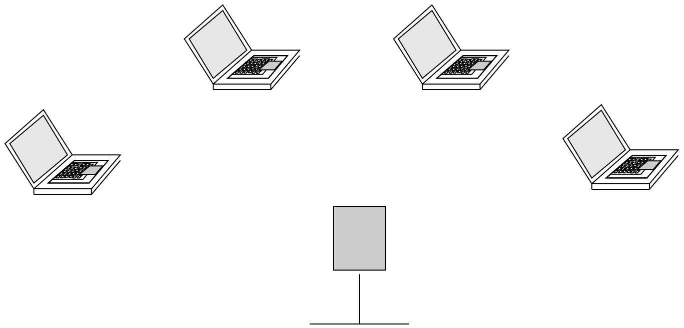
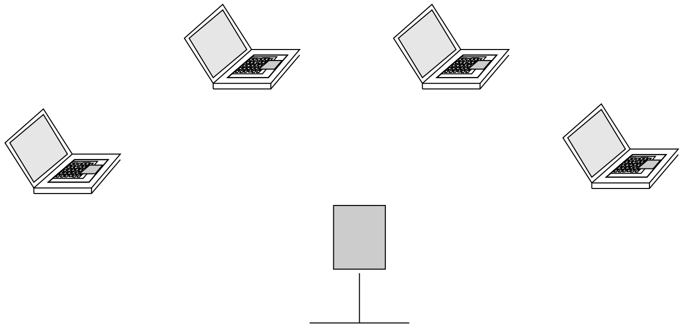
Random Access Networks:
Transmission Costs, Selfish Nodes, and Protocol Design



Joint work with Clement Yuen and Ran Pang

Random Access Networks:



- Collisions - Stability
- Transmission Cost
- Rate Control?

Rate Control in Random Access Networks

Questions

- Is transmission cost sufficient to guarantee stability?
- If not, what additional mechanisms are needed?

Answers

- Transmission cost does not guarantee stability
- Pricing mechanism: stability and system performance

Outline

- Non-Cooperative Game - Idealized Situation
  - (Symmetric) Nash Equilibrium
  - Pricing Mechanism
- Distributed Algorithm
- Model: Slotted Aloha (CSMA/CD)

Non-Cooperative Game - Slotted Aloha Model

- Poisson Arrivals
- Collision - Retransmission
- Probabilistic Retransmissions
- Transmission Cost γ
- Infinite set of hosts

Non-Cooperative Game

- Poisson Arrivals with (Aggregated) Rate $\lambda(u)$, $u \geq 0$
  - Packets have different values
  - $\lim_{u \to \infty} \lambda(u) = 0$
- State n: number of backlogged packets
  - whether to accept a new packet
  - retransmission probability for backlogged packet
- Strategy $\pi = (u, q)$
  - $u = (u(0), u(1), u(2), ...)$
  - $q = (q(1), q(2), ...)$
- Strategy $\pi = (\lambda, q)$
  - $\lambda(n) = \lambda\big(u(n)\big), \qquad n = 0, 1, 2, ...$

Markov Chain Formulation

- Nodes are indistinguishable (symmetric strategies)
- Strategy π: Markov chain $\{n_k; k \geq 0\}$
- Successful transmission of a backlogged packet for given node:

$$e^{-\lambda(n)}(1 - q(n))^{n-1} \approx e^{-\lambda(n)-(n-1)q(n)}$$

- Offered load: $G(n) = \lambda(n) + nq(n)$
- Instantaneous throughput

$$G(n)e^{-G(n)}$$

Markov Chain Formulation

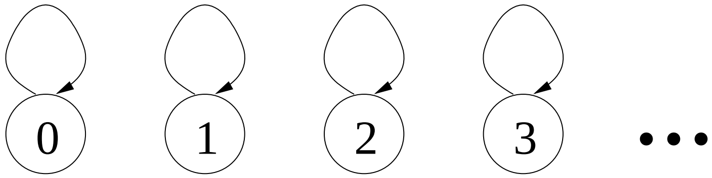
- Cost for successfully transmitting a packet
  - new packet: $R(\pi, n)$
  - backlogged packet: $Q(\pi, n)$
- Retransmission Probabilities $\hat{q}$
  - new packet: $R(\pi, n, \hat{q})$
  - backlogged packet: $Q(\pi, n, \hat{q})$

