Rate Control and Slotted Aloha

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Consider the situation where many nodes communicate over a slotted multiaccess broadcast channel. All packets have the same length and each packet requires one time unit for transmission. All nodes are synchronized such that the transmission of each packet starts at an integer time $t \geq 0$. We refer to the slot beginning at time t as slot t. If two or more nodes transmit a packet in a given time slot, then there is a collision and the receiver obtains no information about the contents or source of the transmitted packets. Packets that experience a collision become backlogged and are retransmitted at a later time. When all nodes involved in a collision immediately retransmit, then another collision would surely occur. Instead, each backlogged node retransmits a packet with a fixed probability q, $0 < q < 1$, in each successive slot until a successful transmission occurs. If just one node sends a packet in a given time slot, the packet is successfully received by the destination node(s). We assume that packets are generated at each node by application(s). Whenever a node receives a new packet from an application, it transmits the packet in the first slot after its arrival, thus risking occasional collisions but achieving a small delay if collisions are rare. The above channel model is the well-known slotted Aloha channel.

If the rate at which applications generate packets is left uncontrolled, then the above channel can get clogged with collisions. Here we consider a rate control mechanism which uses the channel output as feedback information. The intuition behind our approach is that nodes should reduce their rates when the total number of backlogged packets becomes large, thereby avoiding collisions in the future and an even further increase in the backlog. As nodes are remote and the total backlog is not globally known, we form an estimate u, $u \geq 0$, of the system backlog which we update based on the channel output. We use the estimate u as a control signal (congestion measure) to modulate the rate at which nodes are allowed to generate new packets. For the analysis of this rate control scheme we assume an infinite set of nodes. New packets are generated according to a Poisson process where each new packet arrives to a new node. Let $\lambda(u)$ denote the aggregate packet arrival rate which is modulated by a control signal u. We refer to $\lambda(u)$ as the *rate function*. To ensure that nodes increase (decrease) their rate in the even of an idle (collision) slot, we make the following assumption.

Assumption 1 *The rate function $\lambda(u)$ is bounded, continuous and strictly decreasing. Furthermore, there exists a positive constant $u_{\max}$ such that $\lambda(u) = 0$ when $u \geq u_{\max}$.*

Let Z_t be the random variable indicating the total number of transmission attempts in slot t. Note that Z_t includes the transmissions of new packets as well as the retransmissions of backlogged packets. Let $I[\cdot]$ be the indicator function and let $I[Z_t = 0]$, $I[Z_t = 1]$, and $I[Z_t \geq 2]$ indicate an idle slot, a successful transmission, or a collision, in slot t. Let u_t be the control signal that determines the rate of arrivals of new packets in slot t. We then update the control u_t by setting

$$u_{t+1} = \big[u_t + \alpha I[Z_t = 0] + \beta I[Z_t = 1] + \gamma I[Z_t \geq 2]\big]^+, \tag{1}$$

where $[x]^+$ denotes $\max\{x, 0\}, x \in \mathbb{R}$. The parameters α, β and γ are real-valued control updates corresponding to the observed channel outputs of "idle", "success" and "collision" respectively. To ensure that nodes increase (decrease) their rate in the event of an idle (collision) slot, we make the following assumption.

Assumption 2 *We have that $\alpha < 0$ and $\gamma > 0$; while β can take on any real value.*

In the following, we study whether this scheme stabilizes the slotted Aloha channel and how we should choose the control parameters α, β, and γ.

Let n_t be the total number of backlogged packets at the beginning of slot t. Note that we have that

$$n_{t+1} = n_t + Y_t - I[Z_t = 1], \tag{2}$$

where Y_t is a random variable indicating the number of newly arrived packets in slot t. Eq. (1) and (2) describe the dynamics of the control signal and backlog, and the system can be modeled as a Markov chain with a countable state space.

Proposition 1 *The Markov chain given by Eq.* (1) *and* (2) *is positive recurrent.*

Having established stability, in the next step we would like to characterize the expected throughput and delay of the system. We use the following notation. Given a state (n, u), consider the functions $d_n(n, u)$ and $d_u(n, u)$ given by

$$\begin{aligned} d_n(n, u) &\triangleq \mathbf{E}\big(n_{t+1} - n_t \,\big|\, n_t = n, u_t = u\big), \\ d_u(n, u) &\triangleq \mathbf{E}\big(u_{t+1} - u_t \,\big|\, n_t = n, u_t = u\big). \end{aligned}$$

Definition 1 *We call $(n^*, u^*) \in \mathbb{R}^2_+$ an operating point of the system if $d_n(n^*, u^*) = d_u(n^*, u^*) = 0$, We refer to $G^* = \lambda(u^*) + n^* q$ as the offered load at the operating point (n^*, u^*).*

We note that the offered load G^* characterizes both the average throughput and delay at an operating point [1]. We have the following result.

Proposition 2 *There always exists an operating point. Furthermore, when we have that $\lambda\big(\max\{-\alpha, -\beta\}\big) \geq e^{-1}$, then the operating point is unique.*

The next lemma establishes we can set offered load at the operating point by suitably choosing the parameters α, β, and γ.

Lemma 1 *Let G be a given offered load, and let $\alpha < 0$ and $\gamma > 0$ be given control parameters. For*

$$\beta = \frac{\gamma}{G}(G + 1 - e^G) - \frac{\alpha}{G},$$

the offered load G^* at every operating point is such that

$$\begin{aligned} G^* &= G \qquad \text{if} \quad \lambda\big(\max\{-\alpha, -\beta\}\big) \geq e^{-1}, \\ G^* &\leq G \qquad \text{if} \quad \lambda\big(\max\{-\alpha, -\beta\}\big) < e^{-1}. \end{aligned}$$

For a more detailed derivation and discussion of the above results, we refer to [1].

References

[1] Clement Yuen, "Rate Control with Pricing in Contention-Based Access Networks," Master Thesis, Department of Computer Science, University of Toronto, 2002.