Interaction of Rate and Medium Access Control in Wireless Networks: The Single Cell Case

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ABSTRACT

We study the interaction between rate control and medium access control in wireless networks. We start out by developing a discrete time model for the interaction of TCP Reno rate control and IEEE 802.11 medium access control. Considering the operating point of the system, we obtain the well-known characteristics that the throughput decreases as the number of nodes in the network increases and that IEEE 802.11 does not provide per-flow fairness. We then propose and study alternative rate and medium access control schemes which allow to offer per-flow fairness, as well as provide a stable and predictable throughput as the number of nodes increases.

Categories and Subject Descriptors

C.2.5 [**COMPUTER-COMMUNICATION NETWORKS**]: Local and Wide-Area Networks

General Terms

Performance

Keywords

Wireless Ad Hoc Networks, MAC protocols, IEEE 802.11, TCP, Fairness

1. INTRODUCTION

In this paper we investigate the interaction between rate (congestion) and medium access control in wireless networks. To simplify the analysis, we consider the case of a single cell wireless ad hoc network. We study the following questions: (a) Throughput: how does the system throughput scale as the number of nodes in the network increases, and (b) Fairness: how is the throughput shared among flows (connection). We will use the terms flow and connection interchangeably.

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It is well-known that the combination of TCP Reno rate control and IEEE 802.11 as medium access control has several drawbacks in the context of a single cell wireless ad hoc network [1]: (a) the throughput deteriorates as the number of nodes in the network increases and (b) IEEE 802.11 medium access control leads to per-node fairness but not per-flow fairness, i.e. IEEE 802.11 gives each node an equal chance of accessing the channel and hence penalizes flows which share a common network node with other flows. Here, we develop a mathematical model to explain these characteristics. In addition, we develop alternative rate and medium access control schemes that provide per-flow fairness and a stable throughput as the number of nodes increases.

In order to focus on the interaction of rate and medium access control, we make several simplifying assumptions for our analysis. In particular, we assume that (a) packet loss is due to congestion, i.e. we ignore that packet loss might be due to fading or co-channel interference, (b) all nodes transmit at same data rate, i.e. we ignore that different nodes might experiences channel conditions and adapt their tranmission rates accordingly, and (c) all packets are assumed to be of the same (expected) length. Incorporating the above issues in the model is beyond the scope of this paper.

Our analysis provides new insight into the interaction between rate control and medium access control; in particular, it suggests that a simple medium access control scheme is sufficient to provide per-flow fairness. We comment on this in more details in Section 4.

Below we highlight the literature that is most relevant to our work. For the analysis, we use the models proposed by Kelly [4] and Low [5] in order to characterize the dynamics of TCP Reno rate control, and the models proposed by Bianchi [2] and Kumar et al. [3] to characterize the IEEE 802.11 medium access control. The IEEE 802.11 model presented in [2, 3] is derived under the assumption that nodes always have a packet to send (node saturation assumption); however, this assumption is not true when the transmission rates are regulated through TCP Reno rate control and queues periodically become empty. Here, we adapt the model of [3] to a more general setting where nodes can have empty queues. The fact that 802.11 medium access control does not provide per-flow fairness is well-known. Approaches to address this issue have been proposed by Nandagopal et al. [6], Ozugur et al. [7], and Bharghavan et al. [8]. Whereas the approaches in [6, 7, 8] require the implementation of separate queues for each flow and possibly the exchange of some global state information, the approach presented here is fully distributed and uses only a single (shared) queue at each node. In [9], Heusse et al. propose a medium access control scheme to avoid a throughput collapse as the number of nodes increases. The scheme uses channel feedback information (i.e. the number of idle slots between transmission attempts) to adapt the channel access probabilities and stabilize the throughput. Compared with the analysis presented here, the work in [9] does not consider per-flow fairness nor does it study the interaction of the medium access control with a given rate control scheme.

The rest of the paper is organized as follows. In Section 2, we study the interaction of TCP Reno rate control and IEEE 802.11 medium access control. In Section 3, we consider alternate medium access rate control mechanisms to provide per-flow fairness and provide a stable throughput.

2. INTERACTION OF TCP RENO AND IEEE 802.11

Consider a single cell ad hoc network consisting of a set of nodes $\mathcal{N} = \{1, 2, \cdots, N\}$ where all N nodes are within transmission range of each other. Let $\mathcal{M} = \{1, 2, \cdots, M\}$ be the set of active TCP connections, and let $s(m)$ and $d(m)$ be the source and destination node of connection m, respectively. For connection m, let $b_{s,m}$ be the number of active data packets (i.e. packets within the current TCP window) that have not yet been successfully transmitted and are still backlogged in the MAC-layer at node $s(m)$, and let $b_{d,m}$ be the number of backlogged ACK packets at node $d(m)$. Furthermore, let λ_m be the transmission rate of connection m (we provide an expression for λ_m in the next subsection). Using the above definitions, let $x = (\lambda, b_s, b_d)$ where $\lambda = (\lambda_1, ..., \lambda_M)$, $b_s = (b_{s,1}, ..., b_{s,M})$, and $b_d = (b_{d,1}, ..., b_{d,M})$. We use $x = (\lambda, b_s, b_d)$ as the system state.

To study the interaction between TCP Reno and the IEEE 802.11 protocol, we pursue the same approach as in [5] to analyze TCP Reno and model the dynamics as a discrete time system

$$x(t+1) = \phi(x(t)), \qquad t = 1, 2, ...,$$

where $x(t) = (\lambda(t), b_s(t), b_d(t))$ is the state at time t and the function ϕ describes the system dynamics. As in [5], we then study the existence of an operating point x^* such that $x^* = \phi(x^*)$, and use x^* to characterize the system performance in terms of throughput and fairness. We proceed as follows. In Subsection 2.1 we characterize the dynamics of the transmission rates under TCP Reno, and Subsection 2.2 we model the dynamics of the backlog under IEEE 802.11. In Subsection 2.3 we then combine these two models to formulate the system model that we use to analyze the interaction between TCP Reno and IEEE 802.11.

We use the following notation. Let $b_m = b_{s,m} + b_{d,m}$ be the total backlog of connection m and let B_n be the total backlog at node n, i.e. B_n is given by $B_n = \sum_{m \in \mathcal{M}, s(m)=n} b_{s,m} + \sum_{m \in \mathcal{M}, d(m)=n} b_{d,m}$. We assume that each node n has a finite amount of buffer space available given by $\bar{B}_n > 0$.

2.1 TCP Reno Rate Control

For our analysis of TCP Reno, we use the discrete time model proposed by Kelly in [4] and used by Low in [5]. That is, we consider a discrete time system where the length of each time slot is equal to one time unit. For a connection $m \in \mathcal{M}$, let $w_m(t)$ be the window size (in terms of packets) of connection m during time slot t, $t \geq 0$. The transmission rate $\lambda_m(t)$ (in terms of packets per time slot) of connection m at time t is then given by

$$\lambda_m(t) = w_m(t)/D_m(t), \tag{1}$$

where $D_m(t)$ is the round-trip time of connection m, i.e. $D_m(t)$ denotes the number of time slots it takes for connection m to receive a positive ACK for a packet sent at the beginning of slot t.

TCP Reno uses packet loss as a congestion indicator [5], where window size w_m is increased by $\frac{1}{w_m}$ for each acknowledged packet and halved for each packet that is not acknowledged. As in [4], we characterize the likelihood that a given packet belonging to connection m is not acknowledged and the window size w_m is halved at time t by a congestion indicator function P_m. We make the following assumption for the congestion indicator function.

