Wireless Social Community Networks: A Game-Theoretic Analysis

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***Abstract*—Wireless social community networks have been created as an alternative to cellular wireless networks to provide wireless data access in urban areas. By relying on access points owned by users for access, wireless community networks provide a wireless infrastructure in an inexpensive way. However, the coverage of such a network is limited by the set of users who open their access points to the social community. Using this observations, it is not clear to what degree this paradigm can serve as a replacement, or a complimentary service, of existing centralized networks operating in licensed bands (such as cellular networks). This question currently concerns many wireless network operators. In this paper, we study the dynamics of wireless social community networks using a simple analytical model. In this model, users choose their service provider based on the subscription fee and the offered coverage. We first consider the case where users decided whether or not to join the social community network, and study the evolution of the number of members in the community. For this case, we the dynamics of the community depends on the initial coverage (i.e. number of initial subscribers), the subscription fee, the user preferences for coverage, as well as on the access points density. Furthermore, we derive a pricing strategy that allows the wireless social community to reach a high coverage. Next, we study the case where the mobile users can choose between the services provided by a licensed band operator and those of a social community. Using a game-theoretic framework, we show that for specific distribution of user preferences, there exists a Nash equilibrium for this non-cooperative game. Using the Nash equilibrium, we characterize the number of users that subscribe to each service provider.**

I. Introduction

Wireless social community networks have been created as an alternative to cellular wireless networks to provide wireless data access in urban areas. Traditionally, wireless access has been provided by cellular networks that are operated by central authorities (i.e. the owner of the radio band). The advantage of cellular wireless networks is that they can guarantee a high *quality of service* (QoS) in terms of *network coverage*; however this comes at the expense of substantial deployment and maintenance costs. Wireless social community networks operate in the unlicensed band and rely on users having a WiFi access point to provide access. Thus, there is no need for an operator to make substantial initial investment to buy the spectrum license. Furthermore, the *access points* (AP) are inexpensive, easy to deploy and maintain. However, wireless social community might have a poor coverage as the coverage depends on number of users that subscribe to the social community network.

In this paper, we study how effective wireless social communities networks are, where we are in particular interested in the case where a wireless social community networks competes with traditional a licensed band cellular network. To do this, we first investigate how users decide whether or not to join the social community network, and study the evolution of the number of members in the community by modeling users' payoffs as a function of the subscription fee[1], as well as the operators' provided coverage. For this case, we derive pricing strategies to maximize the coverage of the social community network. Next, we study the competition between a social community operator and cellular wireless network using a game-theoretic framework. For this case, we investigate the existence a Nash equilibrium, and characterize the number of users that subscribe to each service provider under a Nash equilibrium. Due to space constraints we present our results without proof and will focus at several instances on particular special cases, as discussed in the following.

The rest of the paper is organized as follows. In Section II, we characterize the properties of users, the licensed band operator and the social community operator. In Section III and IV, we evaluate the dynamics of these networks separately and derive the maximum payoff and the corresponding optimal number of subscribers. In Section V, we model the competition of these two types of network operators and discuss their coexistence. Finally we conclude the paper in Section VII.

II. System Model

Consider two network operators, a traditional *licensed band operator* (LBO or ℓ) and *social community operators* (SCO or s), that compete for providing access to a set of N *users*. Each provider charges a subscription fee P_i, $i \in \{l, s\}$. Users decided at discrete time instances $t = 1, 2, ...$ to which provider they subscribe. If a user subscribes to provider i, $i \in \{l, s\}$, then it pays a subscription fee $P_i[t]$ at time t. Let $n_s[t]$ be the fraction of users that subscribes to the SCO, and let $n_i[t]$ be the fraction for users subscribing to the LBO.