Equilibrium Strategy

- Admissible retransmission vector $\hat{q}$
  - $T_n(\pi, \hat{q})$ is a random variable, $n \geq 0$
  - set of all admissible retransmission vectors: $\mathcal{Q}(\pi)$
- Admissible strategy π
  - $\lambda(0) > 0$
  - $q \in \mathcal{Q}(\pi)$
- Equilibrium strategy
  - $q = \arg\min_{\hat{q} \in \mathcal{Q}(\pi)} Q(\pi, n, \hat{q}), \qquad n \geq 0$
  - $u(n) = R(\pi, n)$
- Symmetric Nash equilibrium

Stable Strategy

- Stable strategy
  - "Expected number of backlogged nodes stays bounded"
- Stable equilibrium strategy
  - Single positive recurrent class, and possibly some transient states



- Questions
  - Does a stable equilibrium strategy exist?
  - Does a unique stable equilibrium strategy exist?
  - What is the performance at a stable equilibrium strategy?

A particular class of Strategies

- Set $\mathcal{F}_\kappa$ of admissible strategies
  - class $\mathcal{N}_c = \{n; n \geq N_0\}$
  - $\lambda(n) + (n-1)q(n) = \kappa, \qquad n \in \mathcal{N}_c$
- Transmitting backlogged packet
  - $e^{-\lambda(n)-(n-1)q(n)} = e^{-\kappa}, \qquad n \in \mathcal{N}_c$
- Cost $Q(\pi, n)$
  - $Q(\pi, n) = \gamma e^{\kappa}, \qquad n \in \mathcal{N}_c$

Existence of a Equilibrium Strategy $\pi \in \mathcal{F}_\kappa$

Proposition 1 *There exists a stable equilibrium strategy* $\pi \in \mathcal{F}_\kappa$ *if and only if the following conditions hold*

(a) $\max_{r \geq 0}(f_\infty(r) - r) \geq 0$,

(b) $\lambda(r_\infty) < \kappa e^{-\kappa}$, *and*

(c) $\lambda(r_0) \geq \kappa$.

Idea: The transmission cost γ needs to be large enough in order to have a equilibrium strategy $\pi \in \mathcal{F}_\kappa$.

Existence of a Stable Equilibrium Strategy

Proposition 2 *If π is a stable equilibrium strategy then there exists a $\kappa > 0$ such that $\pi \in \mathcal{F}_\kappa$.*

Interpretation

- If transmission cost γ is too small then there does not exist a stable equilibrium allocation
- If there exists a stable equilibrium allocation, then there is typically a continuum of stable equilibria (in κ).
- Different values of κ lead to different throughput and delay.

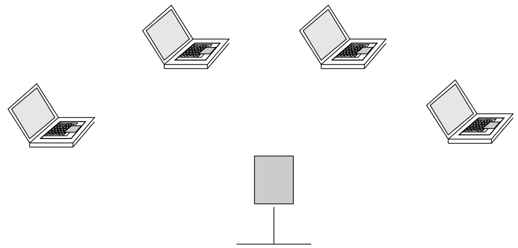
Protocol Design

- Need additional mechanism to guarantee stability
- Would like mechanism for choosing κ
- **Idea:** cost c for successfully transmitted packets
- Can choose c and κ to
  - set throughput/delay (trade-off)
- MAC protocol: choosing κ
  - Pick κ in advance ($\kappa = 1$)
  - Choose c for throughput/delay
  - Determine $q(n)$ (probability of successful retransmission is the same at all states)
  - No node has incentive to deviate (no cheating)
  - MAC standard

Protocol Design

- Assumption
  - Know $\lambda(u)$
  - Can observe state n

Protocol Design



- Rate Control
  - Collision: Increase Price
  - Idle: Decrease Price
- Questions
  - Stable?
  - Operating Point?

