Transmission Costs, Selfish Nodes, and Protocol Design

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***Abstract*— We study how selfish nodes react to transmission costs in wireless networks. Intuitively, it seems that transmission costs should have a stabilizing effect as (rational) nodes will defer packet transmissions when congestion develops and the cost for (successfully) transmitting a packet becomes high. In this paper we investigate whether this intuition is true. We use slotted Aloha to model the communication channel.**

I. Introduction

An important feature of wireless networks is that packet transmissions incur a cost in terms of battery energy. In this paper, we study how this cost affects the way nodes access the network. Intuitively, it seems that transmission costs should have a stabilizing effect as nodes will defer packet transmissions when congestion occurs and the cost for (successfully) transmitting a packet becomes high. In this paper we investigate this intuition and its implication for the design of protocols. For our analysis, we focus on random access networks using slotted Aloha [1]; however our results can easily be extended to random access networks using CSMA or CSMA/CD.

We assume that (a) nodes obtain a reward for successfully transmitting a packet (where different packets can have different rewards) and (b) each transmission attempt incurs a cost (which is the same for all packets). Nodes can decide (a) when to accept a new packet and (b) when to retransmit a backlogged packet. We assume that nodes are rational and make these decisions in order to maximize their net benefit. We model the interaction among nodes as a non-cooperative game, and study the existence and properties of a (symmetric) Nash equilibrium.

The motivation for studying the above situation is to obtain a better understanding of protocol design for wireless networks. In particular, we are interested in the following questions: (a) is the existence of a transmission cost sufficient to guarantee stability, (b) if this is not the case, which additional mechanisms are necessary, and (c) how can these mechanisms be used to optimize system performance. The hope is that studying these questions contributes toward the design of simple and robust protocols.

In our main results, we show that the existence of a transmission cost is not always sufficient to guarantee stability. In particular, a stable equilibrium strategy will not exist if the transmission cost is small. This result suggests that an additional mechanism is required to guarantee stability. We consider such a mechanism where nodes are charged a price (cost) for each successfully transmitted packet. In the case of a wireless LAN, this cost could be charged at the base-station. We show that such pricing mechanism can be used to guarantee system stability. In addition, it can be used to optimize the system performance in terms of throughput and delay.

The rest of the paper is organized as follows. In Section II, we define the non-cooperative game that we consider and the notion of a stable equilibrium strategy. In Section IV, we then study the existence of a stable equilibrium strategy; in particular, we show that there does not exist a stable equilibrium strategy if the cost for transmitting a packet is too small. In Section V, we discuss the implication of this result on protocol design and consider a pricing mechanism to guarantee stability and optimize system performance. We discuss related work in Section VII.

II. Problem Formulation

Consider a random access network using slotted Aloha [1] where each slot has the duration of one time unit, which is equal to the time it takes to transmit a packet (all packets have the same length). At the beginning of slot k, $k \geq 0$, active nodes (*i.e.* nodes which received a new packet in slot $k-1$ or have a backlogged packet at the beginning of slot k) learn the number of nodes that currently have a backlogged packet. Nodes which received a new packet in slot $k-1$ can decide whether to accept or drop the packet. If a new packet is accepted, then it is immediately transmitted in the next slot k. Nodes with a backlogged packet can decide whether or not to retransmit it in slot k. A packet is successfully transmitted if it is the only packet being transmitted during a slot; if two or more packets are transmitted during the same slot then these packets collide and are not successfully transmitted. Packets that experienced a collision become backlogged. We assume that each packet transmission incurs a cost $\gamma > 0$ (which is the same for all nodes and packets). When a new packet is accepted, the node will stay active until it has successfully transmitted the packet; when the packet has been successfully transmitted, then the node receives a reward (where different packets can have different rewards) and leaves the system. Let u denote that value (reward) of a packet. A node which received a new packet and decided to drop it, immediately leaves the system. We assume an infinite set of nodes and an active node does not receive any additional packets (*i.e.* each active nodes has exactly one packet) [1].

The number of nodes that become active during a time slot is given by a independent Poisson random variable with parameter λ_0. The probability that a packet has a value equal to or greater than u is given by $p(u)$, and we let $\lambda(u) = p(u)\lambda_0$, $u \geq 0$. Note that $\lambda(u)$ characterizes the rate at which packets with value equal to or higher than u are generated.

Assumption 1: We assume that the function $\lambda(u)$, $u \geq 0$, is bounded, continuous and non-increasing, and we have $\lim_{u\to\infty} \lambda(u) = 0$.

A. Node Strategies

Knowing the number of nodes with a backlogged packet, each active node can decide on (a) the minimal value a new packet must have in order to be accepted and (b) the probability for retransmitting a backlogged packet. A rational node will accept a new packet only if the expected cost for successfully transmitting the packet is less than or equal to the value of the packet. A node strategy π is then given by the vector pair $\pi = (u, q)$ where $u = (u(0), u(1), ..., u(n), ...)$ and $q = (q(1), q(2), ..., q(n), ...)$. Given the current state n, $u(n)$ indicates the minimal value a new packet must have in order to be accepted and $q(n)$ indicates the probability for retransmitting a backlogged packet. Naturally we have $u(n) \geq 0$ and $q(n) \in [0, 1]$. Note that we allow nodes to retransmit backlogged packets with probability 1 or 0. Abusing the notation, we will also identify a strategy π by $\pi = (\lambda, q)$ where $\lambda(n) = \lambda(u_n)$, $n \geq 0$.

B. Markov Chain Formulation

To simplify the analysis, we assume that nodes are indistinguishable and focus on the situation where all nodes use the same strategy. In this case, a strategy π defines a Markov chain $(n_k; k \geq 0)$ on the state space $\mathcal{N} = \{n \geq 0\}$, where n_k indicates the number of nodes that have a backlogged packet at the beginning of a slot k. The state transition probabilities of the Markov chain are given by

$$P_{n,n'}(\pi) = \begin{cases} e^{-\lambda(n)} nq(n)\big(1-q(n)\big)^{n-1}, & n' = n-1, \\ e^{-\lambda(n)}\Big[\lambda(n)\big(1-q(n)\big)^{n} + ... \\ +\big[1 - nq(n)\big(1-q(n)\big)^{n-1}\big]\Big], & n' = n, \\ \lambda(n)e^{-\lambda(n)}\Big[1 - \big(1-q(n)\big)^{n}\Big], & n' = n+1, \\ \dfrac{e^{-\lambda(n)}\lambda(n)^k}{k!}, & n' = n+k, \\ & k \geq 2, \\ 0, & \text{otherwise.} \end{cases}$$

To simplify notation, we are going to use the (Poisson) approximation that $(1 - q(n))^n \approx e^{-nq(n)}$ (see [1] for a derivation), and obtain the following transition probabilities

$$P_{n,n'}(\pi) = \begin{cases} nq(n)e^{-\lambda(n)-(n-1)q(n)}, & n' = n-1, \\ -\lambda(n)e^{-\lambda(n)-nq(n)} + e^{-\lambda(n)} - ... \\ nq(n)e^{-\lambda(n)-(n-1)q(n)}, & n' = n, \\ \lambda(n)e^{-\lambda(n)}\left[1 - e^{-nq(n)}\right], & n = n+1, \\ \dfrac{e^{-\lambda(n)}\lambda(n)^k}{k!}, & n' = n+k, \\ & k \geq 2, \\ 0, & \text{otherwise.} \end{cases}$$

Let

$$G(n) = \lambda(n) + nq(n), \qquad n \in \mathcal{N},$$

be the offered load at state n [1]. The probability an active node leaves the system at state n (by successful transmitting either a new packet or a backlogged packet) is equal to $G(n)e^{-G(n)}$ [1]. Furthermore, note the following. Given that a node transmits a backlogged packet at state n, the probability that this packet is successfully transmitted is equal to $e^{-\lambda(n)-(n-1)q(n)}$, *i.e.* a retransmission attempt of a backlogged packet by a given node is successful if no other nodes makes a transmission attempt (of either a new packet or a backlogged packet).