Assumption 1. *The congestion indicator function for connection m is a function of the backlog (of all connections) at the source node $s(m)$ and destination node $d(m)$ of connection m, which is given by $B_{s(m)} + B_{d(m)}$. Furthermore, we have that $P_m : [0, \bar{B}_{s(m)} + \bar{B}_{d(m)}] \mapsto [0,1]$ is strictly increasing with $P_m(0) = 0$ and $P_m\left(\bar{B}_{s(m)} + \bar{B}_{d(m)}\right) = 1$.*

The above assumption implies that the less buffer space that is available along the path of connection m, the more likely it is that a data packet, or ACK, is lost and the window w_m is halved. One can show (see [4] for a detailed derivation) that the expected change in the window size w_m (per time slot) at time t is then given by

$$\lambda_m(t) \left\{ \frac{1 - P_m\left(B_{s(m)}(t) + B_{d(m)}(t)\right)}{w_m(t)} - \frac{w_m(t)}{2} P_m\left(B_{s(m)}(t) + B_{d(m)}(t)\right) \right\}. \tag{2}$$

Using the above equation, we can then express the expected change in the transmission rate λ_m as follows. Recall that $b_{s,m}$ is the number of active data packets (i.e. packets within the current TCP window) of connection m that have not yet been successfully transmitted and are still backlogged at the source node $s(m)$ of connection m. As all active packets (i.e. all data packets and ACKs within the current window size) of connection m are backlogged either at node $s(m)$ or $d(m)$, and we have that

$$w_m(t) \approx b_{s,m}(t) + b_{d,m}(t) = b_m(t), \tag{3}$$

where the above equation does not hold strictly (and introduces a modeling error) as it ignores lost packets. For TCP Reno we have that $w_m(t) \geq 1$, $m \in \mathcal{M}$, and in the following we impose the condition that

$$b_m(t) \geq 1, \qquad t \geq 1. \tag{4}$$

Using Eq. (1)-(4), one can show (see [5] for a derivation) that (under suitable assumptions) the expected rate of change in the transmission rate λ_m at time t is given by

$$\frac{\lambda_m(t)^2}{b_m(t)^2} \left[1 - P_m\left(B_{s(m)}(t) + B_{d(m)}(t)\right) - P_m\left(B_{s(m)}(t) + B_{d(m)}(t)\right) \left(\frac{b_m(t)^2}{2}\right) \right].$$

2.2 IEEE 802.11 Medium Access Control

The medium access control of IEEE 802.11 works roughly as follows [3]. Before a transmission attempt, nodes sense the channel for a given time interval L_i (called DIFS - Distributed Inter Frame Space) to detect whether the channel is free (idle). If the channel is sensed to be free, a node will wait for some random time before starting with the transmission as follows. Each node keeps track of a *back-off* timer and a *retry* counter. The back-off timer interval is chosen at random uniformly from the interval $[1, CW]$, where CW is set equal to a given constant $CW_{\min}$ for the first transmission attempt of a packet. The back-off timer is then decremented by one every time the channel is sensed to be idle. When the back-off timer reaches zero, a transmission starts. If a collision occurs (i.e. two or more nodes make a transmission at the same time), CW is doubled (up to some given constant $CW_{\max}$), the retry counter is incremented and the packet is backlogged waiting for next transmission. When the retry counter reaches the maximal *retry limit* K, the packet is discarded.

Bianchi [2] and Kumar et al. [3] have shown that (under suitable assumptions) the IEEE 802.11 protocol can be modeled as follows. Let L_i be the time a node has to detect the channel to be idle before making a transmission attempt. We will refer to this time as an idle period. Let N_a be the number of nodes that have a packet to send. Then after an idle period, each of the N_a nodes makes a transmission attempt with probability $q(N_a)$ where $q(N_a)$ is the unique solution to the following fixed-point equation [3]

$$\begin{cases} \vartheta = 1 - (1-q)^{N_a - 1} \\ q = \frac{1+\vartheta^1+\cdots+\vartheta^K}{\omega_0+\vartheta^1\omega_1+\cdots+\vartheta^K\omega_K}, \end{cases}$$

and $\omega_k = \min\{\frac{2^k CW_{\min}+1}{2}, \frac{CW_{\max}+1}{2}\}$. In the above equation, ϑ characterizes the collision probability observed by a given node. If exactly one node makes a transmission, then the packet is successfully transmitted. We assume that all packets have the same length and the time it takes to transmit one packet is equal to L_p. If two or more nodes make a transmission, then all transmitted packets are lost and become backlogged. Let L_c be the duration of a collision period. We make the following assumption.

ASSUMPTION 2. *We have that* $0 < L_i \le L_p$ *and* $0 < L_c \le L_p$.

The above assumption is based on the IEEE 802.11 standard where the time required to sense an idle channel and a collision does not exceed the transmission delay of a packet [3], i.e. when the RTS/CTS mechanism then we have $L_c < L_p$, otherwise we have $L_c = L_p$.

Note that in the above IEEE 802.11 model, nodes make a transmission attempt only after an idle period of length L_i. Furthermore, an idle period is followed by either (a) another idle period of length L_i in the case where no node makes a transmission attempt, (b) a successful transmission of length L_p in the case where exactly one node makes a transmission attempt, or (c) a collision period of length L_c in the case where two or more nodes make a transmission attempt. This observation suggests that it might be convenient to characterize the system dynamics by considering the times when idle slots start. We pursue this approach in our analysis. More precisely, we mark the times when a new idle period starts as follows. Starting the system at time $t = 0$, let t_k be the time at which the kth idle period of length L_i ends.

Using the above model, we can characterize the probability that a packet is successfully transmitted as follows. Let $B_n(k)$ be the backlog number at node n at time t_k and let the function $I_a(B_n(k))$ be given by

$$I_a(B_n) = \begin{cases} B_n, & 0 \le B_n \le 1, \\ 1, & B_n > 1. \end{cases} \quad (5)$$

Note the $I_a(B_n(k))$ indicates whether node n has a packet to transmit at time t_k.

The probability that node n makes a successful transmission attempt between t_k and t_{k+1} is then equal to

$$\frac{I_a(B_n(k))q(N_a(k))}{1 - I_a(B_n(k))q(N_a(k))} \prod_{l=1}^{N} \Big(1 - I_a(B_l(k))q(N_a(k))\Big),$$

where $N_a(k) = \sum_{n=1}^{N} I_a(B_n(k))$. We refer to $N_a(k)$ as the number of active nodes at time t_k. Note that $I_a(B_n(k))q(N_a(k))$ is the probability that node n makes a transmission attempt in the time interval $[t_k, t_{k+1}]$ and

$$\frac{1}{1 - I_a(B_n(k))q(N_a(k))} \prod_{l=1}^{N} \Big(1 - I_a(B_n(k))q(N_a(k))\Big)$$

is the probability that all other nodes (except node n) do not make a transmission attempt in $[t_k, t_{k+1}]$.

Assuming that $q(N_a(k))$ is small, we can simplify the probability that node n makes a successful transmission attempt between t_k and t_{k+1} to

$$I_a(B_n(k))q(N_a(k))e^{-G(k)}$$

where $G(k) = N_a(k)q(N_a(k))$ and

$$e^{-G(k)} \approx \frac{\prod_{l=1}^{N} \Big(1 - I_a(B_l(k))q(N_a(k))\Big)}{1 - I_a(B_n(k))q(N_a(k))}.$$

We refer to $G(k)$ as the offered load at time t_k.

Consider a connection $m \in \mathcal{M}$ with source node $s(m)$. Given that node $s(m)$ makes a transmission attempt, we assume that the probability that this packet belongs to connection m is equal to $\frac{b_{s,m}}{B_{s(m)}}$ where $b_{s,m}$ is the number of packets of connection m backlogged at node $s(m)$ and $B_{s(m)}$ is the total backlog (over all connections) at node $s(m)$. The probability that a data packet of connection m is successfully transmitted between t_k and t_{k+1} is then given by

$$G_{s,m}(k)e^{-G(k)},$$

where $G_{s,m}(k) = \frac{b_{s,m}(k)}{B_{s(m)}(k)} I_a(B_{s(m)}(k))q(N_a(k))$ and we use the convention that $G_{s,m}(k) = 0$ if $B_{s(m)}(k) = 0$.

Similarly, the probability that an ACK of connection m is successfully transmitted is by node $d(m)$ given by

$$G_{d,m}(k)e^{-G(k)},$$

where $G_{d,m}(k) = \frac{b_{d,m}(k)}{B_{d(m)}(k)} I_a(B_{d(m)}(k))q(N_a(k))$ and we use the convention that $G_{d,m}(k) = 0$ if $B_{d(m)}(k) = 0$.

2.3 System Model

Combining the TCP Reno and IEEE 802.11 model given above, we obtain the following system model that we use for our analysis. Let the time t_k be as defined in the previous subsection and let $x(k) = (\lambda(k), b_s(k), b_d(k))$ be the system state at time t_k. We then describe the system as a discrete time system

$$x(k+1) = \phi(x(k)), \qquad k \geq 1,$$

where the function ϕ characterizes the expected change of the state $x(k)$ between time t_k and t_{k+1}. More precisely, the expected change in the $\lambda_m(k)$, $b_{s,m}(k)$, and $b_{d,m}(k)$, are as given below.

Let $L(k)$ be the expected length of the interval $[t_k, t_{k+1}]$ given by (see [12] for a detailed derivation)

$$L(k) = L_i + G(k)e^{-G(k)}L_p + ... + \left(1 - e^{-G(k)} - G(k)e^{-G(k)}\right)L_c, \quad (6)$$

where $e^{-G(k)}$, $G(k)e^{-G(k)}$ and $1 - e^{-G(k)} - G(k)e^{-G(k)}$, are the probabilities that between t_k and t_{k+1} we have no transmission attempt, a successful transmission, or collision, respectively. Note that the expected number of new data packets of connection m received between time t_k and $t_{k+1}$is then equal to $\lambda_m(k)L(k)$. Using the TCP Reno model of Subsection 2.1, the expected change in the transmission rate of connection m in the interval $[t_k, t_{k+1}]$ is given by

$$\lambda_m(k+1) = \lambda_m(k) + \lambda_m(k)^2 \left[\frac{1 - P_m(k)}{b_m(k)^2} - \frac{P_m(k)}{2}\right] L(k), \quad (7)$$

where the congestion indicator function $P_m(k) = P_m\Big(B_{s(m)}(k) + B_{d(m)}(k)\Big)$ is as given in Assumption 1.

