Pricing Differentiated Services: Stability and Fairness

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"A Business Model for Internet Services"
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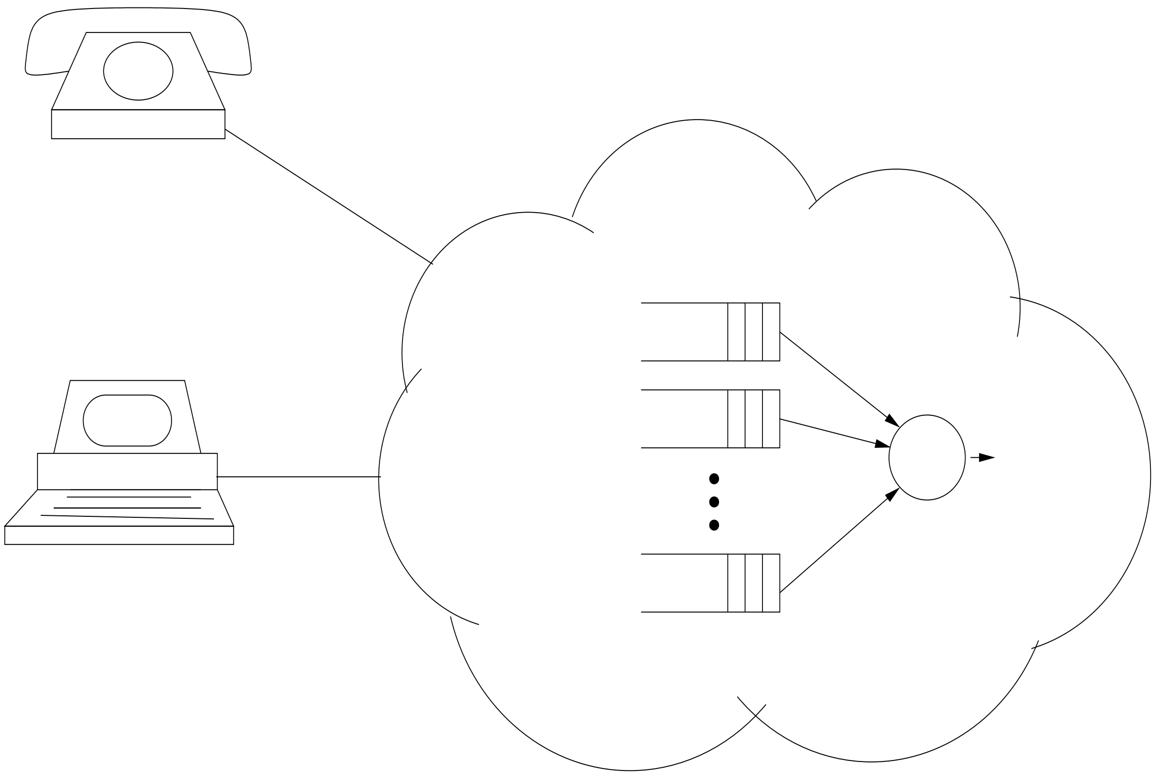
QoS Through Priority Service



- Priority Levels: $1, ..., N$

- Price: u_i per Submitted Packet with Priority i

- Non-Cooperative Game

"Utility - Cost"

Questions

- Stability
  - Equilibrium?
  - Pricing?

- “Fairness”

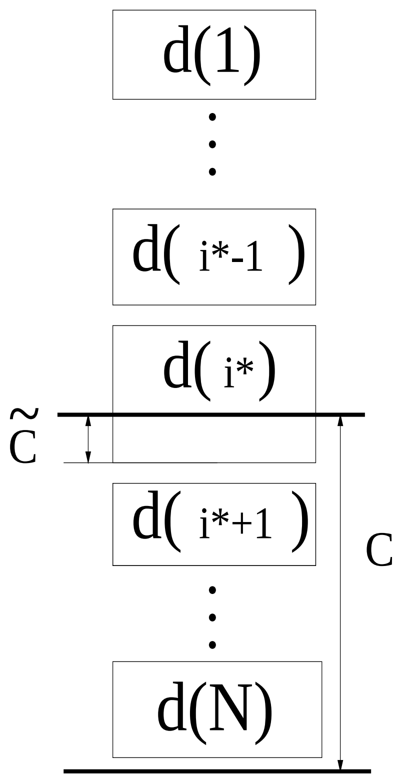
Stability: Single Link Case

- Equilibrium Exists

- Pricing as a Tool to Control Congestion

Link Model

- Slotted Time, Capacity C, No Buffer
- $d = (d(1), d(2), ..., d(N))$



$$P_{tr}(i,d) = 0, \qquad i < i^*$$

$$P_{tr}(i^*,d) = \frac{\tilde{C}}{d(i^*)}, \qquad i = i^*$$

$$P_{tr}(i,d) = 1, \qquad i > i^*$$

Elastic Traffic

- Care only about Throughput

- Customers $1, ..., R$

- Utility:

$$U_r(d_r, P_{tr}(d_r + d_{-r})) = \bar{U}_r(x_r),$$

$$x_r = \sum_{r=1}^{N} P_{tr}(i, d_r + d_{-r}) d_r(i)$$

- $\bar{U}_r$: strictly concave and "smooth"

- Demand $D_r(u)$:

$$\max_{x \geq 0} \bar{U}(x) - xu, \qquad u \geq 0$$

Non-Cooperative Game

- Customer's Problem:

$$\max_{y_r \in \Re_+^N} \left\{ U_r(y_r, P_{tr}(y_r + d_{-r})) - \sum_i y_r(i) u_i \right\}$$

- "Local Nash Equilibrium" $(d_1, ..., d_R)$
  - Equilibrium Exists
  - $P_{tr}(i^*, d) \geq \frac{u_{i^*}}{u_{i^*+1}}$
  - $\sum_r x_r = C$

Competitive Price-Taking Assumption

- Do Not Anticipate Changes in P_{tr}
  - Nash Equilibrium Exists
  - $P_{tr}(i^*) \geq \frac{u_{i^*}}{u_{i^*+1}}$
  - $x_r = D_r(u^*), \qquad u_{i^*} \leq u^* \leq u_{i^*+1}$
  - $\sum_r x_r = C$
- "Fairness"

Quality of Service

- Utility:

$$\bar{U}_r(x_r) - \sum_{i=1}^{N} c_r\left(P_{tr}(i,d)\right) d_r(i)$$

- Competitive Price-Taking Assumption
  - Nash Equilibrium Exists
  - $P_{tr}(i^*) \geq \frac{u_{i^*}}{u_{i^*+1}}$
  - $D_r(u_{i^*+1}) \leq x_r \leq D_r(u_{i^*})$
  - $\sum_r x_r = C$