C. Expected Total Transmission Costs

Next, we consider the Markov reward process indicating the expected cost at this node for successfully transmitting the packet. More precisely, suppose that a given node has a backlogged packet. Furthermore, suppose that the given node retransmits packets with probability $\hat{q}(n)$, $n \geq 1$, and all other nodes use a given strategy π for accepting new packets and retransmitting backlogged packets. To compute the expected cost $Q(n, \pi, \hat{q})$ for the node to successfully transmit the backlogged packet starting from state n when all other nodes use strategy π, we introduce a termination state T and consider the Markov chain with transition probabilities

$$P^T_{n,n'}(\pi, \hat{q}) = \begin{cases} (n-1)q(n)e^{-\lambda(n)-(n-2)q(n)-\hat{q}(n)}, \\ \qquad\qquad n' = n-1, \\ \lambda(n)e^{-\lambda(n)-(n-1)q(n)-\hat{q}(n)} + e^{-\lambda(n)} - ... \\ -(n-1)q(n)e^{-\lambda(n)-(n-2_q(n)-\hat{q}(n)} - ... \\ -\hat{q}(n)e^{-\lambda(n)-(n-1)q(n)}, \qquad n' = n, \\ \lambda(n)e^{-\lambda(n)}\Big[1 - e^{(n-1)q(n)-\hat{q}(n)}\Big], \\ \qquad\qquad n' = n+1, \\ \dfrac{e^{-\lambda(n)}\lambda(n)^k}{k!}, \qquad n' = n+k, \\ \qquad\qquad k \geq 2, \\ 0, \quad \text{otherwise.} \end{cases}$$

and

$$P^T_{n,T} = \hat{q}(n)e^{-\lambda(n)-(n-1)q(n)}.$$

Note that $P^T_{n,T}$ is equal to the probability that the node which uses the retransmission vector $\hat{q}$ is able to successfully retransmit the backlogged packet. Let $P^T_{T,T} = 1$. Furthermore, we associate with each state a cost as follows. For $n \in \mathcal{N}$, let the cost $c(n)$ be given by

$$c(n) = \gamma\hat{q}(n).$$

Note that $c(n)$ is the expected transmission cost at state n. For T, the cost $c(T)$ is equal to 0. Note that the above definitions imply that T is a cost-free termination state.

Let $(s_k; k \geq 0)$ be the Markov chain with transition probabilities given by $P^T(\pi, \hat{q})$. We then have

$$Q(n, \pi, \hat{q}) = E_T\left[\sum_{k=0}^{\infty} c(s_k) \mid s_0 = n\right],$$

where the expectation E_T is taken with respect to the transition probabilities $P^T(\pi, \hat{q})$.

The expect transmission cost $R(n, \pi, \hat{q})$ for a packet that is accepted at state n by a node which uses the retransmission vector $\hat{q}$ when all other nodes use strategy π is then given by

$$\begin{aligned} R(n, \pi, \hat{q}) = & \\ & \gamma + \frac{\lambda(n) e^{\lambda(n)}}{1 - e^{-\lambda(n)}} \left(1 - e^{-nq(n)}\right) Q(n+1, \pi, \hat{q}) + ... \\ & + \sum_{k=2}^{\infty} \frac{\frac{\lambda(n)^k}{k!} e^{\lambda(n)}}{1 - e^{-\lambda(n)}} Q(n+k, \pi, \hat{q}). \end{aligned}$$

For a strategy $\pi = (\lambda, q)$ we use the following notation,

$$Q(n, \pi) = Q(n, \pi, q) \qquad \text{and} \qquad R(n, \pi) = R(n, \pi, q).$$

D. Symmetric Nash Equilibrium and Stable Strategies

Once a node has accepted a packet, we require that the node makes a "proper" effort transmit it; *i.e* nodes are not allowed to delay re-transmission attempts indefinitely and we require that the time between retransmission attempts of a given backlogged packet is finite (with probability 1).

Consider the situation where a given node has a backlogged packet and uses the retransmission vector $\hat{q}$ to retransmit the backlogged packet (where $\hat{q}(n)$ is the probability that the given node will make a retransmission attempt at state n), and suppose that all other nodes use a given strategy π. Starting at the state n, let $T_n(\pi, \hat{q})$ be the first time that the given node (which uses the retransmission vector $\hat{q}$) will make a retransmission attempt of the backlogged packet.

Definition 1: Given a strategy π, we call a retransmission vector $\hat{q}$ admissible with respect to π if for all states $n \geq 1$ we have that $T_n(\pi, \hat{q})$ is a random variable.

Given a strategy π, let $\mathcal{Q}(\pi)$ be the set of all admissible retransmission probability vectors $\hat{q}$ with respect to π.

Definition 2: We call a strategy π admissible if $\lambda(0) > 0$ and $q \in \mathcal{Q}(\pi)$.

Using the above notation, we define an equilibrium strategy as follows.

Definition 3: We call a admissible strategy π an equilibrium strategy if for all $n \geq 0$ we have $q = \arg\min_{\hat{q} \in \mathcal{Q}(\pi)} Q(n, \pi, \hat{q})$ and $\lambda(n) = \lambda(R(n, \pi))$.

The above definition states that an equilibrium strategy π minimizes the expected total retransmission cost for an accepted packet, and only accepts a new packet if its value is equal to or larger than the expected total transmission cost, *i.e.* $u_n = R(n, \pi)$ or equivalently $\lambda(n) = \lambda(R(n, \pi))$.

Note that an equilibrium strategy π defines a symmetric Nash equilibrium: if all nodes adopt strategy π then no node has an incentive to deviate from π as the strategy π maximizes at each state $n \in \mathcal{N}$ the net benefit given by reward minus (expected) cost.

Besides admissible strategies, we are also interested in strategies which guarantee that the expected number of backlogged packets in the system stays bounded. We use the following criteria.

Definition 4: We call a strategy π stable if there exists $\epsilon > 0$ and a integer N_0 such that

$$\lambda(n) - G(n) e^{-G(n)} < -\epsilon, \qquad n \geq N_0.$$

The following lemma states that an equilibrium strategy leads to a well-behaved system.

Proposition 1: If π is a stable equilibrium strategy, then the corresponding Markov chain has a single positive recurrent class and possibly some transient states.

We provide a proof for the above result in Appendix B

In the following, we are interested in the existence of a stable equilibrium strategy.

III. A Particular Class of Strategies

To study the existence of a stable equilibrium strategy we consider the following class of strategies. Recall the definition of a class (of states) for a Markov chain [2].