Using the IEEE 802.11 model of Subsection 2.2, the expected change in the backlog of connection m at node $s(m)$ between time t_k and t_{k+1} is given by

$$b_{s,m}(k+1) = b_{s,m}(k) + \lambda_m(k)L(k) - G_{s,m}(k)e^{-G(k)}, \quad (8)$$

where $\lambda_m(k)L(k)$ is the expected number of new data packets of connection m received between time t_k and t_{k+1}, and $G_{s,m}(k)e^{-G(k)}$ is the probability that a data packet of connection m is successfully transmitted between time t_k and t_{k+1}. Similarly, the expected change in the backlog of connection m at node $d(m)$ between time t_k and t_{k+1} is given by

$$b_{d,m}(k+1) = b_{d,m}(k) + G_{s,m}(k)e^{-G(k)} - G_{d,m}(k)e^{-G(k)}, \quad (9)$$

where $G_{s,m}(k)e^{-G(k)}$ is the probability that a new ACK gets generated at node $d(m)$ and $G_{d,m}(k)e^{-G(k)}$ is the probability that an ACK is successfully transmitted.

To characterize the system performance in terms of throughput and fairness, we analyze the performance at a system operating point.

Definition 1. *We call $x^* \in \mathbb{R}_+^{3M}$ an operating point if $x^* = \phi(x^*)$.*

Note that the system is in equilibrium at an operating point, i.e. the expected change of the system state at an operating point is equal to 0. We are interested in the following questions: (a) does an operating point exist and is it unique and (b) what is the system performance in terms of throughput and fairness at an operating point.

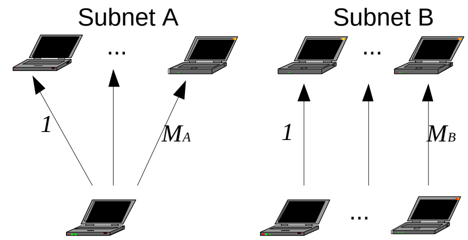

Figure 1: A special case

In the following, we use the operating point to characterize the system performance. Computing the average system performance would require to compute the steady-state probabilities of the Markov chain associated with the system dyanamics, which is not feasible for the above system. In Section 2.5, we will use a numerical case study to investigate how well the operating point analysis characterizes the average system performance.

Definition 1 does not require that the backlog at an operating point $x^* = (\lambda^*, b_s^*, b_m^*)$ takes on an integer value. As the system dynamics ϕ captures the expected change in the state variables, we can interpret $b_{s,m}^*$ as an approximation for the expected backlog of connection m at node $s(m)$, and $b_{d,m}^*$ as an approximation of the expected backlog at node $d(m)$. Furthermore, the node backlog B_n^* at an operating point can be interpreted as an apporximation of the expected backlog at node n and $I_a(B_n^*) \in [0,1]$ as an approximation of the likelihood that node n has a packet to transmit.

In the following, we will refer to $N_a^* = \sum_{n=1}^{N} I_a(B_n^*)$ as the number of active nodes at the operating point x^*.

2.4 Operating Point

Unfortunately, it is difficult to answer the above questions regarding the existence and properties of an operating point for a general network topology. The reason is that the congestion indicator function $P_m(B_{s(m)} + B_{d(m)})$ introduces a nonlinear coupling that makes it difficult to obtain analytical solutions for a general network topology. To overcome this, we consider the network topology given by Fig. 1[1] which consists of two sets of connections, $\mathcal{M}_A = \{1, ..., M_A\}$ and $\mathcal{M}_B = \{1, ..., M_B\}$. We refer to the connections in the set $\mathcal{M}_A$ as subnet A and the connections in set $\mathcal{M}_B$ as subnet B. Note that the connections in subnet A share the same source node n_0. This topology captures the situation where node n_0 acts as a server for all other nodes in subnet A. In the following we assume that $M_A \geq 2$; otherwise subnet A does not exist ($M_A = 0$) or is identical to a connection in subnet B ($M_A = 1$). For this topology, we are interested in the following questions: (a) how does the throughput scale as the number of nodes (connections) in subnet B increases and (b) how is the throughput shared between connections in subnet A and B.

For our analysis, we make the following assumption regarding the congestion indicator function for subnet B.

Assumption 3. *For all connections $m \in \mathcal{M}_B$ we have that $P_m(2) \leq 1/3$.*

[1]For the analysis in the Section 3 we do not need to make this restriction.

The above assumption is a (mild) technical assumption on the buffer space at nodes in subnet B; it states that each node in subnet B has a buffer space to easily accommodate 2 packets. This assumption is needed to prove the existence of system operating point. For connections in subnet A we make the following assumption.

ASSUMPTION 4. *For all connection $m \in \mathcal{M}_A$, the congestion indicator function is a function of the total backlog B_{n_0} at node n_0 such that*

$$P_m(B_{n_0}) = P_{m'}(B_{n_0}) = P(B_{n_0}), \qquad m, m' \in \mathcal{M}_A,$$

where $P : [0, \bar{B}_{n_0}] \mapsto [0, 1]$ is strictly increasing with $P(0) = 0$, and $P(\bar{B}_{n_0}) = 1$. Furthermore, we have that $P(1) \leq 2/3$.

The above assumption is motivated by a result presented in [3] which implies that the total backlog at node n_0 will be much larger than the backlog at any other nodes in subnet A and a packet loss is most likely due to congestion at node n_0. The assumption that $P(1) \leq 2/3$ is again a technical assumption needed to prove the existence of an operating point.

Using the above assumptions, we obtain the following result.

PROPOSITION 1. *Under Assumptions 1-4 there exists an operation point. If $M_B > 0$, then there exists an infinite set of operating points.*

The above result states that there always exists an operating point; but it is not necessarily unique. However, as shown in the next result, the expected number of active nodes N_a^* at an operating point, and hence the offered load $G^* = N_a^* q(N_a^*)$, is unique.

PROPOSITION 2. *Let x^* be an operating point, and let N_a^* be the expected number active nodes and G^* the offered load at x^*. Under Assumptions 1-4, we have $N_a^* = 2(1 + M_B)$ and $G^* = 2(1 + M_B)q(N_a^*)$.*

An important implication of Proposition 2 is that at an operating point, not all nodes are saturated, i.e. not all nodes have packet to send. To see this, note that Proposition 2 states that the number of active nodes (i.e. nodes that have a packet to send) at an operating point is equal to $N_a^* = 2(1+M_B)$ which is strictly smaller than the total number of nodes in the network given by $N = 1+M_A+2M_B$ (see Fig. 1). The interpretation of the result is as follows: while all nodes in subnet B are saturated, only 2 of the $(1 + M_A)$ nodes in subnet A are saturated (i.e. have a packet to send), i.e. most nodes in subnet A are not saturated.

Proposition 2 states that N_a^* and the offered load G^* at an operating point only depends on the number of connections M_B in subnet B. Furthermore, the offered load G^* increases as M_B increases. Using these results, we characterize below the system throughput at an operating point and how this throughput is shared among individual connections. We provide a proof for Proposition 1 and 2 in Appendix A.

2.4.1 Throughput

It is well-known (see for example [12]) that for the IEEE 802.11 model of Subsection 2.2 the system throughput T (in packets per unit time) is a function of the offered load G given by

$$T(G) = \frac{Ge^{-G}}{L(G)}, \qquad (10)$$

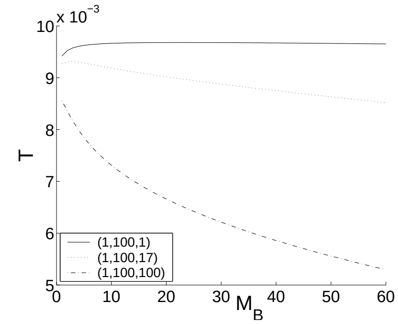

Figure 2: Throughput at the operating point as a function of M_B for different values of (L_i, L_p, L_c).

where $L(G) = L_i + Ge^{-G}L_p + (1 - e^{-G} - Ge^{-G})L_c$. Combining this result with Proposition 2, we immediately obtain the following corollary.

