Machine Learning I
MATH60629

Sequential Decision Making I
Summary
— Week #12

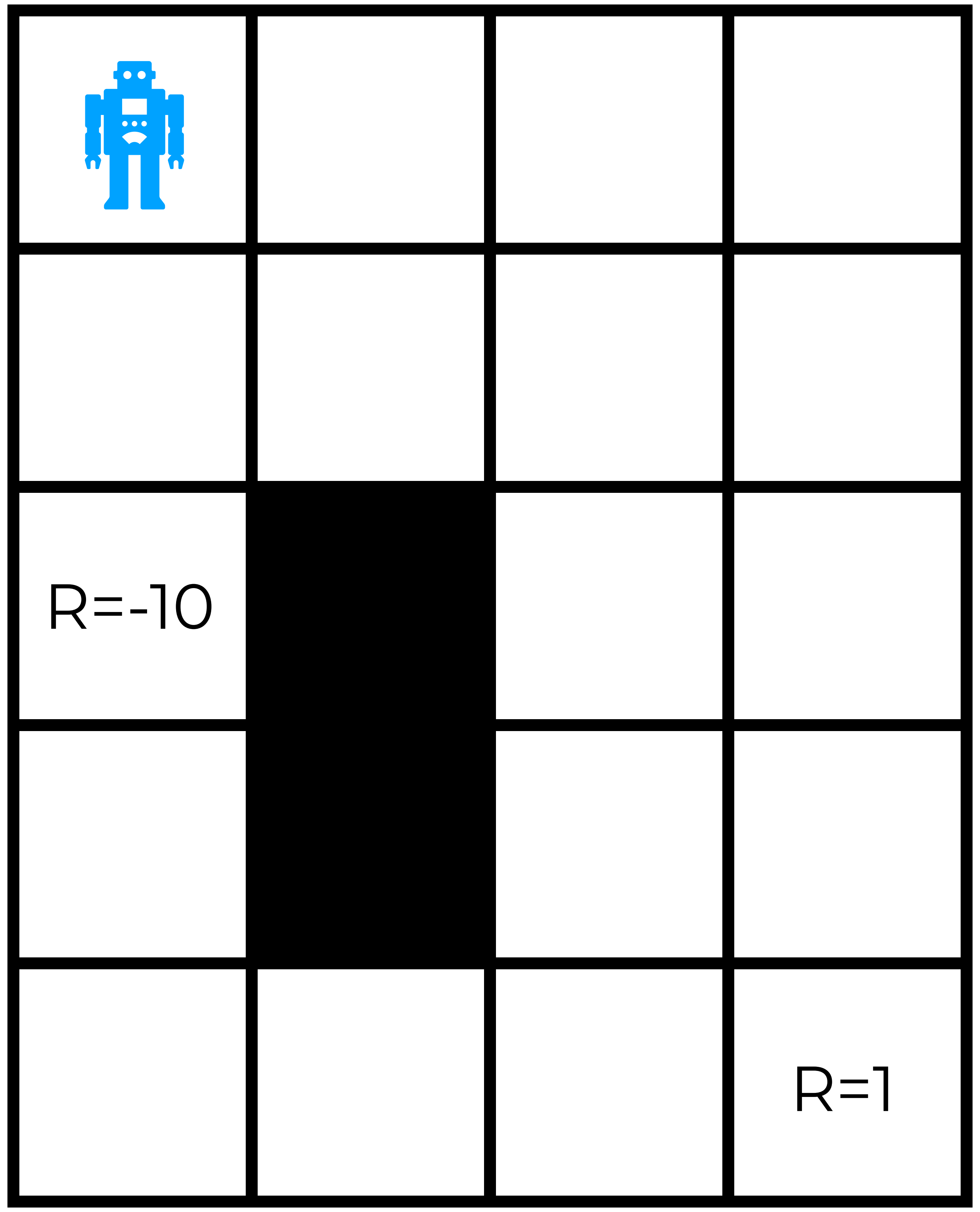