Definition 5: For $\kappa > 0$, let $\mathcal{F}_\kappa$ be the set of all admissible strategies $\pi = (\lambda, q)$ with property that there exists a class $\mathcal{N}_c(\pi)$ of the form $\mathcal{N}_c(\pi) = \{n \geq N_0\}$ such that

$$\lambda(n) + (n-1) q(n) = \kappa,$$

for all states $n \in \mathcal{N}_c(\pi)$ with $q(n) > 0$.

Note that if state n is in the class $\mathcal{N}_c(\pi)$ then state n will only communicate with other states $n' \in \mathcal{N}_c(\pi)$ [2].

In the next section, we will show that the collection $\mathcal{F}_\kappa$, $k > 0$, is sufficient to characterize a stable equilibrium strategy. In addition, we derive necessary and sufficient conditions for the existence of a stable equilibrium strategy $\pi \in \mathcal{F}_\kappa$. Before we proceed, we introduce additional notation that we will use to characterize a stable equilibrium strategy $\pi \in \mathcal{F}_\kappa$.

Note that for a states $n \in \mathcal{N}_c(\pi)$ with $q(n) > 0$ the probability that a given retransmission attempt of a backlogged packet is successful is equal to $e^{-\kappa}$. As by definition a strategy $\pi \in \mathcal{F}_\kappa$ is admissible, we have

$$Q(n, \pi) = \gamma e^\kappa, \qquad n \in \mathcal{N}_c(\pi).$$

For a given strategy $\pi \in \mathcal{F}_\kappa$, the cost for successfully transmitting a packet that was accepted at a state $n \in \mathcal{N}_c(\pi)$ is then given by

$$R(n, \pi) = \gamma + \left(1 - \frac{\lambda(n)}{1 - e^{-\lambda(n)}} e^{-\lambda(n) - nq(n)}\right) \gamma e^\kappa.$$

Using the fact that $\lambda(n) + (n-1)q(n) = \kappa$, we obtain for $n \in \mathcal{N}_c(\pi)$ that

$$R(n, \pi) = \gamma + \left(1 - \frac{\lambda(n)}{1 - e^{-\lambda(n)}} e^{-\lambda(n) - \frac{n}{n-1}(\kappa - \lambda(n))}\right) \gamma e^\kappa.$$

For $r \geq 0$, consider the function

$$f_n(r) = \gamma + \left(1 - \frac{\lambda(r)}{1 - e^{-\lambda(r)}} e^{-\lambda(r) - \frac{n}{n-1}(\kappa - \lambda(r))}\right) \gamma e^\kappa.$$

If π is an equilibrium strategy, then we have

$$\lambda(n) = \lambda(R(n, \pi)),$$

and $R(n, \pi)$ is a solution to the equation

$$r = f_n(r).$$

Next consider the function

$$f_\infty(r) = \lim_{n\to\infty} f_n(r) = \gamma + \left(1 - \frac{\lambda(r)}{1-e^{-\lambda(r)}}e^{-\kappa}\right)\gamma e^{\kappa}.$$

In Appendix D, we show that the equation

$$r = f_\infty(r)$$

has a solution $r \geq 0$ if and only if

$$\max_{r\geq 0}(f_\infty(r) - r) \geq 0.$$

If the above equation has at least one solution $r \geq 0$, then we define

$$r_\infty = \max\{r \geq 0; f_\infty(r) = r\}.$$

It can be shown that when there exists a $r \geq 0$ such that $r = f_\infty(r)$, then r_∞ is well defined. In Appendix D, we show that if the equation $r = f_\infty(r)$ has a solution and $\lambda(r_\infty) < \kappa$, then there exists a solution $r_n \geq 0$ to the equation

$$r = f_n(r), \qquad n \geq 2.$$

Furthermore, it can be shown that if $\max_{r\geq 0}(f_\infty(r) - r) < 0$ then there does not exists an equilibrium strategy $\pi \in \mathcal{F}_\kappa$. This implies that the existence of a solution to $r = f_\infty(r)$ and $\lambda(r_\infty) < \kappa$ are a necessary conditions for the existence of an equilibrium strategy $\pi \in \mathcal{F}_\kappa$.

Finally, we define the function

$$f_0(r) = \gamma + \left(1 - \frac{\lambda(r)}{1-e^{-\lambda(r)}}e^{-\lambda(r)}\right)\gamma e^{\kappa}, \qquad r \geq 0.$$

In Appendix D, we show that the equation

$$r = f_0(r)$$

always has a unique solution $r \geq 0 \geq 0$; let r_0 be the solution to the above equation. The value r_0 has the following interpretation. Let n_0 be given by

$$n_0 = \min\{n \in \mathcal{N}_c(\pi)\}.$$

In Appendix D we show that $q(n_0) = 0$ and

$$R(n_0, \pi) = \gamma + \left(1 - \frac{\lambda(n_0)}{1-e^{-\lambda(n_0)}}e^{-\lambda(n_0)}\right)\gamma e^{\kappa}.$$

This implies that when π is an equilibrium strategy then we have

$$\lambda(n_0) = \lambda(r_0).$$

IV. Existence of a Stable Equilibrium Allocation

Using the above definition, we obtain the following necessary and sufficient conditions for the existence of a stable strategy $\pi \in \mathcal{F}_\kappa$ as follows.

Proposition 2: There exists a stable equilibrium strategy $\pi \in \mathcal{F}_\kappa$ if and only if the following conditions hold

(a) $\max_{r\geq 0}(f_\infty(r) - r) \geq 0$,
(b) $\lambda(r_\infty) < \kappa e^{-\kappa}$, and
(c) $\lambda(r_0) \geq \kappa$.

We provide a proof in Appendix E. Let us briefly comment on the above conditions. As discussed above, condition (a) is a necessary condition for the existence of an equilibrium strategy. Condition (b) is a necessary condition for the existence of a stable equilibrium strategy. Condition (c) has the following interpretation. As noted above, one can show that if $\pi \in \mathcal{F}_\kappa$ is an equilibrium strategy then we have $q(n_0) = 0$ and $\lambda(n_0) = \lambda(r_0)$. If

$$e^{-\lambda(n_0)} = e^{-\lambda(r_0)} > e^{-\kappa}$$

then the probability that no node attempts to transmit a packet is larger at state n_0 than at all other states $n \in \mathcal{N}_c(\pi)$ and it is more economical for a node to retransmit a backlogged packet at stage n_0 than at any other state $n \in \mathcal{N}_c(\pi)$; hence π with $q(n_0) = 0$ is not an equilibrium strategy.

The next result states that is is sufficient to consider the the collection of sets $\mathcal{F}_\kappa$, $\kappa > 0$, to characterize stable equilibrium strategies.

Proposition 3: If π is a stable equilibrium strategy then there exists a $\kappa > 0$ such that $\pi \in \mathcal{F}_\kappa$.

We provide a proof for the above proposition in Appendix C.

Combining Proposition 2 and 3, it follows that there does not always exist a stable equilibrium strategy. In particular, we have the following result.

Corollary 1: If $\sup_{\kappa>0}\{\kappa e^{-\kappa} - \lambda(\gamma e^{\kappa})\} \leq 0$, then there does not exist a stable equilibrium strategy.

Furthermore, one can show that if there exists a stable equilibrium strategy, then there typically exists a continuum of stable equilibrium strategies. These results suggest that a transmission cost is not sufficient to guarantee stability (existence of a stable equilibrium strategy) and to able to predict the system performance (existence of a unique stable equilibrium strategy).