COROLLARY 1. *Under Assumptions 1-4, the system throughput at the operating point for TCP Reno with IEEE 802.11 is equal to $T(G^*)$ where $G^* = N_a^* q(N_a^*)$ and $N_a^* = 2(1 + M_B)$.*

Using the above result, we can compute how the system throughput changes as the number of nodes in subnet B (and hence the number of connections in subnet B) increases. Fig. 2 gives the resulting graph which shows the throughput $T(G^*)$ decreases when the number of connections M_B becomes larger. Note that the degree of throughput degradation depends on the values of L_i, L_c, and L_p, i.e. on the actual implementation of the medium access protocol.

2.4.2 Fairness

The next result shows that network does not provide per-flow fairness.

PROPOSITION 3. *Let m_A be a connection in subnet A and m_B be a connection in subnet B, and let $\lambda^*_{m_A}$ and $\lambda^*_{m_B}$ be the throughput of these two connections at an operating point. Under Assumptions 1-4, we have that*

$$\lambda^*_{m_A} = \frac{1}{M_A} \frac{q(N_a^*)e^{-G^*}}{L(G^*)}$$

and

$$\lambda^*_{m_B} = \frac{q(N_a^*)e^{-G^*}}{L(G^*)},$$

and $\lambda^*_{m_A}/\lambda^*_{m_B} = 1/M_A$.

The above result states that connections in subnet A obtain a smaller throughput than connections in subnet B. In fact, (as all connections in subnet A share the same source node n_0) the combined throughput received by nodes in subnet A is equal to the throughput of a single connection in subnet B. We provide a proof for Proposition 3 in Appendix A.

2.5 Numerical Results

In this subsection, we illustrate the above results using numerical case studies in which we implemented the actual TCP Reno protocol and IEEE 802.11 protocol. We use these numerical results to study how well the performance predictions obtained by the theoretical results of the previous subsection match the actual system performance.

For the IEEE 802.11 medium access control we use the following parameter values: $CW_{\min} = 32$, $CW_{\max} = 1024$ and $K = 7$. The values for L_i, L_p, and L_c, that we used are given in Table 1, along with the theoretical optimal value T_{opt} of the throughput given by Eq. (10) for the different choices of L_i, L_p, and L_c.

Table 1: (L_i, L_p, L_c) and T_{opt}

(L_i, L_p, L_c)	T_{opt}
$(1, 100, 1)$	0.9680
$(1, 100, 17)$	0.9318
$(1, 100, 100)$	0.8654

For the case studies, we set the number M_A of connections in subnet A equal to 3 but vary the number of connections M_B in subnet B in order to investigate how the throughput scales as the number of nodes in the network increases. Using this setup, we simulated the system for 180,000 time slots.

2.5.1 Throughput

Fig. 3 shows the time average system throughput for the different scenarios. The experimental results confirm the results of Subsection 2.4 that the throughput decreases as the number of connections M_B in subnet B increases. In addition, the throughput predicted by the theoretical results in Subsection 2.4 matches quite well the numerical results.

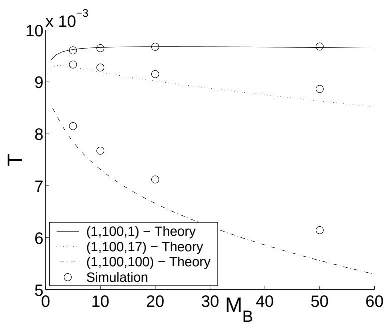

Figure 3: Time average system throughput for different values of (L_i, L_p, L_c).

2.5.2 Fairness

Next we investigate how the throughput is shared among connections for the case where $(L_i, L_p, L_c) = (1, 100, 17)$, and $M_A = 3$ and $M_B = 10$. Fig. 4 shows the trajectory of the transmission rates of one connection in subnet A and one connection in subnet B. The time average transmission rate of the connection from subnet A is equal to 0.17×10^{-3} and the time average transmission rate of the connection in subnet B is equal to 0.417×10^{-3}. Hence the connection of subnet A gets roughly 1/3 of the transmission rate of a connection in subnet B as predicted by the theoretical results in Section 2.4.

2.6 Discussion

We developed a mathematical model to study the interaction between TCP Reno rate control and IEEE 802.11 medium access control.

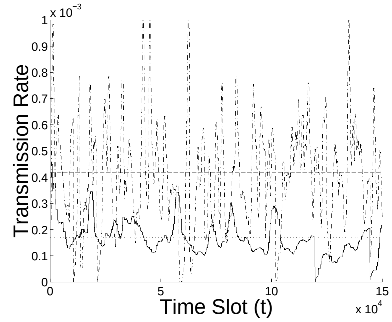

Figure 4: Trajectory of the transmission rates of one connection in subnet A (bottom) and one connection in subnet B (top).

Although we made several simplifying assumptions, the numerical results illustrate that the model indeed predicts well the actual system performance in terms of throughput and fairness. In the next sections, we study an approach to provide per-flow fairness and to avoid the throughput degradation as the number of nodes increases.

3. AN ALTERNATIVE RATE AND MEDIUM ACCESS CONTROL

In this section, we consider an alternative rate and medium access control scheme to provide per-flow fairness and avoid a throughput degradation as the number of nodes increases. The basic idea for the medium access control scheme is the following: a node that serves several connections will typically have a larger backlog than nodes that serve only a few connections. Therefore, in order to provide per-flow fairness we should give nodes with a large backlog a higher chance of accessing the channel and transmitting a packet. As we show below, this simple mechanism is indeed sufficient to obtain per-flow fairness. To avoid a throughput degradation, we consider a rate control mechanism that uses a price (congestion) signal to modulate the packet transmission rate of individual connection. An important feature of both the rate control and medium access control scheme that we consider is that they can be implemented in a fully distributed manner.

3.1 Rate Control

We consider a rate control mechanism where the transmission rate λ_m of connection m depends on a price (control) signal u_m as follows.

ASSUMPTION 5. *The transmission rate $\lambda_m(u_m)$ is a function of u_m where $\lambda_m(u_m)$ is bounded, differentiable, and strictly decreasing such that* $\lim_{u_m \to \infty} \lambda_m(u_m) = 0$.

We use the following approach to compute the signal u_m in a distributed manner. The source node $s(m)$ observes the channel and uses channel feedback information to update u_m. Let $I(\cdot)$ be the indicator function and let $Z(k)$ be a random variable indicating the total transmission attempts between t_k and t_{k+1}. Then $I(Z(k) = 0)$, $I(Z(k) = 1)$ and $I(Z(k) \geq 2)$ indicate that an idle period, a successful transmission, or a packet collision, occurred between time t_k and

t_{k+1}, respectively. Using this channel feedback, the signal u_m is updated as follows

$$u_m(k+1) = [u_m(k) - \alpha I(Z(k)=0) + \beta I(Z(k)=1) + \gamma(u_m(k)) I(Z(k) \geq 2)]^+, \quad (11)$$

where $[u]^+ = \max\{u, 0\}$, $u \in \mathbb{R}$, and α, β and $\gamma(u)$ satisfy the following assumption.

ASSUMPTION 6. *We have $\alpha > 0$ and $\beta \in \mathbb{R}$. Furthermore, γ is a non-negative and strictly decreasing function $\gamma : [0, +\infty) \mapsto [\gamma_{\min}, \gamma_{\max}]$.*

The above update rule has the following intuition. If the channel is idle, then the price should be decreased (and the transmission rate increased) in order to make more efficient use of network bandwidth. If a collision is detected, then the price should be increased in order to reduce the probability of additional collision in the future. Furthermore, connections with a high rate should decrease their rate more aggressively than connections that already have a low rate; hence the function γ should be a decreasing function.

3.2 Medium Access Control

Below we describe more precisely the medium access control scheme that we consider.

Again, nodes sense the channel before making a transmission attempt. After an idle period of length L_i, node n makes a transmission attempt with probability $B_n q$, where B_n is the number of backlogged packets at node n and q, $0 < q < 1$, is a given parameter such that $\bar{B}_n q < 1$ for all nodes $n \in \mathcal{N}$. The condition $\bar{B}_n q < 1$ is needed to prevent a system deadlock[2]. Note that the expected number of transmission attempts (over all nodes) is given by

$$G = \sum_{n=1}^{N} B_n q.$$

If exactly one node makes a transmission attempt, then this packet is successfully transmitted. If two or more nodes make a transmission, then all transmitted packets are lost and become backlogged. Again, we assume that all packets have the same length and the time it takes to transmit one packet is equal to L_p. Furthermore, let L_c be the duration of a collision.