Price Update

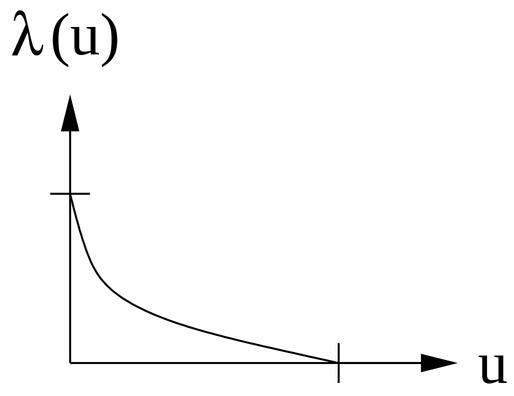
- Price Signal u
- Aggregated Transmission Rate $\lambda(u)$
- Collision: Increase Price
- Idle Slot: Decrease Price
- Price Adaptation: $\alpha < 0$, $\gamma > 0$

$$u_{t+1} = \Big[u_t + \alpha I[Z_t = 0] + \beta I[Z_t = 1] + \gamma I[Z_t \geq 2]\Big]^+$$

- Retransmission Probability: q

Stability - Markov Chain (n_t, u_t)

Assumption 1 *There exist positive constants* λ_{max} *and* u_{max} *such that* $\lambda : \mathbb{R}_+ \to [0,\lambda_{max}]$ *is strictly decreasing, with* $\lambda(u) = 0$ *when* $u \geq u_{max}$.



Stability: Is the Markov chain positive recurrent?

Operating Point

- Mean Drift of Backlog n

$$d_n(n,u) \triangleq \mathbf{E}\big(n_{t+1} - n_t \,\big|\, n_t = n, u_t = u\big)$$

- Mean Drift of Price u

$$d_u(n,u) \triangleq \mathbf{E}\big(u_{t+1} - u_t \,\big|\, n_t = n, u_t = u\big)$$

- Operating Point (n^*, u^*)

$$d_n(n^*, u^*) = d_u(n^*, u^*) = 0$$

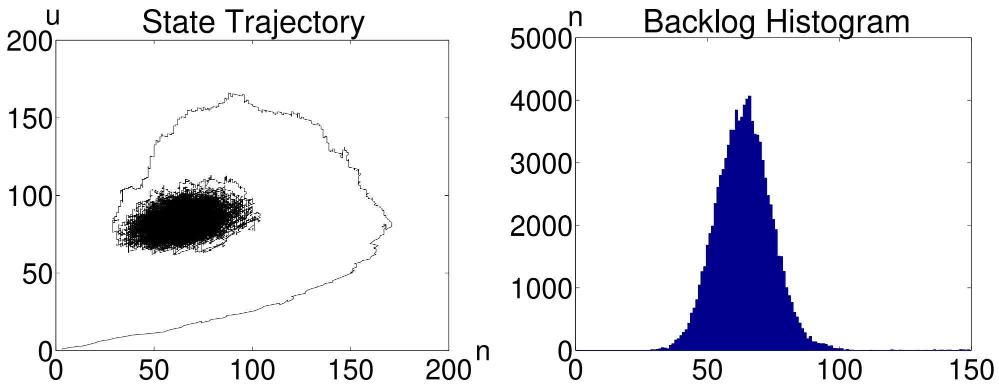
- Questions
  - Does operating point exist?
  - Is there a unique operating point?

Results

- System is stable.
- (Under suitable conditions) There exists a unique operating point (n^*, u^*)
- $G^* = \lambda^* + n^* q$
- We can set G^* by choosing α,β,γ.
  - Throughput $\lambda^* = G^* e^{-G^*}$
  - Backlog n^*
  - Average Delay $D^* = n^* / \lambda^*$
  - $\beta = \frac{\gamma}{G^*}\left(G^* + 1 - e^{G^*}\right) - \frac{\alpha}{G^*}$

Numerical Results

- $G^* = 1$ and $S^* = e^{-1} = 0.368$, $D^* = 171.82$
- $\alpha = -1$, $\gamma = 1$, and $\beta = 0.2817$
- $\lambda(u) = \left[4(1 - u/150)^3\right]^+$
- $q = 0.01$