V. Protocol Design

In this section, we discuss the implications of the above results.

A. Stability

Corollary 1 implies that transmission costs are not always sufficient to guarantee stability. In particular, a stable equilibrium strategy will not exist if the transmission cost γ is too small. This result suggest that an additional mechanism is required to achieve stability. Here, we consider such a mechanism where nodes are charged a price (cost) $c > 0$ for each successfully transmitted packet. In the case of a wireless LAN, this cost could be charged at the base-station. Note that nodes are only charged for successfully transmitted packets, but not for each transmission attempt. For a strategy $\pi \in \mathcal{F}_\kappa$, the cost for successfully transmitting a packet accepted at a state $n \in \mathcal{N}_c(\pi)$ is then given by

$$R^c(n,\pi) = c + \gamma + \left(1 - \frac{\lambda(n)}{1-e^{-\lambda(n)}}e^{-\lambda(n)-nq(n)}\right)\gamma e^{\kappa}.$$

Using the notation of Section III, we define the function $f^c_\infty(r)$ by

$$\begin{aligned} f^c_\infty(r) &= c + \gamma + \left(1 - \frac{\lambda(r)}{1-e^{-\lambda(r)}}e^{-\kappa}\right)\gamma e^{\kappa} \\ &= c + f_\infty(r). \end{aligned}$$

If there exists a $r > 0$ such that $r = f^c_\infty(r)$, let

$$r^c_\infty = \max\{r \geq 0; r = f^c_\infty(r)\}.$$

Furthermore, we define the function $f^c_0(r)$ by

$$f^c_0(r) = c + \gamma + \left(1 - \frac{\lambda(r)}{1 - e^{-\lambda(r)}} e^{-\lambda(r)}\right) \gamma e^\kappa.$$

One can show that there always exists a unique $r^c_0 \geq 0$ such that $r^c_0 = f^c_0(r)$.

We have the following result.

Proposition 4: For every $\kappa > 0$, there exists a price $c > 0$ such that

(a) $\max_{r \geq 0}(f^c_\infty(r) - r) > 0$, and
(b) $\lambda(r^c_\infty) < \kappa e^{-\kappa}$.

The above result states that we can always satisfy the first two condition of Proposition 2 by choosing the cost c hight enough. We omit here a proof for Proposition 4.

The above result can be used to construct an equilibrium strategy as follows. Choose the cost $c > 0$ such that condition (a) and (b) in Proposition 4 are satisfied. If this is the case, using the same argument as given in Appendix D it can be shown that for $n \geq 2$ there exists a $r_n \geq 0$ such that

$$r_n = f^c_n(r_n) = c + f_n(r_n)$$

and $\lambda(r_n) < \kappa$. Using this value r_n, $n \geq 2$, consider the following strategy $\pi = (\lambda, q) \in \mathcal{F}_\kappa$ as follows. Let $\lambda(n) = \lambda(r_n)$ and $\lambda(0) = \lambda(1) = \lambda(r^c_0)$; and let $q(1) = 0$ and

$$q(n) = \frac{\kappa - \lambda(n)}{n - 1}, \qquad n \geq 2.$$

If $\lambda(r^c_0) \geq \kappa$, then using the same argument as given in Appendix E to prove Proposition 2 one can show that this strategy π is indeed a stable equilibrium strategy. If $\lambda(r^c_0) < \kappa$, then (by the same argument as given in Section IV) it is more economical for a node to retransmit a backlogged packet at stage $n = 1$ than at any other state $n \geq 2$, and π with $q(1) = 0$ is not an equilibrium strategy. To overcome this problem, we can use the following approach. In order to make state $n = 1$ "unattractive" for retransmitting a backlogged packet (and make $q(1) = 0$ indeed the optimal probability for retransmitting a packet a backlogged packet at state $n = 1$), the base station could jam the channel with some (small) probability q_j. More precisely, for $\delta > 0$ let the function $f^c_\delta(r)$, $r \geq 0$, be given by

$$f^c_\delta(r) = c + \gamma + \left(1 + \frac{\lambda(r)}{1 - e^{-\lambda(r)}} e^{-\kappa-\delta}\right) \gamma e^\kappa.$$

If conditions (a) and (b) of Proposition 4 are stratified, then it can been shown that there exists a $r^c_\delta \geq 0$ such that $r^c_\delta = f^c_\delta(r^c_\delta)$ and $\lambda(r^c_\delta < \kappa$. If the base station jams the channel (by sending a fictitious packet) at state $n = 1$ with probability

$$q_j = \kappa + \delta - \lambda(r^c_\delta)$$

then the probability for successfully retransmitting a backlogged packet at state $n = 1$ is equal to $e^{\kappa+\delta}$ which is less than for any other state $n \geq 2$. Using this result, one can then show that the strategy π that we constructed above is indeed a stable equilibrium strategy.

It might seem that jamming the channel is too drastic a counter-measure for the case where $\lambda(r^c_0) < \kappa$, and will hurt the system performance; however this is not necessarily the case. Note that when δ is very small, then the price r^c_δ is very close to r^c_∞, and the arrival rate $\lambda(r^c_\delta)$ at state $n = 1$ is very close to the arrival rate $\lambda(n)$, $n \geq 2$. Hence, the overall system performance in terms of throughput is not significantly reduced.

Note that in order to construct a stable equilibrium strategy as outlined above, it is necessary to have exact knowledge of the rate function λ in order to pick the price c and the jamming probability q_J. Of course, in practice this is not the case. We revisit this issue in Section V-D.

B. System Performance

In the previous section, we discussed how pricing can be used to stabilize the system. Next, we study whether the above pricing mechanism can also be used to optimize system performance. As noted above, for a strategy $\pi \in \mathcal{F}_\kappa$, we have

$$\lim_{n \to \infty} G(n) e^{G(n)} = \kappa e^{-\kappa},$$

and $\kappa e^{-\kappa}$ can be used to characterize the maximal sustainable throughput of the system under a stable equilibrium strategy $\pi \in \mathcal{F}_\kappa$. It is well known that the sustainable throughput is maximized for $\kappa = 1$ and is equal to $e^{-\kappa}$. Therefore, the optimal choice of κ is $\kappa = 1$. Having decided on κ, the next step is to choose the price c. We have the following result.

Proposition 5: For every $\kappa > 0$ and every rate $\lambda_t = \lambda(r_t) < \kappa e^{-\kappa}$, there exists a price $c > 0$ such that

$$r_t = c + f_\infty(r_t).$$

The above result states that the price c can be used to set the (asymptotic) throughput λ_t of the system, and that any throughput that is less than $\kappa e^{-\kappa}$ can be attained. In particular, by setting $\kappa = 1$ any throughput $\lambda_t < e^{-1}$ can be obtained. Hence, the above pricing mechanism can be used to optimize the system performance. Note that choosing c introduces a trade-off between throughput and delay. In order to maximize the throughput we should choose λ_t close to κe^κ; however, in order to minimize the delay we should choose λ_t to be small. We note that this throughput/delay trade-off is well-known.

It is important to note that the above results can also be used to design rate control algorithms, and MAC-layer protocols, which achieve a desired performance (in terms and throughput and delay) in the case where nodes cooperate and are not selfish. In this case, the "cost" c has the interpretation of a congestion control signal.