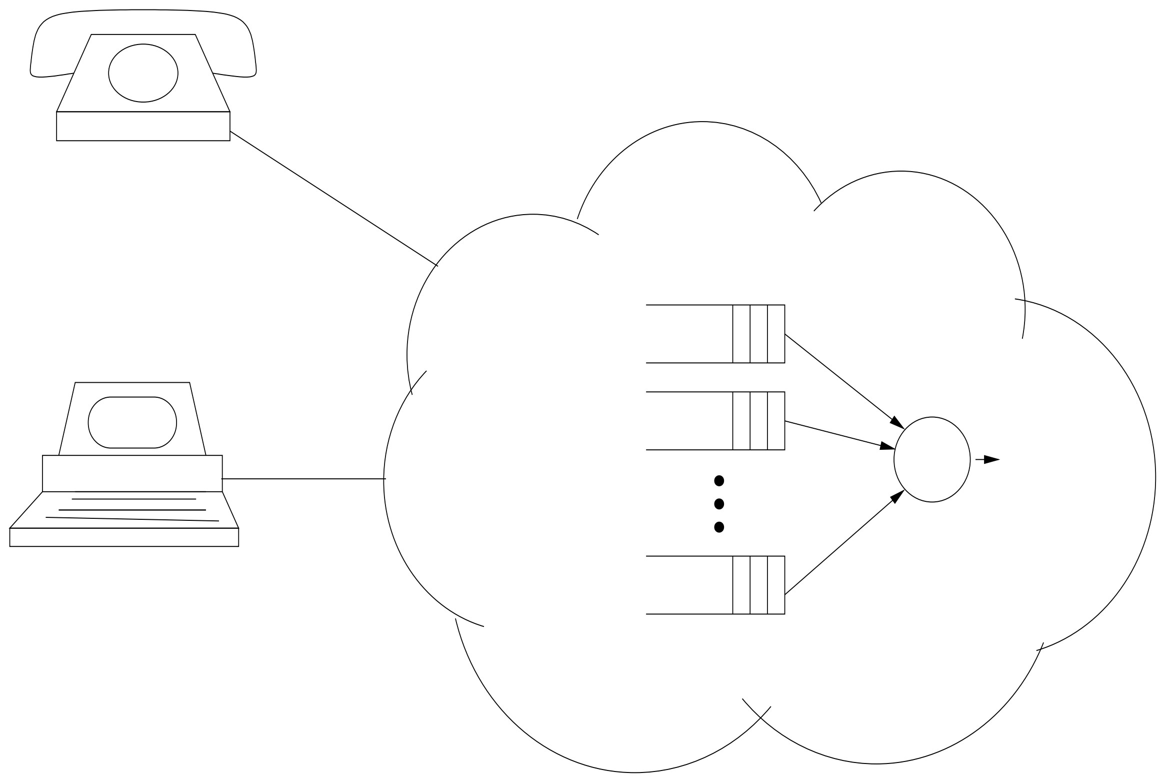
- Differentiated QoS

Fairness: “Idealized” Service

- Unique Equilibrium

- Pricing is “Min-Max” Fair

“Idealized” Priority Service



- Priorities: $\mathcal{I} = [u_{\min}, \infty)$

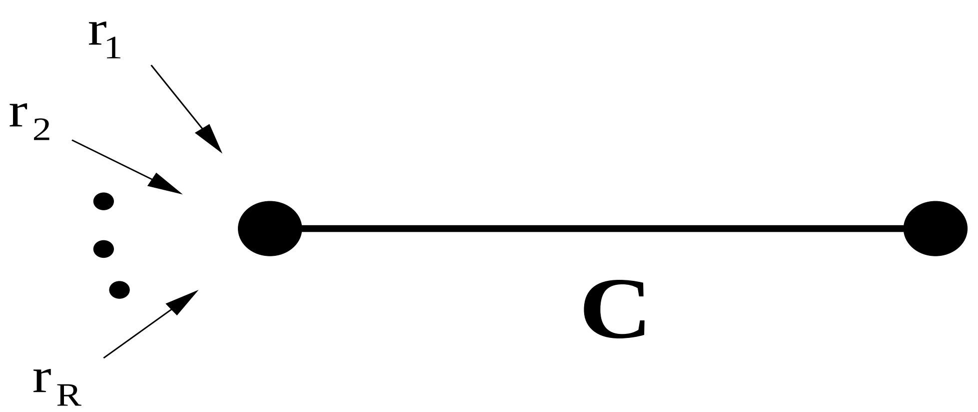
- Price: u per Submitted Packet with Priority u

- Non-Cooperative Game

  “Utility - Cost”

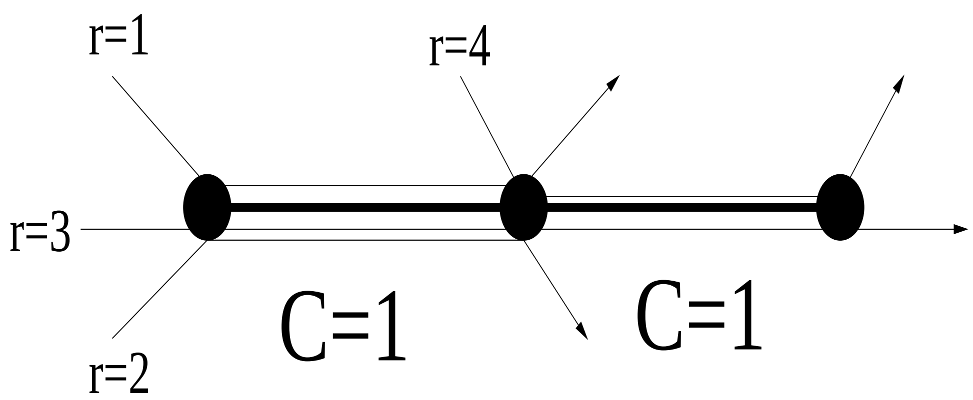
"Max-Min" Fair Rate Allocation

- Single Link



$$x_r = \frac{C}{R}$$

- Network

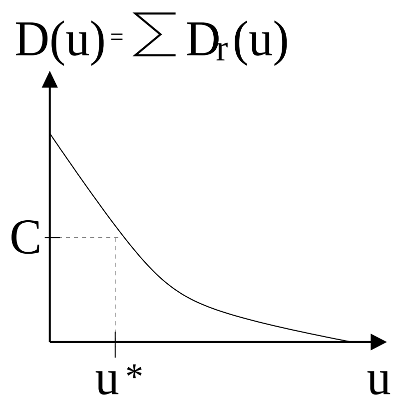


$$x_1 = x_2 = x_3 = \frac{1}{3}, \quad x_r = \frac{2}{3}$$

- **Max**imize the rate of the customer with the **min**imal rate, subject to the capacity constraints.