Recall

Three main components

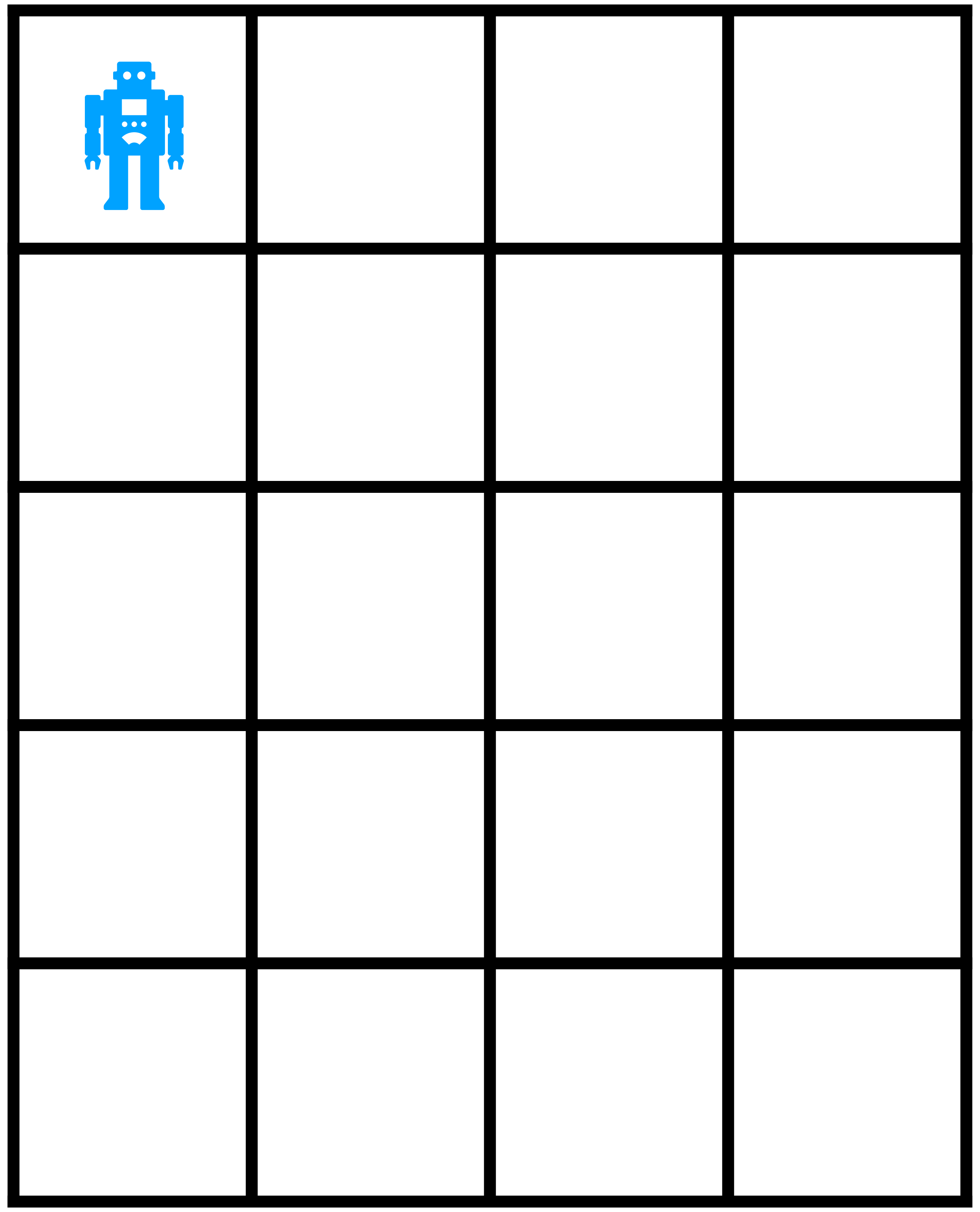
- Task (T)
- Performance measure (P)
- Experience (E)