C. Standardization of the MAC-Layer Protocol

As noted in Section IV, if there exists a stable equilibrium strategy then there typically exist a continuum of stable equilibrium strategies which might lead to very different systems performances in terms throughput and delay.Therefore, the choice of a particular equilibrium strategy is important.

However, nodes can not unilaterally influence the choice of a particular equilibrium strategy, and nodes would have to agree a priori on a equilibrium strategy in order to achieve a desired system performance; clearly this is not practical. Alternatively, nodes could relay on an "institution" to choose the parameter κ associated with an equilibrium strategy (see previous section) and standardize the MAC layer protocol accordingly (note that κ can be set by suitably defining the retransmission probabilities $q(n)$). Standardizing the MAC protocol (either through an institution or some other mechanism) is necessary (to guarantee stability) and desirable (to obtain good system performance), and is in the interest of rational nodes (to obtain good system performance). Furthermore, one can show that a MAC-layer protocol chosen as outlined in Section V-A will implement a unique stable equilibrium strategy. This has the following important implications. As π is an equilibrium strategy, no node has an incentive to deviate from it; hence the protocol is robust toward "cheating". Furthermore, as the equilibrium strategy is unique, one can predict (and therefore optimize) the system performance as outlined in Section V-B.

D. *Imperfect Information*

The results of Section V-B states that it is possible to achieve a desired system performance by suitably choosing c and κ. However, this requires a priori knowledge of the rate function λ and the transmission cost γ, which is unrealistic. In addition, we assumed that nodes have perfect system information and know the total number of currently backlogged packets. Again, this is unrealistic. A natural question that arises then in this context is whether there exists a mechanism that can be used to achieve a desired system performance (in terms of throughput and delay) without requiring (a) knowledge of the rate function λ and transmission cost γ, and (b) knowledge of the total number of currently backlogged packets. Surprisingly, this seems possible. In [3], we discuss such an approach where the system iteratively updates the cost c based on the ternary channel feedback information ("idle", "successful","collision") in order to obtain a desired system performance. However, the mechanism proposed in [3] has been derived assuming that nodes cooperate; studying it within a game-theoretic framework is future research.

VI. Delay-Sensitive Traffic

An important feature of the model used in this paper is that nodes have delay-insensitive traffic, *i.e.* when choosing the retransmission probabilities nodes are only concerned about transmission costs but not delay. The extension of the analysis to delay-sensitive traffic seems possible, but poses additional challenges that need to be carefully evaluated. We are currently carrying out this extension.

VII. Related Work

There is considerable interest in studying the behavior of selfish nodes in wireless networks. Below we highlight some of this literature. Closely related to the analysis presented here is the work by MacKenzie and Wicker in [4] which uses the same channel model as used above. However, the work in [4] assumes that the arrival rate of new nodes is fixed (*i.e.* nodes are not allowed to drop packets when the total transmission costs are high); here, the arrival rate is a function of the expected total transmission cost.

Also related to the above work, is the work of Altman et al. presented in [5]. However, [5] uses a different channel model where the number of nodes is finite, and uses numerical case studies to characterize the system performance. An advantage of the model presented in this paper is that it allows a formal analysis. In particular, the analysis presented above confirms several of the experimental results obtained in [5]; for example, it confirms the observation made in [5] that introducing a cost c can be used to improve system performance.

The issue of selfish nodes in wireless random access networks (IEEE 802.11 hotspots), and the role of MAC-layer standards, is also considered in [6] where a mechanism for dealing with "cheaters" (*i.e* nodes which do not follow the standard) is proposed and analyzed. Note that for delay-insensitive traffic, the problem of "cheaters" does not arise as the MAC-layer protocol of Section V-C implements a Nash equilibrium.

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Appendix

A. *Properties of Admissible Strategies and Equilibrium Strategies*

In this section we derive some basic properties of admissible strategies and equilibrium strategies.

Lemma 1: If π is admissible, then we have

$$\lambda(n) + q(n) > 0, \qquad n \geq 0.$$

Proof: We prove the lemma by contradiction. Suppose that there exists a state n such that $q(n) = \lambda(n) = 0$. Note that in this case we have $T_n(\pi, q) = \infty$ with probability 1 and π is not admissible. ■

Lemma 2: If π is an equilibrium strategy then we have

$$\big(1 - q(n)\big) + \lambda(n) > 0, \qquad n \geq 2.$$

Proof: Suppose that there exists a state n_0 such that $\lambda(n_0) = 0$ and $q(n_0) = 1$. Note, this implies that $Q(n_0, \pi) =$

∞ and $R(0,\pi) = \infty$ (as by assumption we have $\lambda(0) > 0$). As

$$\lim_{u\to\infty} \lambda(u) = 0,$$

it follows that

$$\lambda(0) \neq \lambda\big(R(0,\pi)\big),$$

and π is not an equilibrium strategy. ■

The above lemma states that an equilibrium strategy is deadlock-free.

B. Properties of Stable Strategies

In this section we derive properties of stable strategies that we use to prove our main results.

Lemma 3: If π is a stable equilibrium strategy, then there exists a integer N_0 such that $q(n) > 0$ for all $n \geq N_0$.

Proof: Note that for all $n \geq 1$ with $q(n) = 0$, we have $G(n) = \lambda(n)$. Therefore it follows that for n such that $q(n) = 0$, we have

$$G(n)e^{-G(n)} = \lambda(n)e^{-\lambda(n)} \leq \lambda(n).$$

If π is a stable strategy, then there exists a integer N_0 with $G(n)e^{-G(n)} > 0$. This implies that for all states $n \geq N_0$ we have $q(n) > 0$. ■

Lemma 4: If strategy π is a stable equilibrium strategy, then for every pair of states n and n', $n < n'$, we have that n' is accessible from n.

Proof: Note that the lemma is true if we have that $\lambda(n) > 0$. Therefore, suppose that $\lambda(n) = 0$ and let

$$n'' = \max\{m < n : \lambda(m) > 0\}.$$

Note that such a state n' exists as the strategy π is an equilibrium strategy and hence is admissible. Furthermore, because the strategy π is admissible, we have

$$q(m) > 0, \qquad n'' < m \leq n.$$

It then follows that that the state n'' is accessible from n. Furthermore, because $\lambda(n'') > 0$ we have that the state n' is accessible from state n''. Combining these results, it follows that state n' is accessible from n. ■

Lemma 5: If strategy π is a stable equilibrium strategy, then the corresponding Markov chain has a single class and possibly some transient states.

Proof: As π is a stable strategy, by Lemma 3 there exists a integer N_0 such that $q(n) > 0$ for all $n \geq N_0$. Combining this result with Lemma 4, all states $n, n' \geq N_0$ communicate. Also from Lemma 4, we have if $n < n'$ then state n' is accessible from n; hence all states $n \geq N_0$ are accessible from all other states. The result then follows. ■

Proposition 1 of Section II-D, states that if π is a stable equilibrium strategy then the has a single positive recurrent class and possibly some transient states. Using Lemma 5, we can use the following criteria to prove Proposition 1 [7].