“Min-Max” Fair Pricing

- Single Link



$$x_r = D_r(u_r) = D_r(u^*)$$

- Network:
  Minimize the price of the customer with the **max**imal price, subject to the capacity constraints.

Summary

- Priority Service with Pricing
  - Pricing to Control Congestion
  - Differentiated QoS
  - Min-Max Fair Pricing
- Link Model
- Algorithm for Choosing d_r

Related Work

- A. Odlyzko:
  Paris Metro Pricing

- A. Gupta/D. Stahl/A. Whinston:
  Dynamic Pricing

- R. Cocchi/S. Shenker/D. Estrin/L. Zhang:
  Experimental Results

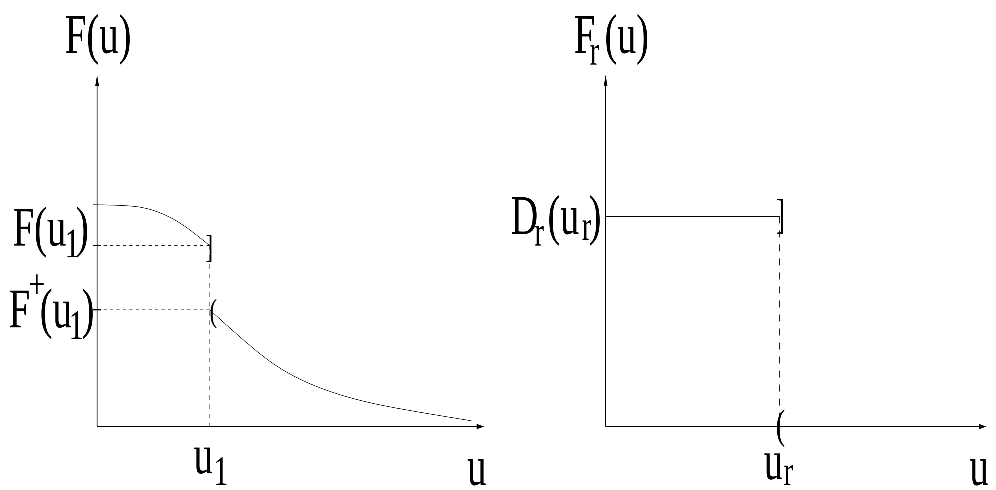
- S. Chen and K. Park:
  Cost only Performance Related

- J. MacKie-Mason and H. Varian:
  Smart Market

- F. Kelly, A. Maulloo, D. Tan:
  Proportional Fairness

Mathematical Model

- Single Link
  Aggregate Allocation: $F(u) = \sum_{r=1}^{R} F_r(u)$.



Transmission Probabilities: $P_{tr}(u, F)$

- Network

  - Network: $l \in \mathcal{L}$, C_l, H_l

  - Route: $(l_{r,1}, ..., l_{r,K_r})$

  - Allocation: $\underline{F} = (F_1, ..., F_R)$

  - Link Trans. Prob.: $P_{tr,l}(u, \underline{F})$

  - Trans. Prob.:

$$P_{tr,r}(u, \underline{F}) = \Pi_{k=1}^{K_r} P_{tr,l_{r,l_k}}(u, \underline{F})$$

- System Of Equations

$$
\begin{aligned}
F_{r,l_{r,1}}(u,\underline{F}) &= F_r(u) \\
F_{r,l_{r,k}}(u,\underline{F}) &= \int_u^\infty P_{tr,l_{r,k-1}}(v,\underline{F})(-dF_{r,l_{r,k-1}}(v)), \quad k = 2,...,K_r \\
P_{tr,l}(\underline{F}) &= P_{tr}\left(\sum_{r\in H_l} F_{r,l}(\underline{F})\right), \quad l \in \mathcal{L}.
\end{aligned}
$$