Supervised learning

- Experience a fixed data set
  - Fit a model using this data
  - Use the model to make predictions about unseen data
  - Eventually: Predictions may be used downstream


R=-10
R=1

Initial example with grid world



- Each cell is a state (S)
- Actions indicate which movements are possible: $A := \{L, R, U, D\}$
- Rewards encode the task: $R(s)$
- Transition probabilities encode the outcome of an action: $P(s' \mid s, a)$

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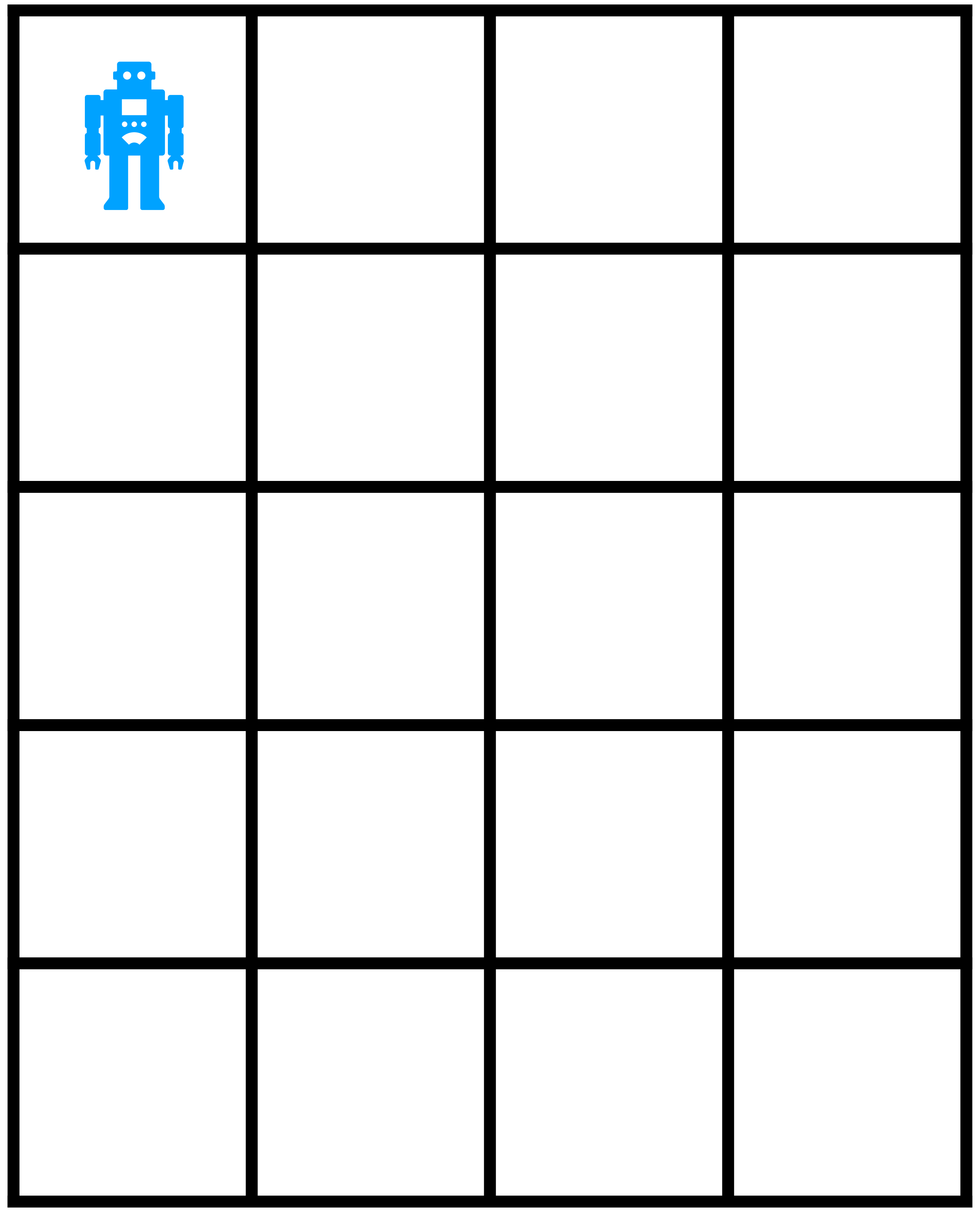