Proposition 6: If strategy π is a stable equilibrium strategy, then the corresponding Markov chain has a single positive-recurrent class and possibly some transient states if and only if there exist a non-negative function $V(n)$, $n \in \mathcal{N}$, positive constants δ and b, and a finite set $\bar{\mathcal{N}} \subset \mathcal{N}$ such that

$$\begin{aligned} &\text{(i)} \quad E\big(V(n_{k+1}) - V(n_k) \mid n_k = n\big) \leq -\delta, \qquad n \notin \bar{\mathcal{N}}, \\ &\text{(ii)} \quad E\big(V(n_{k+1}) - V(n_k) \mid n_k = n\big) < b, \qquad n \in \bar{\mathcal{N}}, \end{aligned}$$

where the expectation is taken with respect to the Markov chain with transition probabilities $P(\pi)$ as given in Section II-B.

Note that when π is a stable strategy, then for the function $V(n) = n$, $n \geq 0$, we have

$$E\big(V(n_{k+1}) - V(n_k) \mid n_k = n\big) = \lambda(n) - G(n)e^{G(n)}, \quad n \geq 0,$$

where $G(n) = \lambda(n) + nq(n)$. Hence, the above criteria is satisfied as π is a stable strategy and Proposition 1 is true.

Lemma 6: If π is a stable strategy, then we have

$$\lim_{n\to\infty} q(n) = 0.$$

Proof: If π is a stable strategy, then there exists a integer N_0 such that $G(n)e^{-G(n)} - \lambda(n) > \epsilon > 0$. As

$$\lim_{G\to\infty} Ge^{-G} = 0,$$

it follows that there exists a $G_{max} < \infty$ such that for $n \geq N_0$ we have $G(n) < G_{max}$. Using this result, we obtain that for $n \geq N_0$, we have

$$q(n) = \frac{G(n) - \lambda(n)}{n} < \frac{G_{max}}{n}$$

and

$$\lim_{n\to\infty} q(n) < \lim_{n\to\infty} \frac{G_{max}}{n} = 0.$$

■

Lemma 7: If π is a stable strategy, then there exists $G_{min} > 0$ and a integer N_0 such that

$$G(n) \geq G_{min}, \qquad n \geq N_0.$$

Proof: If π is a stable strategy, then there exists a integer N_0 such that

$$G(n)e^{-G(n)} - \lambda(n) > \epsilon > 0, \qquad n \geq N_0.$$

This implies that for $n \geq N_0$, we have

$$G(n)e^{-G(n)} > \epsilon, \qquad n \geq N_0.$$

As

$$\lim_{G\to 0} Ge^{-G} = 0,$$

the result follows. ■

Lemma 8: If π is a stable strategy, then there exists $\tilde{\epsilon} > 0$ and a integer $\tilde{N}_0$ such that

$$\tilde{G}(n)e^{-\tilde{G}(n)} - \lambda(n) > \tilde{\epsilon}, \qquad n \geq \tilde{N}_0,$$

where

$$\tilde{G}(n) = \lambda(n) + (n-2)q(n), \qquad n \geq 2.$$

Proof: If π is a stable strategy, then there exists $\epsilon > 0$ and a integer N_0 such that

$$G(n)e^{-G(n)} - \lambda(n) > \epsilon, \qquad n \geq N_0.$$

Choose a positive constant δ_1 and choose G_1 such that

$$(G-\delta_1)e^{-G+\delta_1} < \frac{\epsilon}{4}, \qquad G \geq G_1.$$

Note that such a G_1 exists as

$$\lim_{G\to\infty} Ge^{-G} = 0.$$

Next consider the function $h_\delta(G)$ given by

$$h_\delta(G) = \left|Ge^{-G} - (G-\delta)e^{-G+\delta}\right|, \qquad G \in [\delta, G_1].$$

Note that for every δ the function h_δ has a maximum on $[\delta, G_1]$; let K_δ be given by

$$K_\delta = \max_{G\in[\delta,G_1]} h_\delta(G).$$

Furthermore, note that

$$\lim_{\delta\to 0} K_\delta = 0.$$

Therefore, there exists a δ_2 such that

$$\left|Ge^{-G} - (G-\delta_2)e^{-G+\delta_2}\right| \leq \frac{\epsilon}{2}, \qquad G \in [\delta_2, G_1].$$

As π is a stable strategy, by Lemma 7 there exists a $G_{min} > 0$ such that $G(n) \geq G_{min}$ for $n \geq N_0$. Now set $\delta_0 = \min\{\delta_1, \delta_2, G_{min}\}$. Combining these results, we obtain

$$\left|G(n)e^{-G(n)} - (G(n)-\delta_0)e^{-G+\delta_0}\right| \leq \frac{\epsilon}{2}, \qquad n \geq N_0.$$

Note that

$$G(n) - \tilde{G}(n) = q(n), \qquad n \geq 2.$$

By Lemma 6, we have

$$\lim_{n\to\infty} q(n) = 0,$$

and there exists a integer $\tilde{N}_0$, such that

$$G(n) - \tilde{G}(n) \leq \delta_0, \qquad n \geq \tilde{N}_0.$$

Using this result, we have for $n \geq \tilde{N}_0$ that

$$\begin{aligned} &\tilde{G}(n)e^{-\tilde{G}(n)} - \lambda(n) \\ &> G(n)e^{-G(n)} - \left|\tilde{G}(n)e^{-\tilde{G}(n)} - G(n)e^{-G(n)}\right| - \lambda(n) \\ &\geq \frac{\epsilon}{2}, \end{aligned}$$

and the result follows. ∎

Lemma 9: If π is a stable strategy, then the Markov chain with transition probabilities $P^T(\pi, q_0)$ with $q_0(n) = 0, n \geq 1$, is stable on $\mathcal{N}$.

Proof: Note that when $q_0(n) = 0$, $n \geq 1$, then $P^T_{n,T}(\pi, q_0) = 0$, $n \geq 0$. Therefore, in order to show that the Markov chain with transition probabilities $P^T(\pi, q_0)$ is stable on $\mathcal{N}$ it suffices to show that there exists $\epsilon > 0$ and a integer N_0 such that

$$\tilde{G}(n)e^{\tilde{G}(n)} - \lambda(n) > \epsilon, \qquad n \geq N_0.$$

That this is indeed the case is the statement of Lemma 6 and the result follows. ∎

For a given stable equilibrium strategy, let $\mathcal{N}_c(\pi)$ the the single recurrent class, and let the state n_0 be defined by

$$n_0 = \min\{n \in \mathcal{N}_c(\pi)\}.$$

Lemma 10: We have $q(n_0) = 0$.

Proof: If this is not true, then the state $n' = n_0 - 1$ is accessible from n_0. By Lemma 4 we have that all states $n \in \mathcal{N}_c(\pi)$ are accessible from n' and hence n' belongs also to the set $\mathcal{N}_c(\pi)$ contradicting the fact that $n_0 = \min\{n \in \mathcal{N}_c(\pi)\}$. ∎

C. Proof of Proposition 3

Proposition 3 is equivalent to the statement that if π is a stable equilibrium strategy and $\mathcal{N}_c(\pi)$ is its single recurrent class, then there exists $\kappa > 0$ such that

$$\lambda(n) + (n-1)q(n) = \kappa, \qquad n \geq n_0,$$

where $n_0 = \min\{n \in \mathcal{N}_c(\pi)\}$.

