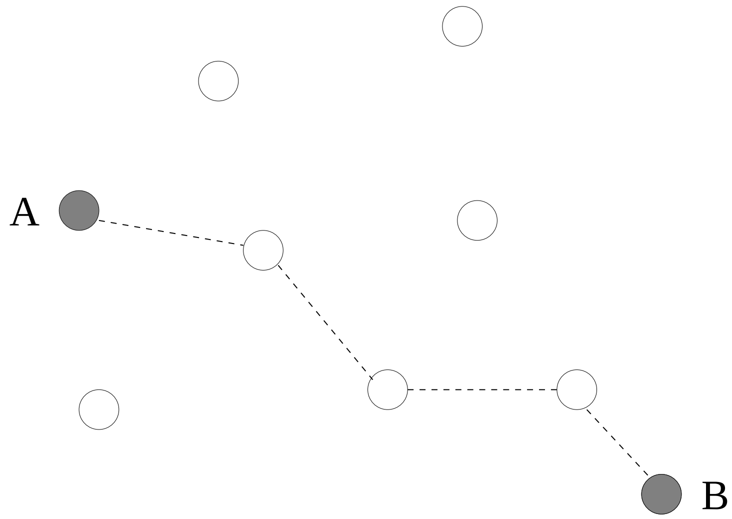
- Routing
- Protocol
- Security
- Network Performance

Other Applications of this Model

- Peer-to-Peer Computing
- Content Delivery Networks
- Application Service Providers
- and other Peer-to-Peer Applications

Model/Terminology



- Routes are Fixed
- C_r: Capacity of Node r
- $U_r(x_r)$: Utility Function of Node r

Model/Terminology

- x_r: Transmission Rate of Node r
- y_r: Traffic forwarded by Node r
- μ_r: Price Node r charges for forwarding Packets
- λ_r: Price Node r has to pay for sending Packets
- $D_r(p)$: Demand Function of Node r

$$D_r(p) = \arg \max_{x_r \geq 0} \Big\{ U_r(x_r) - x_r p \Big\}, \qquad p \geq 0.$$

- $I_r(\mu_r, \mu_{-r})$: Incoming Traffic at Node r under Price μ_r

Total Utility

- $U_r(x_r)$: Utility of Node r
- $y_r\mu_r$: Revenue for forwarding Packets
- $-x_r\lambda_r$: Cost of Sending Packets

Objective Function

Given λ_r, $\underline{\mu}_{-r}$

$$\max_{x_r, y_r, \mu_r \geq 0} \Big\{ U_r(x_r) - x_r \lambda_r + y_r \mu_r \Big\},$$

subject to

$$x_r + y_r \leq C_r$$

$$y_r \leq I_r(\mu_r, \mu_{-r})$$

Related Work

Frank Kelly et al.

- End Hosts
- Network Routers/Links

Ad Hoc Network

$$\max_{x_r, y_r, \mu_r \geq 0} \left\{ U_r(x_r) - x_r \lambda_r + y_r \mu_r \right\},$$

subject to

$$x_r + y_r \leq C_r$$

$$y_r \leq I_r(\mu_r, \mu_{-r})$$

User Adaptation Model

Step 1: Given $\lambda_r^{(k)}$, $i_r^{(k)}$, and $\mu_r^{(k)}$, choose $x_r^{(k+1)}$ and $y_r^{(k+1)}$

Case 1: $$x_r^{(k+1)} = \arg\max_{x_r \geq 0} \left\{ U_r(x_r) + (C_r - x_r)\mu_r^{(k)} - x_r \lambda_r^{(k)} \right\}$$

Case 2: $$x_r^{(k+1)} = \arg\max_{x_r \geq 0} \left\{ U_r(x_r) + i_r^{(k)} \mu_r^{(k)} - x_r \lambda_r^{(k)} \right\}$$

Case 3: ...