In the following we assume that the LBO always provides a coverage $Q_l[t] = 1$, i.e., all users that subscribe to the LBO

[1]Note that the *subscription fee* corresponds to the *price* users have to pay. Hence, we use the two terms interchangeably in the paper.

always have access to a base station. On the other hand, the coverage $Q_s[t]$ provided by the SCO at time t depends on the number of users $n_s[t]$ that subscribe to the SCO. Here, we use the following simple relation to model the situation. We assume that

$$Q_s[t] = \min\{1, \lambda n_s[t]\},$$

where λ is a non-negative constant modeling the density of the access points owned by users (see also Fig. 1). For example, a large λ captures the case where access points are very dense as it might be the case in a city center. Given the subscription

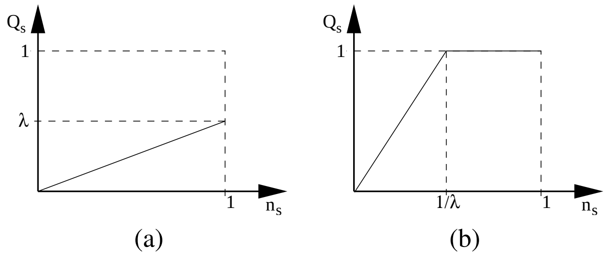

Fig. 1. Relation between fraction of subscribers n_s and the social community coverage Q_s: (a) $0 < \lambda \leq 1$, (b) $1 < \lambda$.

fee P_i and coverage Q_i of a provider $i \in \{l, s\}$, the benefit that a given user v obtains by subscribing to provider i is given by

$$u_v^i = a_v Q_i - P_i, \tag{1}$$

where a_v is a non-negative parameter that characterizes the user sensitivity with respect to coverage. In the following we assume a large user population and that the users' sensitivity towards coverage is uniformly distributed in $[\alpha, \beta]$, $\alpha \geq 0$. As a result, we let the fraction of users with a sensitivity towards coverage that is larger than a given value $x \in [\alpha, \beta]$ be given by $\frac{\beta - x}{\beta - \alpha}$. The payoff of operator i, $i \in \{\ell, s\}$, at time t is given by

$$u_i[t] = \quad N \cdot n_i[t] \cdot P_i - c_i, \quad i \in \{\ell, s\}, \tag{2}$$

where $N n_i[t]$ is the total number of users subscribing to provider i at time t and c_i is the operating cost of provider i per unit time slot.

III. Optimal Pricing Strategy of LBO

Consider the situation where the LBO is the only wireless access provider in a given area. For this case, we are interested in determining the optimal price P_l^*, that the LBO should charge per unit time in order to maximize its revenue. Note that under a given price P_l, only user for which the payoff u_v^l given by Equation (1) is non-negative subscribes to the LBO, and the fraction of users n_l that subscribes to the LBO under the price P_l is given by

$$n_\ell = \frac{1}{\beta - \alpha} \left(\beta - \max\{\alpha, P_\ell\}\right). \tag{3}$$

The resulting payoff of the LBO is given by

$$u_\ell = \frac{N}{\beta - \alpha} (\beta - \max\{\alpha, P_\ell\}) \cdot P_\ell - c_\ell \tag{4}$$

The following lemma shows the optimal price of LBO.

Lemma 1: The optimal price P_l^* is given by

$$P_\ell^{opt} = \max\{\alpha, \frac{\beta}{2}\}. \tag{5}$$

The fraction of users n_l^* that subscribes to the LBO under the price P_l^* is given by

$$n_l^* = \max\{1, \frac{1}{2} \frac{\beta}{\beta - \alpha}\}.$$

IV. Optimal Pricing Strategy of SCO

Next we consider the situation where the SBO is the only wireless provider in a given area. For this case, we are again interested in determining the optimal price P_s^* the SCO should charge per unit time in order to maximize its revenue. Here we assume that at time t users observe the coverage $Q_s(t-1)$ at time $t-1$ and the subscription fee $P_s[t]$ at time t. Using this information, a user v then subscribes to the SCO if

$$u_v^s[t] = a_v Q_s(t-1) - P_s[t] \geq 0.$$

Under a fixed price $P_s[t] = P_s$, $t \geq 0$, the fraction of users $n_s[t]$ that subscribe to the SCO at time t, and hence the coverage $Q_s[t]$ of the SCO at time t, is then a function of the coverage $Q_s(t-1)$ at the previous time step. In particular, we have that

$$Q_s[t] \quad = \quad \min\{1, \frac{\lambda}{\beta - \alpha}(\beta - \max\{\alpha, \frac{P_s}{Q_s[t-1]}\})\}$$