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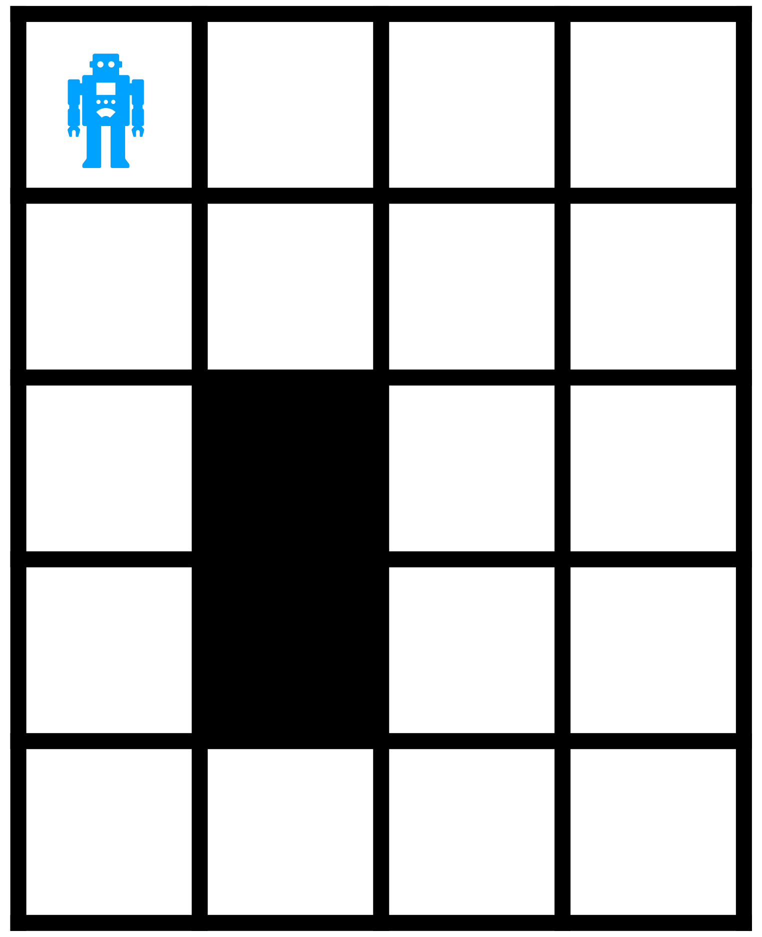
Planning

This week we discuss a version of RL where these are observed

Markov Decision Process (MDP)

- Provide a framework for decision-making under uncertainty
  - Markov process with decisions and utilities
  - Assumes stationarity (i.e., transitions are fixed across time)