Suppose that the statement is not true, then there exist state $n', n'' \in \mathcal{N}_c(\pi)$ with $q(n) > 0$ and $q(n') > 0$ such that

$$\lambda(n') + (n'-1)q(n') > \lambda(n'') + (n''-1)q(n'').$$

Let

$$\kappa_{inf} = \inf_{n\in\mathcal{N}_c(\pi)} \{\lambda(n) + (n-1)q(n)\}.$$

Note that

$$\lambda(n') + (n'-1)q(n') > \kappa_{inf}.$$

Furthermore, note that

$$\begin{aligned} Q(n',\pi) \quad > \quad & \gamma e^{-\lambda(n')-(n'-1)q(n')} + ... \\ & + \left(1 - e^{-\lambda(n')-(n'-1)q(n')}\right)\left(\gamma + \gamma e^{\kappa_{inf}}\right). \end{aligned}$$

As $\lambda(n') + (n'-1)q(n') > \kappa_{inf}$, there exists $\delta > 0$ such that

$$\begin{aligned} &\gamma e^{-\lambda(n')-(n'-1)q(n')} + ... \\ &+ \left(1 - e^{-\lambda(n')-(n'-1)q(n')}\right)\left(\gamma + \gamma e^{\kappa_{inf}}\right) \\ &\quad > \gamma e^{\kappa_{inf}+\delta} \end{aligned}$$

and

$$Q(n', \pi) > \gamma e^{\kappa_{inf}+\delta}.$$

Consider the retransmission vector $\hat{q}$ given by

$$\hat{q}(n) = \begin{cases} 1, & \lambda(n) + (n-1)q(n) \leq \kappa_{inf} + \delta \\ 0, & \text{otherwise} \end{cases}$$

Note that

$$Q(n', \pi, \hat{q}) \leq \gamma e^{\kappa_{inf}+\delta}$$

and

$$Q(n', \pi) > \gamma e^{\kappa_{inf}+\delta} \geq Q(n', \pi, \hat{q}).$$

This implies that π is not an equilibrium strategy.

D. Properties of Strategies in $\mathcal{F}_\kappa$

In this section, we derive some properties of strategies in the set $\mathcal{F}_\kappa$, $\kappa > 0$, that we use to prove our main results.

Consider a strategy $\pi \in \mathcal{F}_\kappa$, and let $\mathcal{N}_c(\pi)$ be the recurrent class; then there exists a integer N_0 such that $n \in \mathcal{N}_c(\pi)$ if $n \geq N_0$. Note that for every state $n \in \mathcal{N}_c(\pi)$ with $q(n) > 0$, we have

$$e^{-\lambda(n)-(n-1)q(n)} = e^{-\kappa}, \qquad n \geq N_0.$$

We then have the following result.

Lemma 11: For every strategy $\pi \in \mathcal{F}_\kappa$, we have

$$\lim_{n\to\infty} q(n) = 0.$$

Proof: Note that

$$q(n) = \frac{\kappa - \lambda(n)}{n-1}.$$

As

$$\lambda(n) \leq \lambda(0) < \infty,$$

it follows that

$$\lim_{n\to\infty} q(n) \leq \lim_{n\to\infty} \frac{\kappa - \lambda(0)}{n-1} = 0.$$

■

Lemma 12: We have

$$\lim_{n\to\infty} G(n)e^{-G(n)} = \kappa e^{-\kappa}.$$

Proof: Note that in order to prove the lemma, it suffices to prove that

$$\lim_{n\to\infty} G(n) = \kappa.$$

Note that $(G(n) - \kappa) = q(n)$. As by Lemma 11, we have that

$$lim_{n\to\infty} q(n) = 0,$$

the result follows. ■

Lemma 13: If $\pi \in \mathcal{F}_k$ then we have

$$Q(n,\pi) = \gamma e^{\kappa}, \qquad n \in \mathcal{N}_c(\pi).$$

Proof: By definition, for all states $n \in \mathcal{N}_c(\pi)$ we have $\lambda(n) + (n-1)q(n) = \kappa$. This implies that the probability that a transmission attempt of a backlogged packet at a state $n \in \mathcal{N}_c(\pi)$ is successful is equal to $e^{-\kappa}$. As by definition the strategy π is admissible, we have $Q(n,\pi) = \gamma e^{\kappa}$ for all states $n \in \mathcal{N}_c(\pi)$. ■

For a state $n \in \mathcal{N}_c(\pi)$, the expected cost for successfully transmitting a packet is then given by

$$R(n,\pi) = \gamma + \left(1 - \frac{\lambda(n)}{1-e^{-\lambda(n)}} e^{-\lambda(n)-nq(n)}\right)\gamma e^{\kappa}.$$

Using the fact that $\lambda(n) + (n-1)q(n) = \kappa$, we obtain that for $n \in \mathcal{N}_c(\pi)$ that

$$R(n,\pi) = \gamma + \left(1 - \frac{\lambda(n)}{1-e^{-\lambda(n)}} e^{-\lambda(n)-\frac{n}{n-1}(\kappa-\lambda(n))}\right)\gamma e^{\kappa}.$$

Consider now the function

$$f_n(r) = \gamma + \left(1 - \frac{\lambda(r)}{1-e^{-\lambda(r)}} e^{-\lambda(r)-\frac{n}{n-1}(\kappa-\lambda(r))}\right)\gamma e^{\kappa}.$$

If π is an equilibrium strategy, then we have

$$R(n,\pi) = f_n(R(n,\pi))$$

and $R(n,\pi)$ is a solution to the equation

$$r = f_n(r), \qquad r \geq 0.$$

In the following we investigate whether this equation has a solution.

Lemma 14: There exist a $r \geq 0$ such that $r = f_n(r)$ if and only if

$$\max_{r\geq 0}(f_n(r) - r) \geq 0.$$

Proof: Clearly, if $\max_{r\geq 0}(f_n(r) - r) < 0$ then there does not exists a $r \geq 0$ such that $r = f_n(r)$. Suppose that $\max_{r\geq 0}(f_n(r) - r) > 0$. Then there exists a $r \geq 0$ such that $f_n(r) > r$. As

$$\lim_{r\to\infty} f_n(r) = \gamma + \left(1 - e^{-\frac{n}{n-1}\kappa}\right)\gamma e^{\kappa} < \infty.$$

and the function $f_n(r)$ is continuous, this implies that there exists a $r \geq 0$ such that $r = f_n(r)$. The result then follows. ■

Lemma 15: Let $n, n' \geq 2$ be such that $n < n'$. If $\kappa - \lambda(r) > 0$, then we have $f_n(r) > f_{n'}(r)$.

Proof: Note that

$$f_n(r) - f_{n'}(r) = \\ -\gamma\frac{\lambda(r)e^{\kappa}e^{-\lambda(r)}}{1-e^{-\lambda(r)}}\left(e^{-\frac{n}{n-1}(\kappa-\lambda(r))} - e^{-\frac{n'}{n'-1}(\kappa-\lambda(r))}\right).$$

Note that

$$\frac{n}{n-1} > \frac{n'}{n'-1}, \qquad n < n'.$$

Combining this result with the fact that $\kappa - \lambda(r) > 0$, it follows that

$$\frac{n}{n-1}(\kappa - \lambda(r)) > \frac{n'}{n'-1}(\kappa - \lambda(r)), \qquad n < n',$$

and the result follows. ■

Lemma 16: Let $n, n' \geq 2$ be such that $n < n'$. If $r_{n'} \geq 0$ is such that $r_{n'} = f_{n'}(r_{n'})$ and $\kappa - \lambda(r_{n'}) > 0$, then there exist $r_n > r_{n'}$ such that $r_n = f_n(r_n)$.