Step 2: Choose $\lambda_r^{(k+1)}$

$$\mu_r^{(k+1)} = \left[\mu_r^{(k)} + \alpha \Big(D_r\big(\lambda^{(k)} + \mu_r^{(k)}\big) + i_r^{(k)} - C_r \Big) \right]^+$$

Network Performance in Equilibrium

If demand is very elastic, *i.e.* if

$$\left| \frac{D_r(p)}{D_r'(p)} \right| \approx 0,$$

then, in equilibrium, the bandwidth allocation maximizes the social welfare,

$$\max \sum_r U_r(x_r)$$

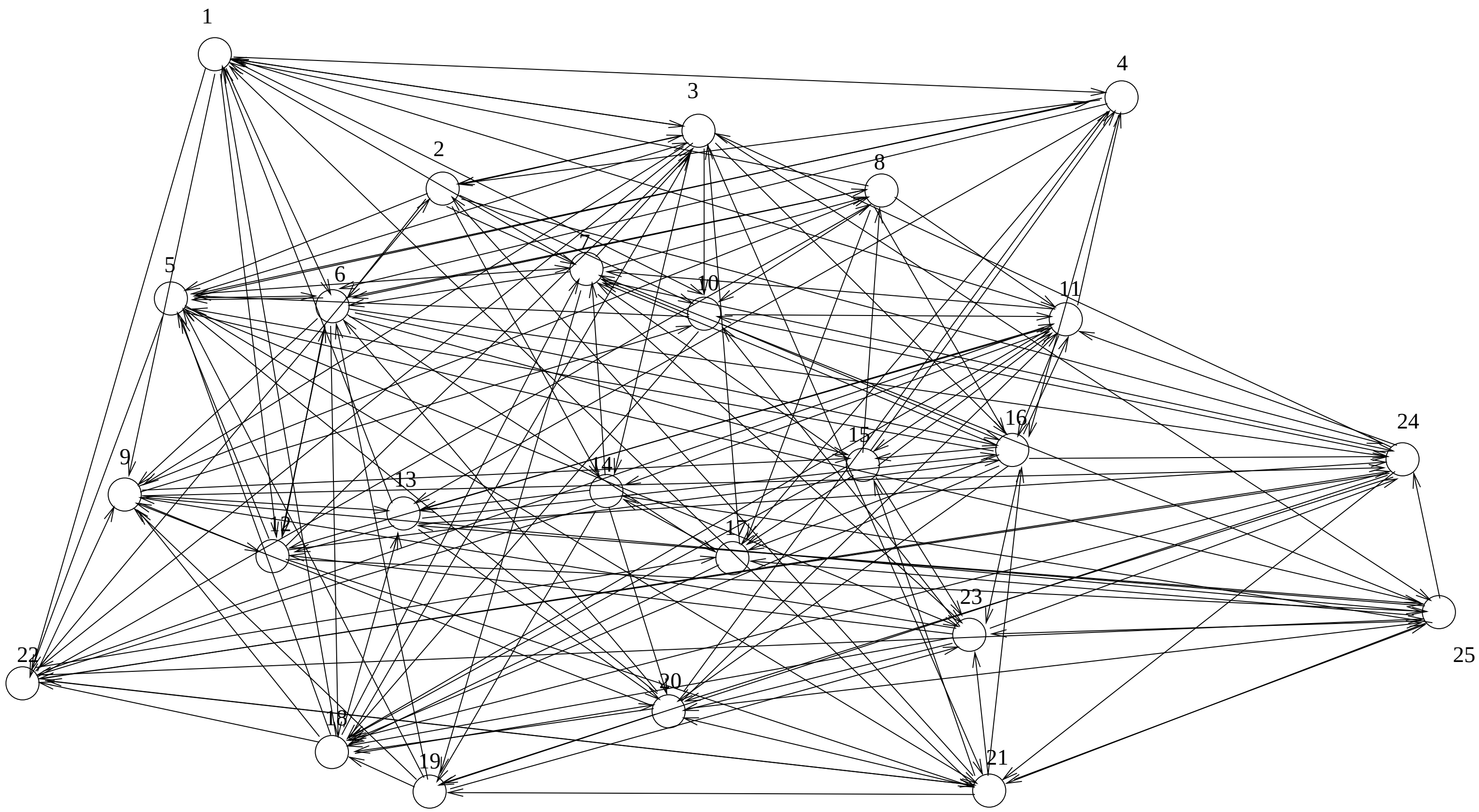
subject to the capacity constraints

$$x_r + y_r \leq C_r, \qquad r = \{1, ..., R\}$$