As before, let t_k be the time when the kth idle period of length L_i ends. Consider a connection $m \in \mathcal{M}$ with source node $s(m)$. Given that node $s(m)$ makes a transmission attempt, we again assume that the probability that this packet belongs to connection m is equal to $\frac{b_{s,m}}{B_{s(m)}}$ where $b_{s,m}$ the number of packets of connection m backlogged at node $s(m)$ and $B_{s(m)}$ is the total backlog (over all connections) at node $s(m)$. Under this model, the probability that a data packet of connection m makes a transmission attempt between time t_k and t_{k+1} is given by

$$G_{s,m}(k) = b_{s,m}(k) q.$$

[2]An alternative approach to prevent a system deadlock is as follows. After an idle slot of length L_i, node n makes a tranmission attempt with probabiltiy $q_n = \min\{B_n q, 1-\epsilon\}$, where ϵ, $0 < \epsilon < 1$. Note that this approach does not require a priori knowledge of the buffer sizes $\bar{B}_n$, $n \in \mathcal{N}$. However the analysis of this scheme is more involved.

Similarly, the probability that an ACK of connection m makes a transmission attempt at time t_k is given by

$$G_{d,m}(k) = b_{d,m}(k) q,$$

and the probability of a transmission attempt (by any connection) at time t_k is equal to

$$G(k) = \sum_{m=1}^{M} \Big(G_{s,m}(k) + G_{d,m}(k) \Big) = \sum_{n=1}^{N} B_n(k) q.$$

Again, we will refer to $G(k)$ as the offered load at time t_k.

Assuming that the parameter q is small, one can approximate the probability that a data packet of connection m makes a successful transmission attempt between t_k and t_{k+1} by

$$\frac{G_{s,m}(k)}{1 - G_{s,m}(k)} \prod_{i=1}^{M} (1 - G_{s,i}(k))(1 - G_{d,i}(k)) \approx G_{s,m}(k) e^{-G(k)}, \quad (12)$$

and the probability that an ACK of connection m is successfully transmitted by

$$\frac{G_{d,m}(k)}{1 - G_{d,m}(k)} \prod_{i=1}^{M} (1 - G_{d,i}(k))(1 - G_{d,i}(k)) \approx G_{d,m}(k) e^{-G(k)}. \quad (13)$$

3.3 System Model

For the above rate and medium access control scheme, we define the system state as $x = (u, b_s, b_d)$ where $u = (u_1, ..., u_M)$ indicates the price associated with connection m, and the backlog vectors b_s and b_d are as previously defined. Note that at state $x = (u, b_s, b_d)$ the transmission rate of connection m is equal to $\lambda_m = \lambda_m(u_m)$.

The expected change in the price u_m of connection m between time t_k and t_{k+1} is given by

$$u_m(k+1) = [u_m(k) - \alpha e^{-G}(k) + \beta G(k) e^{-G(k)} + ... + \gamma(u_m(k))(1 - e^{-G(k)} - G(k) e^{-G(k)})]^+, \quad (14)$$

where $e^{-G(k)}$, $G(k)e^{-G(k)}$ and $1 - e^{-G(k)} - G(k)e^{-G(k)}$, are the probabilities that between t_k and t_{k+1} we have no transmission attempt, a successful transmission, or collision, respectively.

The expected change in the backlog $b_{s,m}$ between time t_k and t_{k+1} is given by

$$b_{s,m}(k+1) = b_{s,m}(k) + \lambda_m(k) L(k) - G_{s,m}(k) e^{-G(k)}, \quad (15)$$

and the expected change in the backlog $b_{d,m}$ between time t_k and t_{k+1} is given by

$$b_{d,m}(k+1) = b_{d,m}(k) + G_{s,m}(k) e^{-G(k)} - G_{d,m}(k) e^{-G(k)}. \quad (16)$$

Using Eq. (14)-(16), we can again model the dynamics as a discrete-time system

$$x(k+1) = \phi(x(k)), \quad k \geq 1,$$

and use the operating point $x^* = \phi(x^*)$ to characterize the system performance. The above model leads to a simpler mathematical structure of the system dynamics and we can analyze the above system for a general network topology.

Before we study the existence and properties of an operating point, we introduce the following result for the function $T(G)$ given by Eq. (10) that we use to choose the parameters q and the function γ.

LEMMA 1. *Under Assumption 2, there exists a unique* $G^+ \geq 0$ *such that*

$$G^+ = \arg\max_{G \geq 0} T(G).$$

Furthermore, we have that $G^* \in (0,1)$.

The lemma states that there exists a unique offered load G^+ that maximizes the function $T(G)$. The proof of Lemma 1 is straightforward and we omit a detailed derivation.

Using the above lemma, we make the following assumption on the retransmission parameter q and the function γ.

ASSUMPTION 7. *We have that*

(a) $q \leq \frac{L_i}{L(G^+)}$,

(b) $2\sum_m \lambda_m(\alpha) > T(G^+)$,

(c) $\alpha \leq \beta < \frac{\alpha}{G^+}$,

(d) $\gamma_{\min} \geq \frac{(\alpha - \beta G^+)e^{-G^+}}{(1-e^{-G^+} - G^+e^{-G^+})}$.

The above conditions (c) and (d) ensures that $\gamma_{\min} > 0$, and condition (b) implies that the unconstrained arrival rate $\sum_m \lambda_m(0)$ exceeds the maximal throughput $T(G^+)$. Note that Assumption 7 can always be satisfied by (i) choosing q small enough, (ii) appropriately choosing α and β, and (iii) choosing $\gamma_{\min}$ large enough.

3.4 Operating Point

We have the following result for the above system.

PROPOSITION 4. *Under Assumption 2 and Assumptions 5 - 7, there exists a unique operating point* x^*. *Furthermore, there exists a unique price* $u^* \geq 0$ *such that for all connections* $m \in \mathcal{M}$ *we have* $u_m^* = u^*$.

We provide a proof for Proposition 4 in Appendix B.

Having established the existence of a unique operating point, we next characterize the throughput at the operating point x^* and how the throughput is shared among different connections.

3.4.1 Throughput

To characterize the throughput at the operating point x^*, we use the following notation. For $G \geq 0$ and $y \geq 0$, let the function $f(G,y)$ be given by

$$f(G,y) = -\alpha e^{-G} + \beta G e^{-G} + y(1 - e^{-G} - Ge^{-G}).$$

Note that $f(G,y)$ is equal to the expected change in the price u_m as given by Eq. (14) for the offered load G and $\gamma_m(u_m) = y$. One can derive the following result for the function $f(G,y)$.

LEMMA 2. *Let Assumptions 6 - 7 hold. Then for* $y \geq \frac{(\alpha-\beta G^+)e^{-G^+}}{(1-e^{-G^+}-G^+e^{-G^+})}$, *there exists a unique* $\hat{G}(y) \geq 0$ *such that*

$$f(\hat{G}(y), y) = 0.$$

Furthermore, we have that $\hat{G}(y) \in (0, G^+]$ *and* $\hat{G}(y)$ *is strictly decreasing in* y.

Using Lemma 2, we immediately obtain the following result for the throughput at the operating point.

LEMMA 3. *Let Assumption 2 and Assumptions 5 - 7 hold. Then the offered load* G^* *at the operating point* x^* *satisfies the following condition,*

$$0 < G_{\min} < G^* < G_{\max} \leq G^+$$

where $G_{\min} = \hat{G}(\gamma_{\max})$ *and* $G_{\max} = \hat{G}(\gamma_{\min})$, *and* $\gamma_{\min}$ *and* $\gamma_{\max}$ *are as given in Assumption 6. Furthermore, the throughput* $T(G^*)$ *at the operating point* x^* *is such that* $T(G_{\min}) < T(G^*) < T(G_{\max})$.

Lemma 3 implies that the offered load G^* at the operating point depends on the control parameters α, β, and the function $\gamma(\cdot)$, but not on other parameters such as the number of connections M or the rate functions $\lambda_m(u_m)$, $m \in \mathcal{M}$. This is important as it means that one can tune the throughput $T(G^*)$ to lie within in a desired range $[T(G_{\min}), T(G_{\max})] \subset [0, T(G^+)]$ without having any prior knowledge of connection number M or rate functions $\lambda_m(u_m)$. Moreover, the throughput is lower bounded by $T(G_{\min})$ (and does not collapse) as the number of nodes in the network increases.

In Subsection 3.5, we provide an algorithm for choosing the parameters α and β, and the function γ, in order to obtain a desired throughput range $[T(G_{\min}), T(G_{\max})] \subset [0, T(G^+)]$.

3.4.2 Fairness

Proposition 4 states that at the operating point all connections see the same price u^*, i.e we have that $u_m^* = u^*$. $m \in \mathcal{M}$. As the throughput of connection m at the operating point is equal to $\lambda_m(u^*)$, this implies that per-flow fairness is obtained by assigning each connection same rate function $\lambda_m(u_m)$. However, more general bandwidth sharing (fairness) properties can be obtained by suitably choosing the rate functions λ_m, $m \in \mathcal{M}$. In particular, the rate function can be used in order to provide differentiated quality of service in terms of throughput. We will illustrate this in Subsection 3.6.