In the following, we study (a) how the coverage $Q_s[t]$ evolves over time under a fixed price P_s and (b) what price P_s^* the SCO should charge in order to maximize its revenue. Before we start our analysis, we observe the following results.

Lemma 2: If $Q_s[0] = 0$ and $P_s[t] = P_s > 0$, $t \geq 0$, then we have that $Q_s[t] = 0$, $t \geq 0$.

Lemma 3: If $P_s[t] = P_s > 0$, $t \geq 0$, and $\frac{P_s}{Q_s[0]} \leq \alpha$ then $Q_s[t] = \min\{1, \lambda\}$, $t \geq 0$.

A. Dynamics of the SCO under a Fixed Price P_s

In this subsection we assume that the SCO charges a fixed price P_s and study for this case how the coverage $Q_s[t]$ evolves as a function of the initial coverage $Q_s[0]$. For this analysis, we focus on the case where $\lambda \in (0, 2)$ and $\beta \geq \frac{2}{2-\lambda}\alpha$. The analysis, and system behavior, for the general case is similar to this situation. In the following, let $Q_{s,1}$ and $Q_{s,2}$ be given as follows,

$$Q_{s,1} = \frac{\beta\lambda - \sqrt{\beta^2\lambda^2 - 4(\beta - \alpha)P_s\lambda}}{2(\beta - \alpha)}$$

and

$$Q_{s,2} = \frac{\beta\lambda + \sqrt{\beta^2\lambda^2 - 4(\beta - \alpha)P_s\lambda}}{2(\beta - \alpha)}.$$

The following results characterize the dynamics of the coverage $Q_s[t]$ for the above case. In our analysis, we distinguish different cases depending on the price P_s (see also Fig. 2). We first consider the case where the price P_s is low.

Lemma 4: Suppose that

$$P_s \in [0, \beta - \frac{1}{\lambda}(\beta - \alpha)].$$

If $Q_s[0] < Q_{s,1}$ then $\lim_{t\to\infty} Q_s[t] = 0$; otherwise $\lim_{t\to\infty} Q_s[t] = \min\{1, \lambda\}$.

The next result is for the case where the price P_s is higher than $\beta - \frac{1}{\lambda}(\beta - \alpha)$ but smaller than $\frac{\beta^2\lambda}{4(\beta-\alpha)}$.

Lemma 5: Suppose that

$$\beta - \frac{1}{\lambda}(\beta - \alpha) < P_s < \frac{\beta^2\lambda}{4(\beta - \alpha)}.$$

If $Q_s[0] < Q_{s,1}$ then we have that $\lim_{t\to\infty} Q_s[t] = 0$. If $Q_s[0] > Q_{s,1}$ then $\lim_{t\to\infty} Q_s[t] = Q_{s,2}$. Finally, if $Q_s[0] = Q_{s,1}$ then we have $Q_s[t] = Q_{s,1}$, $t \geq 0$.

The third case is the situation where the price P_s is exactly equal to $\frac{\beta^2\lambda}{4(\beta-\alpha)}$.

Lemma 6: Suppose that

$$P_s = \frac{\beta^2\lambda}{4(\beta - \alpha)}.$$

If $Q_s[0] < Q_{s,1}$ then $\lim_{t\to\infty} Q_s[t] = 0$. If $Q_s[0] \geq Q_{s,1}$ then $\lim_{t\to\infty} Q_s[t] = Q_{s,1} = Q_{s,2} = \frac{\beta\lambda}{2(\beta-\alpha)}$.