- $S = 0.367$ and $D = 170.28$

Summary

- Price-Based Rate Control
- Stability
- Performance
- Do not need to know
  - State n
  - Rate function $\lambda(u)$
  - Retransmission probability q
- Model

Delay Differentiation and Dynamic Retransmission Probabilities

- Delay Differentiation: q_c, $c = 1, ..., C$
- Dynamic Retransmission Probabilities: $q(u)$
  - $\lim_{u \to \infty} \lambda(u) = 0$
  - $q(u) = e^{-bu}$, $b > 0$
  - $q(u) = (1 + bu)^{-r}, r > 1$ and $b > 0$.
- Delay Differentiation and Dynamic Retransmission Probabilities

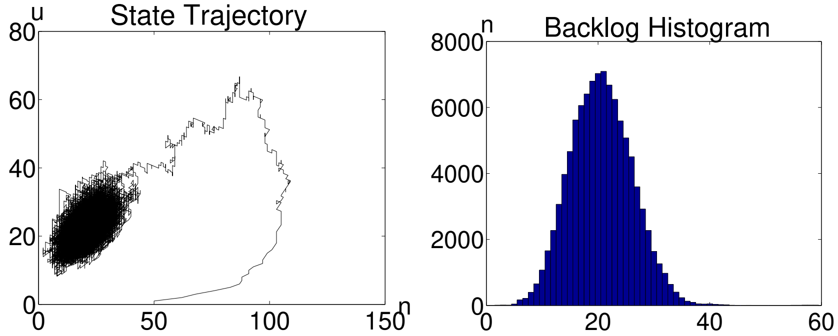
Dynamic Retransmission Probabilities

Aggregated Arrival Rate $\lambda(u)$

$$\lambda(u) = \frac{40}{(1+u)^{1.5}}$$

Retransmission Probability $q(u)$

$$q(u) = \frac{1}{(1+u)^{1.1}}$$



Throughput $S = 0.368$

Finite Number of Nodes

- Finite Number of Nodes

$$\lambda(u) = \sum_{m=1}^{M} \lambda_m(u).$$

- Nodes can have several backlogged packets
- Backlog-Dependant Retransmission Probabilities

$$q_m(n_m) = \begin{cases} n_m q_m, & n_m q_m \leq 1 - \epsilon, \\ 1 - \epsilon, & \text{otherwise}, \end{cases}$$

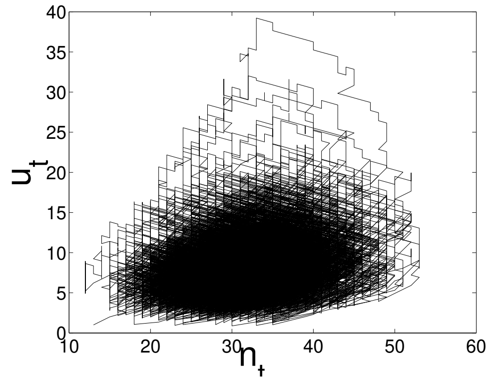
- Backlog-Independent Retransmission Probabilities, q_m.

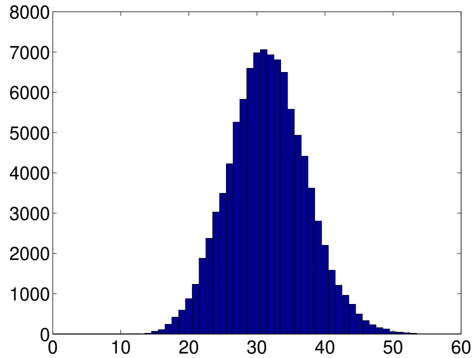
Backlog-Dependant Retransmission Probabilities

Assumption: "Price tends to increase when all nodes are saturated and retransmit with probability $1 - \epsilon$."