3.5 Choosing the Control Parameters

Below we provide an algorithm for choosing the control parameters α and β, and the function γ, to obtain the following system properties: (i) the operating point is unique, and (ii) the system throughput lies in a given range $[T(G_{\min}), T(G_{\max})]$, $G_{\min} < G_{\max} \leq G^+$.

(1) Choose a small $\alpha > 0$.

(2) Choose a small β such that $\alpha \leq \beta < \frac{\alpha}{G^+}$.

(3) Choose a non-negative strictly decreasing function $\gamma(u_m)$ satisfying the following conditions

  (a) $-2 < \gamma'(u_m) < 0$,

  (b) $\gamma_{\min} = \frac{\alpha e^{-G_{\max}} - \beta G_{\max} e^{-G_{\max}}}{1-e^{-G_{\max}} - G_{\max}e^{-G_{\max}}}$,

  (c) $\gamma_{\max} = \frac{\alpha e^{-G_{\min}} - \beta G_{\min} e^{-G_{\min}}}{1-e^{-G_{\min}} - G_{\min}e^{-G_{\min}}}$,

Condition (3-a) is needed to guarantee that the price signals converge; a discussion of this issue is beyond the scope of this paper.

One can show that choosing $G_{\min}$ and $G_{\max}$ introduces a trade-off with respect to performance and the speed of convergence, i.e. if $G_{\min}$ is very close to $G_{\max}$ then the possible throughput range $[T(G_{\min}), T(G_{\max})]$ will be small and $T(G^*)$ will always be close to the optimal throughput $T(G_{\max})$, however in this case $\gamma_{\min}$ will be close to $\gamma_{\max}$ which can lead to a slow convergence of the prices u_m.

3.6 Numerical Results

In this subsection, we illustrate the above results using a numerical case study for the network topology of Fig. 1 where we use the above algorithm for choosing the control parameters α and β, and the function γ.

Table 2 indicates the desired range of the system throughput for the different scenarios that we considered, where the goal was to obtain a system throughput that is close to theoretical maximum throughput T_{opt} given in Table 1.

Table 2: (L_i, L_p, L_c) versus $G_{\min}$, $G_{\max}$, $T(G_{\min})$ and $T(G_{\max})$

(L_i, L_p, L_c)	$G_{\min}$	$G_{\max}$	$T(G_{\min})$	$T(G_{\max})$
$(1, 100, 1)$	0.76	0.7710	9.68×10^{-3}	9.68×10^{-3}
$(1, 100, 17)$	0.29	0.3010	9.32×10^{-3}	9.32×10^{-3}
$(1, 100, 100)$	0.12	0.1310	8.65×10^{-3}	8.65×10^{-3}

Using the algorithm of the previous subsection, we choose the following control parameters α and β, and functions γ.

Table 3: (L_i, L_p, L_c) versus control parameters

(L_i, L_p, L_c)	α	β	$\gamma(u_m)$
$(1, 100, 1)$	0.05	0.06	$0.0022 \times e^{-u_m} + 0.0095$
$(1, 100, 17)$	0.005	0.01	$0.0059 \times e^{-u_m} + 0.0394$
$(1, 100, 100)$	0.005	0.03	$0.0706 \times e^{-u_m} + 0.1161$

The transmission parameter q is chosen to be equal to 0.001 and the rate functions $\lambda_m(u_m)$ are given by

$$\lambda_m(u_m) = \left[k_A\left(3e^{-(u_m-2)} - 1\right)\right]^+, m \in \mathcal{M}_A \quad (17)$$

$$\lambda_m(u_m) = \left[k_B\left(3e^{-(u_m-2)} - 1\right)\right]^+, m \in \mathcal{M}_B, \quad (18)$$

where the parameters k_A and k_B can be used to obtain different bandwidth sharing properties. For the case study, we use the setting $k_A = k_B = 0.01$ to obtain per-flow fairness. in addition we also use $k_A = 0.02$ and $k_B = 0.01$ so that a connection in subnet B should receive twice the rate of connections in subnet A. We use this setup to investigate whether the scheme us indeed able to provide differentiated quality of service in terms of throughput.

We simulate the system for 60,000 time slots. In each time slot t of the simulation, we generate new packets for connection m according to Poisson random variable with mean of $\lambda_m(u_m(t))$.

3.6.1 Throughput

Fig. 5 shows the time average system throughput for the different scenarios where we chose $k_A = k_B = 0.01$; the results for $k_A = 0.01$ and $k_B = 0.02$ were identical. The numerical results illustrate that the rate control mechanism is indeed able to provide a stable throughput as the number of nodes in the network increase. Also, the theoretical results predict well the actual time average system throughput.

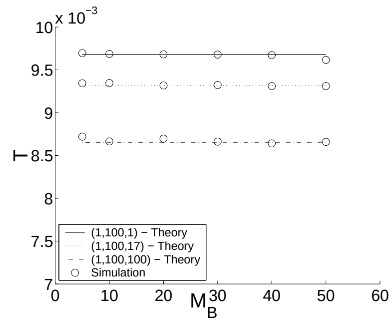

Figure 5: Time average system throughput for different values of (L_i, L_p, L_c).

3.6.2 Fairness

Fig. 6 illustrates the trajectory of the throughput of for one connection in the set $\mathcal{M}_A$ and one connection in the set $\mathcal{M}_B$ for the case where $(L_i, L_p, L_c) = (1, 100, 17)$, and $M_A = 3$ and $M_B = 10$. For $k_A = 0.01 = 0.01$, the time average throughput for a connection in subnet A is equal to 0.703×10^{-3} while the time average throughput of connection in subnet B is equal to 0.734×10^{-3}, and the two connections receive roughly the same throughput as intended.

For $k_A = 0.01$ and $k_B = 0.02$, the time average throughput for a connection in subnet A is equal to 0.386×10^{-3} while the time average throughput of connection in subnet B is equal to 0.801×10^{-3}, and the connection in subnet B receive roughly twice the throughput of connections in subnet A as intended.

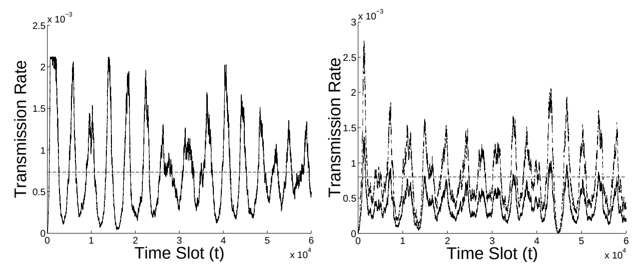

Figure 6: Trajectory of the transmission rates of one connection in subnet A (bottom) and one connection in subnet B for $k_A = k_B = 0.01$ (left) and $k_A = 0.01$ and $k_B = 0.02$ (right).

4. CONCLUSION

The above analysis provides new insight into the interaction between rate control and medium access control. In particular, it suggests that (a) a simple medium access scheme (as given in Section 3) is sufficient to provide per-flow fairness and (b) a stable throughput (as the number of nodes increases) can be achieved by suitably controlling the arrival rate of new packets to the system.

The above analysis has two important drawbacks: it considers only single cell networks and it requires an alternate rate control to TCP Reno in order to provide a stable throughput (which is not a realistic assumption). However, it seems possible to overcome both issues. In particular, using the above analysis one can design an active queue management scheme that works together with TCP Reno as a way to ensure a stable throughput as the number of nodes increases. Moreover, these mechanisms can be applied and analyzed for the case of a multihop ad hoc networks. These results will be presented in a forthcoming paper.

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APPENDIX

A. PROOF OF PROPOSITION 1, 2, AND 3

We use the following result to prove Proposition 1 - 3.

LEMMA 4. *Under Assumptions 1-4, the state $x^* \in \mathbb{R}_+^{3M}$ is an operating point for the system given by Eq. (7) - (9) if only if for all connections $m \in \mathcal{M}$ we have*

$$(a) \quad \lambda_m^* = \frac{G_{s,m}^* e^{-G^*}}{L(G^*)} = \frac{G_{d,m}^* e^{-G^*}}{L(G^*)} > 0$$

and

$$(b) \quad b_m^* = b_{s,m}^* + b_{d,m}^* = \sqrt{\frac{2(1 - P_m(B_{s(m)}^* + B_{d(m)}^*))}{P_m(B_{s(m)}^* + B_{d(m)}^*)}}.$$

PROOF. Let x^* be an operating point. By Assumption 2, we have $L(G^*) > 0$. Using Eq. (8) and (9), we obtain that at x^* we have

$$\lambda_m^* = \frac{G_{s,m}^* e^{-G^*}}{L(G^*)} = \frac{G_{d,m}^* e^{-G^*}}{L(G^*)}.$$

Note $\lambda_m^* > 0$ as otherwise we have that $G_{s,m}^* = G_{d,m}^* = 0$, and $b_{s,m}^* = b_{d,m}^* = 0$, which contradicts Eq. (4).