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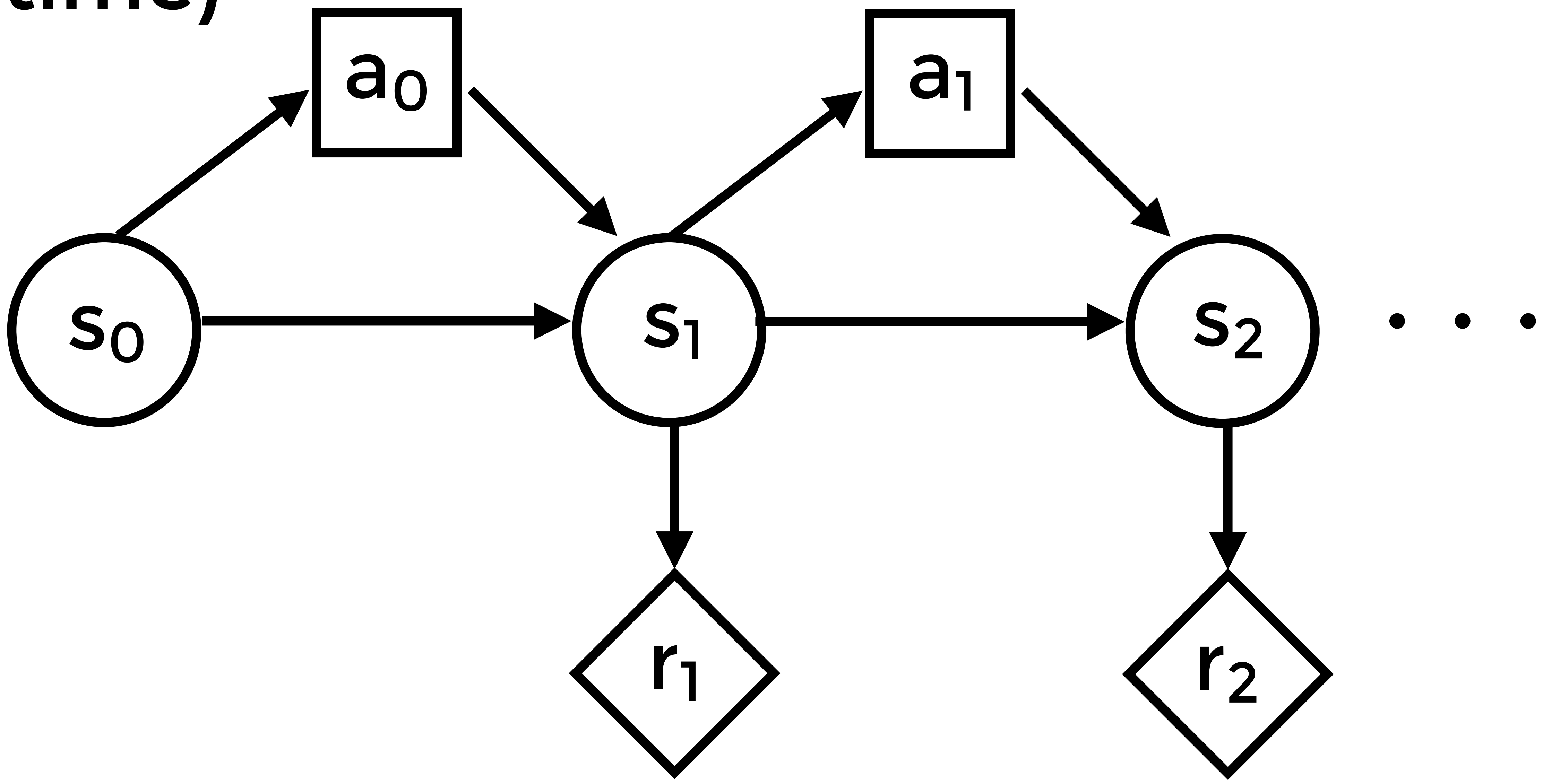
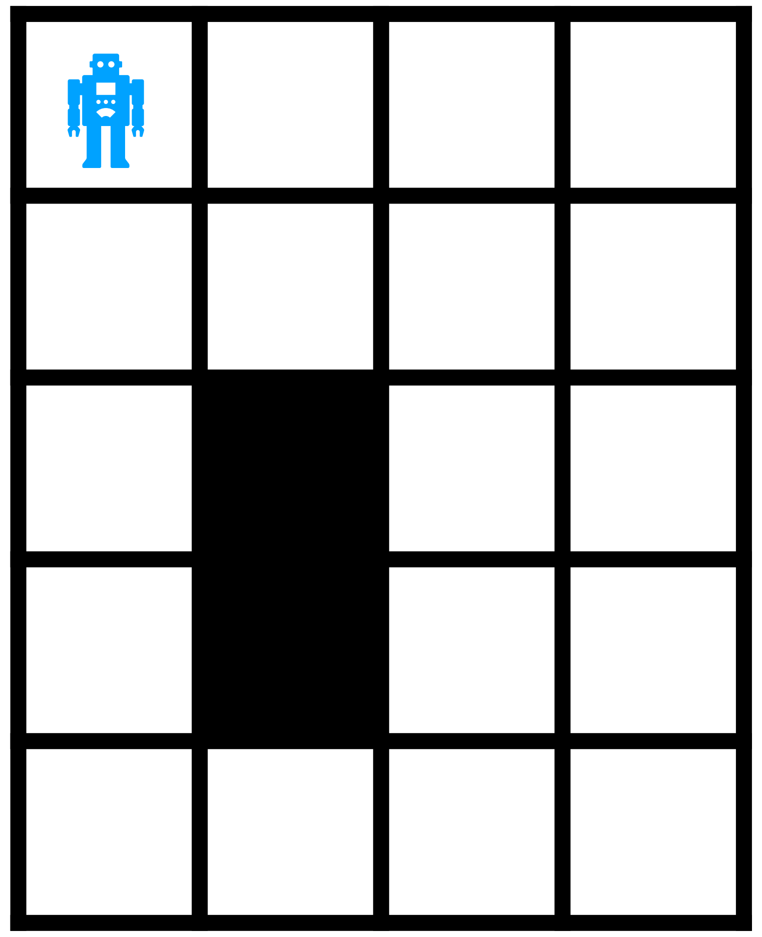
- Square nodes: decisions
- Circle nodes: States
- Diamond nodes: utility