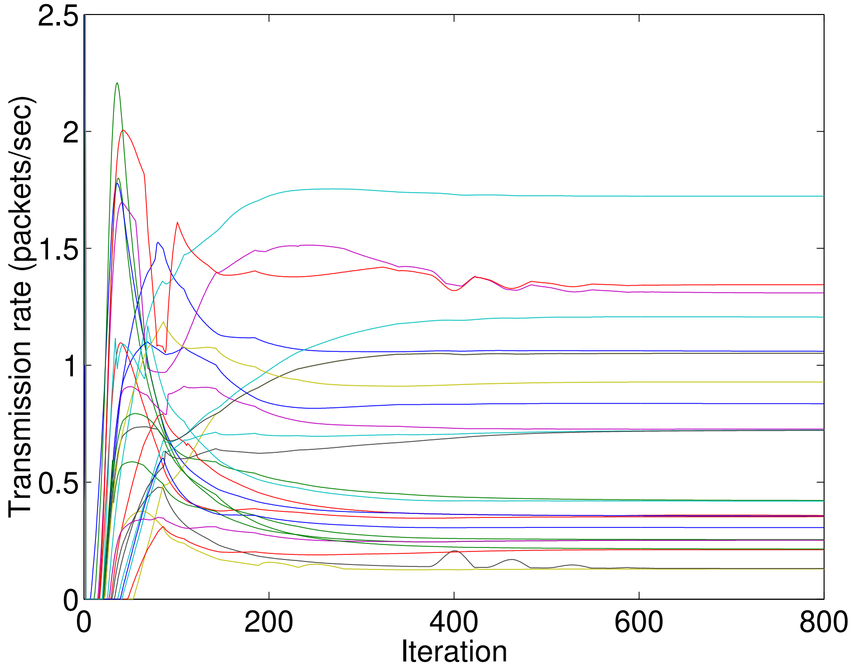
This implies that

- bandwidth allocation $(x_1^*, ..., x_R^*)$ is unique
- price vector $(\mu_1^*, ..., \mu_R^*)$ is not necessarily unique

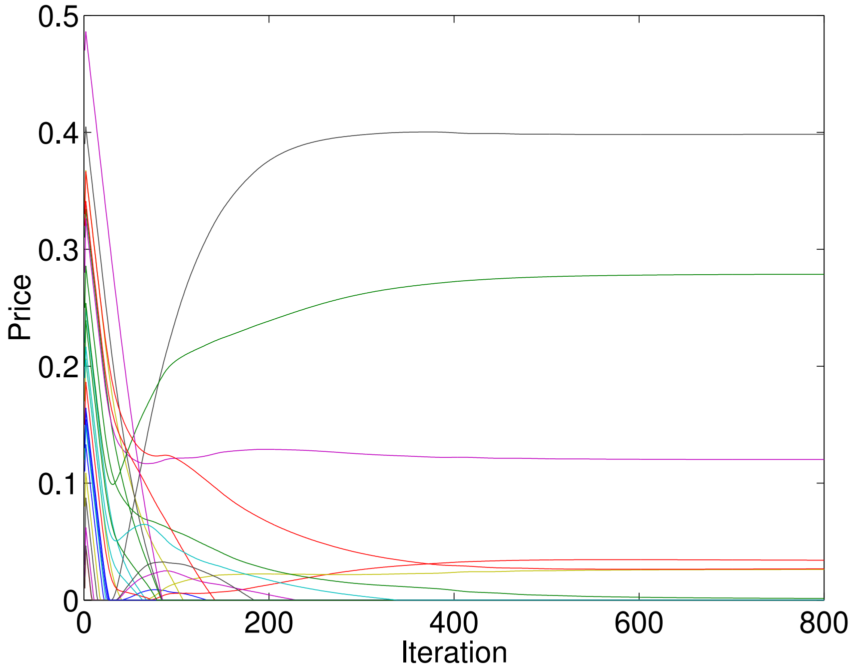
Numerical Example with 25 Nodes



Transmission Rate x_r



Price μ_r for Forwarding Packets



Conclusions/Extensions

- Resource Allocation in Peer-to-Peer Systems
- Maximizes Social Welfare
- Battery Power

$$\max\left\{U_r(x_r) + y_r\mu_r - x_r\lambda_r - p_r(x_r + y_r)\right\},$$

- Interference
- INFOCOM 2003