Furthermore, using Eq. (7) we obtain that at x^* we have

$$(\lambda_m^*)^2 \frac{1 - P_m\left(B_{s(m)}^* + B_{d(m)}^*\right)}{(b_m^*)^2} L(G^*) - \ldots$$
$$-(\lambda_m^*)^2 \frac{P_m\left(B_{s(m)}^* + B_{d(m)}^*\right)}{2} L(G^*) = 0,$$

and it follows that

$$b_m^* = b_{s,m}^* + b_{d,m}^* = \sqrt{\frac{2(1 - P_m(B_{s(m)}^* + B_{d(m)}^*))}{P_m(B_{s(m)}^* + B_{d(m)}^*)}}.$$

The above results establish that the conditions (a) and (b) of the lemma are necessary conditions. Furthermore, using Eq. (8) and (9) it is straightforward to check that the conditions of the lemma are sufficient, i.e. if the conditions hold then x^* is indeed an operating point. □

Using the above results, we can prove Proposition 1, 2 and 3 as follows.

PROOF. Using Lemma 4, at an operating point x we have $G_{s,m} = G_{d,m} > 0$. It follows that (see Subsection 2.2)

$$\frac{b_{s,m}}{B_{s(m)}} I_a(B_{s(m)}) = \frac{b_{d,m}}{B_{d(m)}} I_a(B_{d(m)}) > 0. \tag{19}$$

The above equation implies that for all connections $m \in \mathcal{M}$, we have at an operating point that

$$b_{s,m} > 0, \quad b_{d,m} > 0. \tag{20}$$

Let us first consider a given connection $m \in \mathcal{M}_A$. By definition we have $s(m) = n_0$ and $b_{d,m} = B_{d(m)}$. Using equation Eq. (19), we have at an operating point that

$$\frac{b_{s,m}}{B_{n_0}} I_a(B_{n_0}) = I_a(B_{d(m)}).$$

Recall that

$$B_{n_0} = \sum_{m \in \mathcal{M}_A} b_{s,m}. \tag{21}$$

It follows that $b_{s,m} < B_{n_0}$ and $\frac{b_{s,m}}{B_{n_0}} < 1$. Combining the above results we obtain that

$$I_a(B_{d(m)}) = \frac{b_{s,m}}{B_{n_0}} I_a(B_{n_0}) < 1. \tag{22}$$

Eq (22) implies that (see definition of I_a)

$$I_a(B_{d(m)}) = B_{d(m)} = b_{d(m)}$$

and we have

$$b_{s,m} = B_{n_0} b_{d,m} / I_a(B_{n_0}). \tag{23}$$

Using Assumption 4, we can then write the condition for b_m in Lemma 4 as

$$b_m = (B_{n_0}/I_a(B_{n_0}) + 1)b_{d,m} = \sqrt{\frac{2(1 - P(B_{n_0}))}{P(B_{n_0})}}. \quad (24)$$

As the above equation holds for any connection $m \in \mathcal{M}_A$, it follows that for all connections $m', m'' \in \mathcal{M}_A$, we have $b_{m'} = b_{m''}$, $b_{s,m'} = b_{s,m''}$, and $b_{d,m'} = b_{d,m''}$. It then follows that

$$b_{d,m} = \frac{I_a(B_{n_0})}{M_A}, \quad m \in \mathcal{M}_A. \quad (25)$$

Using the above result, we can rewrite Eq. (24) as $\frac{B_{n_0}+I_a(B_{n_0})}{M_A} = \sqrt{\frac{2(1-P(B_{n_0}))}{P(B_{n_0})}}$.

Let the two function $f_A(B_{n_0})$ and $g_A(B_{n_0})$ be given as follows,

$$f_A(B_{n_0}) = \frac{B_{n_0} + I_a(B_{n_0})}{M_A}$$

and

$$g_A(B_{n_0}) = \sqrt{\frac{2(1 - P(B_{n_0}))}{P(B_{n_0})}}.$$

If $B^*_{n_0}$ is the backlog at node n_0 at an operating point, then we have that

$$f_A(B^*_{n_0}) = g_A(B^*_{n_0}).$$

In the following, we construct such a solution. Note $g_A(B_{n_0})$ is strictly decreasing in B_{n_0} with $g(0) = \infty$ and $g_A(\infty) = 0$. Note $M_A \geq 2$. Furthermore, for $B_{n_0} \leq 1$ we have

$$f_A(B_{n_0}) = \frac{B_{n_0} + B_{n_0}}{M_A} \leq \frac{2}{M_A} \leq 1$$

and

$$g_A(B_{n_0}) > g(1) = \sqrt{\frac{2(1 - P(1))}{P(1)}} \geq 1,$$

where the last inequality from Assumption 3 which states that $P_m(1) \leq 2/3$. The above two equations imply that there does not exist a $B_{n_0} \leq 1$ such that $f_A(B_{n_0}) = g_A(B_{n_0})$. Hence, it suffices to consider $B_{n_0} > 1$ in order to construct a solution the equation $f_A(B_{n_0}) = g_A(B_{n_0})$. Note that for $B_{n_0} \geq 1$ we have

$$f_A(B_{n_0}) = \frac{(B_{n_0} + 1)}{M_A},$$

and f_A is continuous and strictly increasing in B_{n_0} with $f_A(1) = \frac{2}{M_A}$ and $f_A(\infty) = \infty$. Combining these results with the fact that g_A is continuous and strictly decreasing in B_{n_0} with $g_A(1) \geq 1$ it follows that there exists a unique $B_{n_0} > 1$ such that $f_A(B_{n_0}) = g_A(B_{n_0})$.

Let $B^*_{n_0}$ be the unique solution to $f_A(B^*_{n_0}) = g_A(B^*_{n_0})$, then for any connection $m \in \mathcal{M}_A$ we have at an operating point x^* that $b^*_{s,m} = \frac{B^*_{n_0}}{M_A}$, $b^*_{d,m} = \frac{1}{M_A}$, and $b^*_m = b^*_{s,m} + b^*_{d,m}$, are the unique solutions to Eq. 24 and Eq. 25.

Having characterized the backlog at an operating point for connections $m \in \mathcal{M}_A$, consider next a given connection $m \in \mathcal{M}_B$. In this case, we have $b_{s,m} = B_{s(m)}$ and $b_{d,m} = B_{d(m)}$, and we can rewrite Eq. 19 as

$$I_a(b_{s,m}) = I_a(b_{d,m}). \quad (26)$$

Let the functions f_b and g_b be given as follows

$$f_B(b_m) = b_m, \qquad b_m \geq 0$$

and

$$g_B(b_m) = \sqrt{\frac{2(1 - P_m(b_m))}{P_m(b_m)}}, \qquad b_m \geq 0.$$

Then the condition on b_m in Lemma 4 can be rewritten as $f_B(b_m) = g_B(b_m)$.

Using Assumption 3 which states that $P_m(2) \leq 1/3$ for $m \in \mathcal{M}_B$, we obtain that $g_B(2) = \sqrt{\frac{2(1-P_m(2))}{P_m(2)}} \geq 2$. Note $g_B(\infty) = 0$ and $g_B(b_m)$ is continuous and strictly decreasing for $b_m \geq 0$. Using the above results, it follows that there exists a unique $b^*_m > 2$ such that $f_B(b_m) = g_B(b_m)$. Combining these results with Eq. 26, we obtain that at an operating point the backlog $b^*_{s,m}$ and $b^*_{d,m}$ of a connection $m \in \mathcal{M}_b$ is such that know

$$\begin{cases} b^*_{s,m} \geq 1, \\ b^*_{d,m} \geq 1, \\ b^*_{s,m} + b^*_{d,m} \geq 2. \end{cases} \quad (27)$$

Note that there is an infinite set of values $b^*_{s,m}$ and $b^*_{d,m}$ which satisfy the above conditions.

The above two cases demonstrate that there always exists at least one operating point and if $M_B > 0$ then operating points are multiple.

Also note that at an operating point, we have $I_a(B_{n_0}) = 1$ and $I_a(B_{d(m)}) = 1/M_A$, $m \in \mathcal{M}_A$; $I_a(B_{s(m)}) = I_a(B_{d(m)}) = 1$, $m \in \mathcal{M}_B$.

Having characterized the backlog at an operating point, we obtain that the number of active nodes N^*_a at an operating point is given by $N^*_a = \sum_{n=1}^{N} I_a(B^*_n) = 2(1 + M_B)$, and the offered load at an operating point is given by $G^* = N^*_a q(N^*_a) = 2(1 + M_B)q(N^*_a)$.