Proof: By Lemma 15, we have that

$$f_n(r_{n'}) > f_{n'}(r_{n'}) = r_{n'}.$$

As

$$\lim_{r\to\infty} f_n(r) = \gamma + \left(1 + e^{-\frac{n}{n-1}\kappa}\right)\gamma e^{\kappa} < \infty,$$

and $f_n(r)$ is continuous, it follows that there exists $r_n > r_{n'}$ such that $r_n = f_n(r_n)$. ■

Next consider the function

$$f_\infty(r) = \lim_{n\to\infty} f_n(r) = \gamma + \left(1 - \frac{\lambda(r)}{1-e^{-\lambda(r)}} e^{-\kappa}\right)\gamma e^{\kappa}.$$

The following result is obtained by the same argument as given for Lemma 14; we omit a detailed proof.

Lemma 17: There exists $r_\infty \geq 0$ such that $r = f_\infty(r)$ if and only if

$$\max_{r\geq 0}(f_\infty(r) - r) \geq 0.$$

If the above equation has at least one solution $r \geq 0$, then we define

$$r_\infty = \max\{r \geq 0; f_\infty(r) = r\}.$$

The next two lemmas can be obtained using the same argument as given for Lemma 15 and Lemma 16; we omit detailed proofs.

Lemma 18: If $\kappa - \lambda(r) > 0$, then we have for $n \geq$ that $f_n(r) > f_\infty(r)$

Lemma 19: If there exists $r \geq 0$ is such that $r = f_\infty(r)$ and $\kappa - \lambda(r_\infty) > 0$, then for $n \geq 2$ there exist $r_n > r_\infty$ such that $r_n = f_n(r_n)$.

Finally, we define the function

$$f_0(r) = \gamma + \left(1 - \frac{\lambda(r)}{1 - e^{-\lambda(r)}} e^{-\lambda(r)}\right) \gamma e^{\kappa}, \qquad r \geq 0.$$

Note that when π is an equilibrium strategy then we have $\lambda(n_0) = \lambda(r_0)$, where $n_0 = \min\{n \in \mathcal{N}_c(\pi)\}$.

Lemma 20: There exists a unique $r \geq 0$ such that $r = f_0(r), \qquad r \geq 0$.

Proof: Note that

$$f_0(0) = \gamma + \left(1 - \frac{\lambda(0) e^{-\lambda(0)}}{1 - e^{-\lambda(0)}}\right) \gamma e^{\kappa} > 0.$$

Furthermore, as by Assumption $\lim_{r \to \infty} \lambda(r) = 0$, we have

$$\lim_{r \to \infty} f_0(r) = \gamma < \infty.$$

As the function $f_0(r)$ is continuous, it follows that there exists a $r \geq 0$ such that $r = f_0(r)$.

To show that there exists a unique $r \geq 0$ such that $r = f_0(r)$, it suffices to show that the function $f_0(r)$ is non-increasing. However, this is equivalent to showing that the function $\lambda(r) e^{-\lambda(r)} / (1 - e^{-\lambda(r)})$ is non-decreasing in r. This is indeed the case, as the function $x e^{-x} / (1 - e^{-x})$ is non-increasing in x. This can easily be verified by considering the first derivative.

■

E. Proof of Proposition 2

First suppose that the conditions hold. Then we can construct a stable equilibrium strategy $\pi \in \mathcal{F}_\kappa$ as follows. If condition (b) holds, then by Lemma 19 for $n \geq 2$ there exists $r_n \geq 0$ be such that $r_n = f_n(r_n)$ and $\lambda(r_n) < \kappa e^{-\kappa}$.Set $\lambda(n) = \lambda(r_n)$, $n \geq 2$, and set $\lambda(0) = \lambda(1) = \lambda(r_0)$. Then choose $q(n)$, $n \geq 1$, as follows: set $q(1) = 0$ and

$$q(n) = \frac{\kappa - \lambda_(r_n)}{n - 1}, \qquad n \geq 2.$$

Note that π is a stable strategy. As $\lambda(0) = \lambda(r_0) > 0$ and the strategy is stable, it follows that π is admissible. For all $n \geq 2$, the probability for a successfully transmission attempt of a backlogged packet is equal to $e^{-\kappa}$. For $n = 1$, the probability is equal to $e^{-\lambda(r_0)} < e^{-\kappa}$. Combining these two observations, it follows that

$$q = \arg \min_{\hat{q} \in \mathcal{Q}(\pi)} Q(n, \pi, \hat{q}), \qquad n \geq 1,$$

and π is an stable equilibrium strategy.

Next, we have to show if one of the conditions does not hold, then there does not exists a stable equilibrium strategy $\pi \in \mathcal{F}_\kappa$.

Suppose that condition (a) does not hold and that there exists a stable equilibrium strategy $\pi \in \mathcal{F}_k$. This implies that there exists a integer N_0 such that for all $n \geq N_0$ there exists $r_n \geq 0$ such that $r_n = f_n(r_n)$. However, as

$$f_\infty(r) = \lim_{n \to \infty} f_n(r)$$

and

$$\lim_{r \to \infty} f_\infty(r) - r = -\infty,$$

it follows that there exists a bound $R < \infty$ such that $r_n < R$ for $n \geq N_0$ and the set $\{r_n; n \geq N_0\}$ has at least one limit point. Let r_∞ be such a limit point. As the function f_∞ is bounded, it the follows that

$$\lim_{r \to r_\infty} f_\infty(r) = f_\infty(r_\infty) = r_\infty.$$

However, this contradicts our assumption that condition (a) does not hold. Similarly, one can show that there does not exist a stable equilibrium strategy if condition (b) does not hold.

Next, suppose that the conditions do not hold. Clearly, if $\lambda_\infty \geq \kappa e^{-\kappa}$ then there can not exists a stable strategy $\pi \in \mathcal{F}_\kappa$.

Finally, suppose that condition (c) does not hold and there exists a stable equilibrium strategy $\pi \in \mathcal{F}_\kappa$. Now consider the state $n_0 = \min\{n \in \mathcal{N}_c(\pi)\}$. Recall that $q(n_0) = 0$ (Lemma 10) and as π is an equilibrium strategy we have $\lambda(n_0) = \lambda(r_0)$. If $n_0 \geq 1$, then the probability of a successful transmission of a backlogged packet at state n_0 is equal to $e^{-\lambda(r_0)}$ and the probability of success in all other states in $\mathcal{F}_\kappa$ is equal to $e^{-\kappa}$. It follows that for $\hat{q}$ with $\hat{q}(n_0) = 1$ and $\hat{q}(n) = 0$, $n \neq n_0$, we have that

$$Q(n, \pi, \hat{q}) = \gamma e^{\lambda(r_0)} < \gamma e^{\kappa} = Q(n, \pi, q), \qquad n \in \mathcal{N}_c.$$

If $n_0 = 0$, one can show that $\lambda(1) < \lambda(r_0)$ and using the same argument as above it follows that π is not an equilibrium strategy. Hence, if condition (c) does not hold then there does not exist a stable equilibrium strategy $\pi \in \mathcal{F}_\kappa$.