Finally, we consider the case where P_s is high, i.e. if P_s is larger than $\frac{\beta^2\lambda}{4(\beta-\alpha)}$.

Lemma 7: If

$$P_s > \frac{\beta^2\lambda}{4(\beta - \alpha)}$$

then we have that $\lim_{t\to\infty} Q_s[t] = 0$.

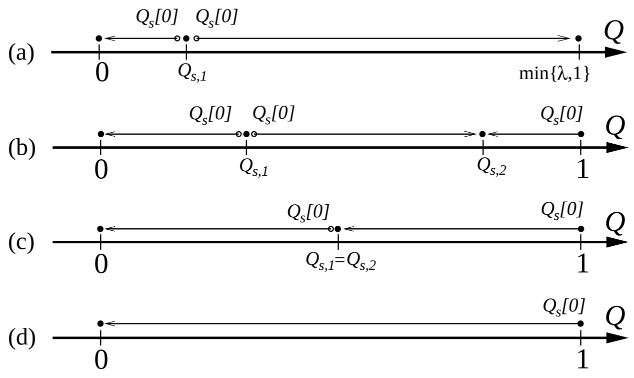

Fig. 2. Dynamics of SCO for $\lambda < 2$ and $\beta \geq \frac{2}{2-\lambda}\alpha$: (a) $0 < P_s \leq \beta - \frac{1}{\lambda}(\beta - \alpha)$, (b) $\beta - \frac{1}{\lambda}(\beta - \alpha) < P_s < \frac{\beta^2\lambda}{4(\beta-\alpha)}$, (c) $P_s = \frac{\beta^2\lambda}{4(\beta-\alpha)}$, (d) $P_s > \frac{\beta^2\lambda}{4(\beta-\alpha)}$.

B. Optimal Static Price

In the previous section, we presented how SCO can select a price to have various final coverage. In this section and with following two theorems we derive the optimal static prices with which the social community operator can maximize its payoff at one of the defined final coverage presented in Lemmas 4 to 6. Recall that $\lambda \in (0, 2)$ and $\beta \geq \frac{2}{2-\lambda}\alpha$.

Theorem 1: If the initial coverage $Q_s[0]$ is smaller than $\lambda\frac{\beta}{\beta-\alpha} - 1$, the SCO should select a price P_s such that $0 < P_s \leq \beta - \frac{1}{\lambda}(\beta - \alpha)$. The best value of P_s which maximizes the SCO payoff is then $P_s^{opt} = Q_s[0] \cdot (\beta - \frac{1}{\lambda}(\beta - \alpha) \cdot Q_s[0])$. The final fraction of subscribed users and coverage are $n_s = 1$ and $Q_s = \min\{1, \lambda\}$, respectively.

Theorem 1 corresponds to the convergence scenario presented in Lemma 4. Since for any price bigger than $\beta - \frac{1}{\lambda}(\beta - \alpha)$ the $Q_{s,1}$ is smaller than $\lambda\frac{\beta}{\beta-\alpha} - 1$, the SCO must select a small price as identified in Lemma 4. Taking into account its initial coverage, the SCO calculates the above price to get all users subscribed to the service.

Theorem 2: If the initial coverage $Q_s[0]$ is bigger than $\lambda\frac{\beta}{\beta-\alpha} - 1$, the optimal static price is $\beta - \frac{1}{\lambda}(\beta - \alpha)$ for $\beta < \frac{3}{\lambda}\alpha$ and the final fraction of subscribed users is $n_s^{opt} = 1$. If $\beta \geq \frac{3}{\lambda}\alpha$, the optimal price is $P_s^{opt} = \frac{2\lambda}{9}\frac{\beta^2}{(\beta-\alpha)}$ and the final fraction of subscribed users is $n_s^{opt} = \frac{2}{3}\frac{\beta}{\beta-\alpha}$.