Furthermore, for a connection $m_A \in \mathcal{M}_A$ we have at an operating point that

$$G^*_{d,m_A} = \frac{b^*_{d,m_A}}{B^*_{d(m_A)}} I_a(B^*_{d(m_A)})q(N^*_a) = q(N^*_a)/M_A$$

and

$$\lambda^*_{m_A} = \frac{q(N^*_a)e^{-G^*}}{M_A L(G^*)}.$$

Similarly, for a connection $m_B \in \mathcal{M}_B$, we have at an operating point that $G^*_{d,m_B} = I_a(B^*_{d(m_B)})q(N^*_a) = q(N^*_a)$ and $\lambda^*_{m_B} = \frac{q(N^*_a)e^{-G^*}}{L(G^*)}$.

□

B. PROOF OF PROPOSITION 4

The next lemma establishes that all connections have the same price (congestion) signal at an operating point.

LEMMA 5. *Under Assumption 6, if x^* is an operating point then there exists $u^* \geq 0$ such that*

$$u^*_m = u^*, \quad m \in \mathcal{M}.$$

PROOF. We prove the lemma by contradiction. Suppose that the result is not true and there exist connections $i, j \in \mathcal{M}$ such that $u^*_i > u^*_j$. By Assumption 6, we then have that

$$\gamma(u^*_i) < \gamma(u^*_j).$$

Furthermore, using Eq. (11) and Definition 1 we have at an operating point x^* that

$$\max\{-\alpha,-u_i^*\}e^{-G^*}+\beta G^*e^{-G^*}+\gamma(u_i^*)(1-e^{-G^*}-G^*e^{-G^*})=$$
$$\max\{-\alpha,-u_j^*\}e^{-G^*}+\beta G^*e^{-G^*}+\gamma(u_j^*)(1-e^{-G^*}-G^*e^{-G^*})=0.$$

Combing the above results, we obtain that

$$\max\{-\alpha,-u_i^*\}>\max\{-\alpha,-u_j^*\}. \qquad (28)$$

Suppose that $u_i^*>u_j^*\geq\alpha$. In this case, we have that

$$\max\{-\alpha,-u_i^*\}=-\alpha=\max\{-\alpha,-u_j^*\}$$

and Eq. (28) does not hold.

Similarly, if $\alpha\geq u_i^*>u_j^*$ then we have

$$\max\{-\alpha,-u_i^*\}=-u_i^*<-u_j^*=\max\{-\alpha,-u_j^*\},$$

and Eq. (28) does not hold.

Finally, if $u_i^*>-\alpha\geq u_j^*$ then we have

$$\max\{-\alpha,-u_i^*\}=-\alpha\leq-u_j^*=\max\{-\alpha,-u_j^*\},$$

and Eq. (28) again does not hold.

The above results establish that Eq. (28) never holds; hence we obtain a contradiction and the lemma is true. □

The next lemma shows that for the price u^* of Lemma 5 we have that $u^*\in(\alpha,\infty)$.

LEMMA 6. *Let Assumptions 2, 5, and 7 hold. If x^* is an operating point and u^* is such that $u_m^*=u^*$, $m\in\mathcal{M}$, then we have that $\alpha<u^*<\infty$.*

PROOF. We first show that $u^*>\alpha$. Suppose that this is not true. Using Eq. 15 and (16), we have that

$$\lambda(u^*)=\sum_{m\in\mathcal{M}}\lambda_m(u^*)=\frac{G^*e^{-G^*}}{2L(G^*)}=\frac{1}{2}T(G^*), \qquad (29)$$

where the function $T(G)$ is as given by Eq. (10). By Assumption 7, we have that $\lambda(\alpha)>T(G^+)$ and

$$T(G^*)=2\lambda(u^*)\geq2\lambda(\alpha)>T(G^+).$$

The above inequality contradicts the definition of G^+ and it follows that $u^*>\alpha$.

Next, we show that $u^*<\infty$. Suppose that $u^*=\infty$. Then we have that

$$\lambda(u^*)=\sum_{m\in\mathcal{M}}\lambda_m(u^*)=0,$$

and

$$\max\{-\alpha,-u^*\}e^{-G^*}+\beta G^*e^{-G^*}+\gamma(u^*)(1-e^{-G^*}-G^*e^{-G^*})$$
$$=\max\{-\alpha,-u^*\}=-\alpha<0.$$

However, this contradicts the fact that x^* is an operating point and it follows that $u^*<\infty$. □

Using the above lemmas, we show that there exists a unique operating point.

LEMMA 7. *Under Assumptions 2, and 5 - 7, there exists a unique operating point x^*.*

PROOF. Using Lemma 5 and 6, we have at an operating point x^* that

$$-\alpha e^{-G}+\beta Ge^{-G}+\gamma(u^*)(1-e^{-G}-Ge^{-G})=0.$$

Let the function $\rho:[0,\infty)\mapsto[0,\infty)$ be such that for $y=\rho(G)$ we have that

$$\lambda(y)=\sum_{m\in\mathcal{M}}\lambda_m(y)=2T(G).$$

One can show that under Assumption 5 and 7 the function ρ is well-defined.

Let the function $f:[0,\infty)\mapsto\mathbb{R}$ be given by

$$f(G)=-\alpha e^{-G}+\beta Ge^{-G}+\gamma(\rho(G))(1-e^{-G}-Ge^{-G}).$$

Note that in order to show that an operating point exists, we have to show that there exists a $G^*\geq0$ such that $f(G^*)=0$. Below we construct such a solution.

Let $f'(G)$ be the derivate of $f(G)$. We have that

$$\begin{aligned}f'(G)&=\alpha e^{-G}+\beta e^{-G}(1-G)+...\\&+\gamma'(\rho(G))\rho'(G)(1-e^{-G}-Ge^{-G})+\gamma(\rho(G))Ge^{-G},\end{aligned}$$

where $\rho'(G)$ is the derivative of $\rho(G)$.

One can show that $\rho(G)$ is strictly decreasing for $0\leq G<G^+$; and we have that $\gamma'(\rho(G))\rho'(G)<0$, $0\leq G<G^+$. Combining this with the fact that by Assumption 7 we have $\beta>0$, it follows that $f'(G)>0$, $\quad 0\leq G<G^+$. Furthermore, we have $f(0)=-\alpha<0$.

Recall that by Lemma 1 we have $G^+<1$. Furthermore, by Assumption 5, we have that $\rho(G)<\infty$ for $G\in[G^+,1]$. Combining this with the fact that by Assumption 7 we have $\gamma_{\min}\geq\frac{-(\alpha+\beta G^+)e^{-G^+}}{(1-e^{-G^+}-G^+e^{-G^+})}$, we obtain

$$\begin{aligned}f(G)&=\alpha e^{-G}+\beta Ge^{-G}+\gamma(\rho(G))(1-e^{-G}-Ge^{-G})\\&>\alpha e^{-G^+}+\beta G^+e^{-G^+}+\gamma_{\min}(1-e^{-G}-Ge^{-G})\\&\geq\alpha e^{-G^+}+\beta G^+e^{-G^+}+...\\&\quad+\frac{-(\alpha+\beta G^+)e^{-G^+}}{(1-e^{-G^+}-G^+e^{-G^+})}(1-e^{-G^+}-G^+e^{-G^+})\\&=0,\qquad G^+\leq G\leq1.\end{aligned}$$

Finally, by Assumption 7 we have $\alpha\leq\beta$ and if follows that

$$\begin{aligned}f(G)&=\alpha e^{-G}+\beta Ge^{-G}+\gamma(\rho(G))(1-e^{-G}-Ge^{-G})\\&\geq\alpha e^{-G}+|\alpha|Ge^{-G}+\gamma(\rho(G))(1-e^{-G}-Ge^{-G})\\&>\alpha e^{-G}+|\alpha|e^{-G}+\gamma(\rho(G))(1-e^{-G}-Ge^{-G})\\&=\gamma(\rho(G))(1-e^{-G}-Ge^{-G})>0,\quad G>1. \qquad (30)\end{aligned}$$

Using the above results, we have that $f(0)<0$ and $f(G)$ is strictly increasing for $G\in(0,G^+)$. Furthermore, we have that $f(G)>0$ for $G\geq G^+$. As under Assumption 5 and 6 the function $f(G)$ is continuous, it follows that there exists a unique $G^*\in(0,G^+)$ such that $f(G^*)=0$. Using this unique solution G^* we can construct an operating point x^* by setting

$$\begin{aligned}b_{s,m}^*&=b_{d,m}^*=\frac{G_m^*}{2q}=\frac{G^*\frac{\lambda_m(u^*)}{\lambda(u^*)}}{2q},\\u_m^*&=u^*=\rho(G^*).\end{aligned}$$

□