Case Study

Node	Bandwidth	Delay
1	tolerant	tolerant
2	tolerant	intolerant
3	intolerant	tolerant
4	intolerant	intolerant

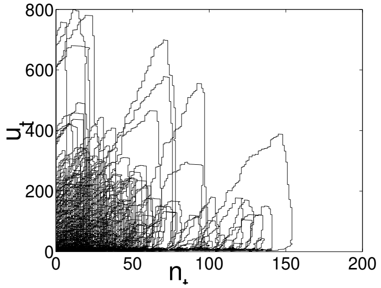


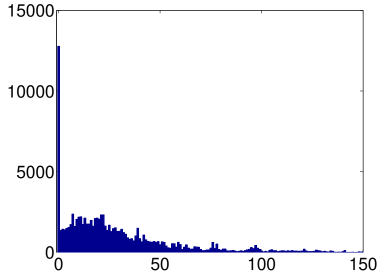


Node m	S_m	D_m
1	0.021	186.7
2	0.021	19.9
3	0.206	116.5
4	0.210	11.8

Backlog-Independent Retransmission Probabilities

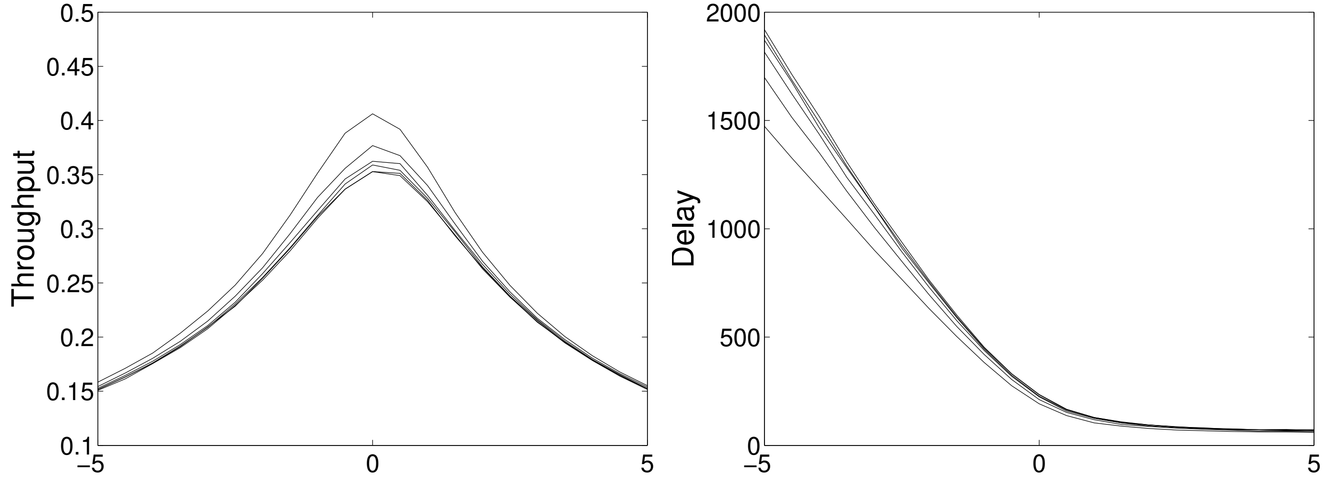
Assumption: "Price tends to increase when each nodes has at least one backlogged packet"





Node m	S_m	D_m
1	0.006	93.0
2	0.006	85.4
3	0.069	417.6
4	0.068	46.0

Infinite Node Approximation



Extensions: End-to-End Rate Control

- Integration with Price-Based Rate Control for Point-to-Point Networks
  - Marking Scheme by by Athuraliya and Low.

Conclusions

- Random Access Networks with Transmission Costs
- Selfish Nodes
- Price-Based Rate Control
  - Operating Point
  - Delay and Throughput Differentiation
  - End-to-End Rate Control

Related Work

- Selfish Users - Retransmission Probabilities
  - MacKenzie and Wicker
- Selfish Users - Experimental
  - Altman, El Azouzi, Jimenez
- Cheating
  - Raja, Hubaux, Aad

Related Work

- Price-Based Rate Control
  - Frank Kelly, Steven Low,.....
- Rate Control and Slotted Aloha
  - Kleinrock and Lam
  - Mittal and Venetsanopoulos
- TCP over 802.11
  - Cali *et al.*
- Price-Based Rate Control for Random Access Networks
  - Jin and Kesidis
  - Battiti *et al.*