Since $Q_s[0]$ is bigger than $\beta - \frac{1}{\lambda}(\beta - \alpha)$ the SCO can select the price bigger than $\beta - \frac{1}{\lambda}(\beta - \alpha)$ and its final coverage will be $Q_{s,2}$ (i.e., Lemma 5). Considering the boundary conditions and the behavior of SCO payoff, we conclude that if the distribution of user types is narrow ($\beta < \frac{3}{\lambda}\alpha$) then the best strategy of the SCO is to choose the price equal to $\beta - \frac{1}{\lambda}(\beta - \alpha)$ where its final coverage is $Q_{s,2} = \min\{1, \lambda\}$, hence Lemma 4. While if the distribution of user types is wide enough ($\beta \geq \frac{3}{\lambda}\alpha$) the optimal price which can maximize the SCO payoff is then equal to $P_s^{opt} = \frac{2\lambda}{9}\frac{\beta^2}{(\beta-\alpha)}$. It is worth mentioning that the SCO can select its price without taking into account the value of $Q_s[0]$ since its initial coverage is always bigger than $Q_{s,1}$.

C. Optimal Dynamic Price

Let us now assume that the SCO adjusts its price P_s at time t to follow the evolution of its network. The essential difference between static and dynamic pricing is that with dynamic pricing the SCO can maintain a lower price until a desired coverage is reached and then fine-tune the price. Since ΔQ_s must be strictly positive, we derive the following condition on the dynamic price:

$$P_s[t] < -\frac{1}{\lambda}(\beta - \alpha)Q_s^2[t-1] + \beta Q_s[t-1] \qquad (6)$$

Using the above dynamic price strategy, the SCO can maintain the increase of the coverage by selecting appropriate dynamic price $P_s[t]$ at each decision time t. If $\beta < \frac{3}{\lambda}\alpha$ the SCO will converge to $Q_s = \min\{\lambda, 1\}$ where all users subscribe to the SCO ($n_s = 1$) whereas for $\beta \geq \frac{3}{\lambda}\alpha$ it will converge to $Q_{s,2} = \frac{2\lambda}{3}\frac{\beta}{\beta-\alpha}$, where $\frac{2}{3}\frac{\beta}{\beta-\alpha}$ fraction of users subscribes to service.

V. Competition between a SCO and a LBO

In this section we consider the situation where a single LBO and SCO co-exist in a given area, and compete for mobile users to subscribe to their service. We model this situation as a two-player non-cooperative *pricing game* where two operators are the *players* [1]. The operators compete through their subscription price and the the *strategy* of operator i in the game is given by its price P_i.

Again we assume that users make decision at discrete time steps $t = 1, 2, \cdots$. Recall the definition of the utility u_v^i that user v achieves when it subscribes to a provider $i \in \{l, s\}$. Given subscription fees P_l and P_s, and observing the coverage $Q_s[t-1]$ of the SCO at time $t-1$, user v will choose at time t the provider i which leads to the largest utility u_v^i at time t. Of course, user v will only subscribe to this provider if the resulting utility is non-negative; otherwise the user will not subscribe to any provider. Let $n_l[t]$ and $n_s[t]$ the resulting fraction of users that subscribe to the LBO and the SCO, respectively.

Given fixed prices P_l and P_s, we call $Q_s(P_l, P_s)$ an equilibrium coverage if for $Q_s[0] = Q_s(P_l, P_s)$ we have under P_l and P_s that $Q_s[t] = Q_s(P_l, P_s)$, $t \geq 0$. Similarly, we define the equilibrium markets shares $n_l(P_l, P_s)$ and $n_s(P_l, P_s)$, and the corresponding equilibrium payoffs $u_l(P_l, P_s)$ and $u_s(P_s, P_l)$ of the LBO and the SCO, respectively. Using the above definitions, a Nash equilibrium for the above game is given as follows.

Definition 1: The price pair (P_l^*, P_s^*) constitutes a Nash equilibrium if for each operator $i \in \{l, s\}$ we have

$$u_i(P_i^*, P_j^*) \geq u_i(P_i, P_j^*), \forall P_i \geq 0. \tag{7}$$

At a Nash equilibrium, none of the operators has an incentive to unilaterally change its subscription fee as this will not increase its payoff. In the following we study whether there exists a Nash equilibrium for the above game. To simplify the analysis, we assume that $\alpha = 0$.

Theorem 3: Suppose that $\alpha = 0$. If $\lambda \in (0, 3)$ then there exists a unique Nash equilibrium given by

$$(P_\ell^*, P_s^*) = \left(\frac{\beta}{2} \cdot \frac{1 - Q_s^*}{1 - \frac{Q_s^*}{4}}, \frac{\beta Q_s^*}{4} \cdot \frac{1 - Q_s^*}{1 - \frac{Q_s^*}{4}} \right) \tag{8}$$

where $Q_s^* = 2 - \sqrt{4 - \lambda}$. The fraction of users that subscribe to the SCO at the Nash equilibrium is given by $n_s^* = \frac{1}{\lambda} Q_s^* = \frac{1}{2+\sqrt{4-\lambda}}$, and the fraction of users that subscribe to the LBO is given by $n_l^* = \frac{2}{2+\sqrt{4-\lambda}}$.

If $\lambda \geq 3$, then there exists a unique Nash equilibrium $(P_l^* = 0, P_s^* = 0)$ with $Q_s(P_l^*, P_s^*) = 1$. However the fraction of users that subscribe the each operator are not uniquely determined. In particular, any market share n_l^* and n_s^* such that $n_s^* \geq 1/\lambda$ and $n_l^* + n_s^* = 1$ may be realized at a Nash equilibrium.

A. Discussion

The above analysis implies that there always exists a unique Nash equilibrium. Furthermore, for $\lambda \in (0, 3)$ the market share of each provider is uniquely determined. The market share that each operator grabs at a Nash equilibrium depends on the parameter λ, i.e. the density of access points. The market share of the SCO tends to increase as λ increases, and for $\lambda \geq 3$ the might be able to grap the whole market.

It is interesting to note that the prices (P_l^*, P_s^*) charged at a Nash equilibrium tend to decrease as λ increases. This suggests that the SCO influences the pricing behavior of a LBO, and that the presence of a SCO in area with a dense network of WiFi access points might be able to significantly reduce the fees charged for wireless access. We note that the above results are intuitive, suggesting that the simple model that we used for our analysis is indeed able to capture the main features of the competition between a LBO and SCO.

VI. Related Work

The wireless community networks over unlicensed band have been recently deployed by some ISPs such as Free [4] in France or FON, a worldwide WiFi community operator funded by Google and Skype [3]. A charging model for wireless social community networks without a centralized authority is proposed by Efstathiou *et al.* [5]. Their solution relies on reciprocity among subscribers. In [6], Zemlianov and de Veciana evaluate using a stochastic geometric model, the cooperation between licensed band WAN and WLAN service providers. A complete evaluation of our model for $\lambda = 1$ is presented in [7].

VII. Conclusions

In this paper we have analyzed the dynamics of social community and licensed band operators using a game-theoretic approach. We have presented and evaluated an analytical model in two scenarios: (1) a *monopoly*, in which a unique operator offers the wireless access, and (2) a *duopoly*, in which both operators compete for subscribers. We have obtained the pricing strategies that maximize the payoff of the operators in both settings. We have considered two pricing strategies: static and dynamic pricing for the social community operator. We have also derived the equilibrium points of the SCO coverage and determined the price which achieves the maximal SCO payoff. We have recognized that the SCO payoff in monopoly is not only affected by the distribution of user types, but also by its initial provided coverage. We have concluded that the SCO should first bootstrap its network with low prices to reach a fair coverage before adjusting its price to maximize its revenue. This conclusion nicely matches the behavior of real wireless social communities [3]. We have finally considered the co-existence of a LBO and a SCO and computed the potential Nash equilibria for such game.

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