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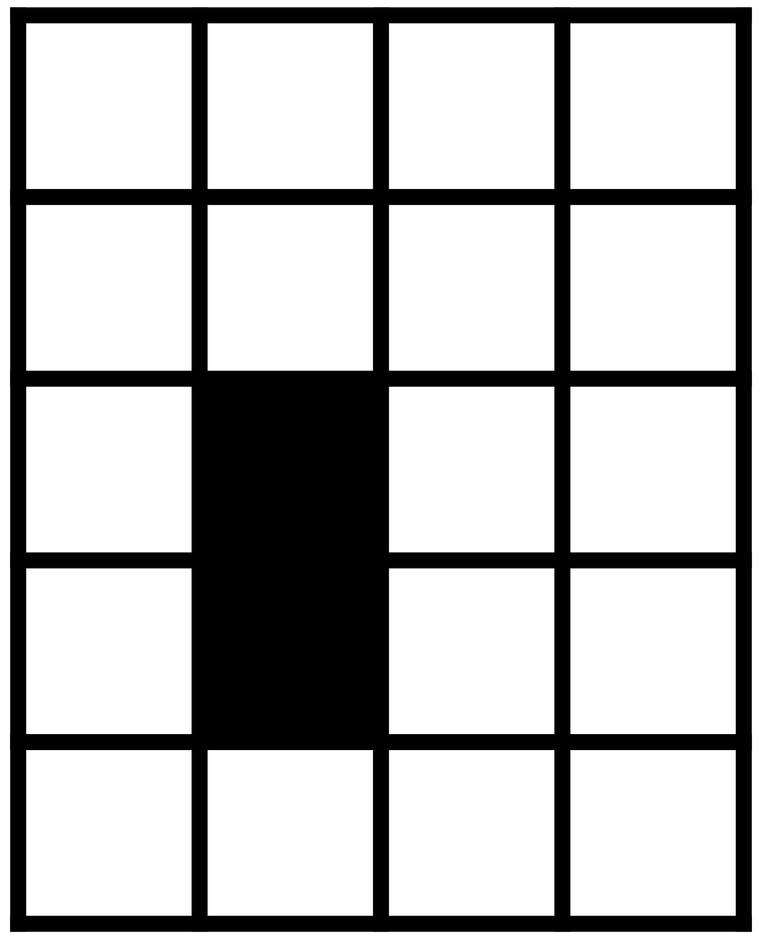
$\langle A, S, P, R, \gamma \rangle$

- A: set of actions
- P(S' | S,A): transition probabilities
- R(S): reward function
- γ : discount factor $\in [0, 1]$



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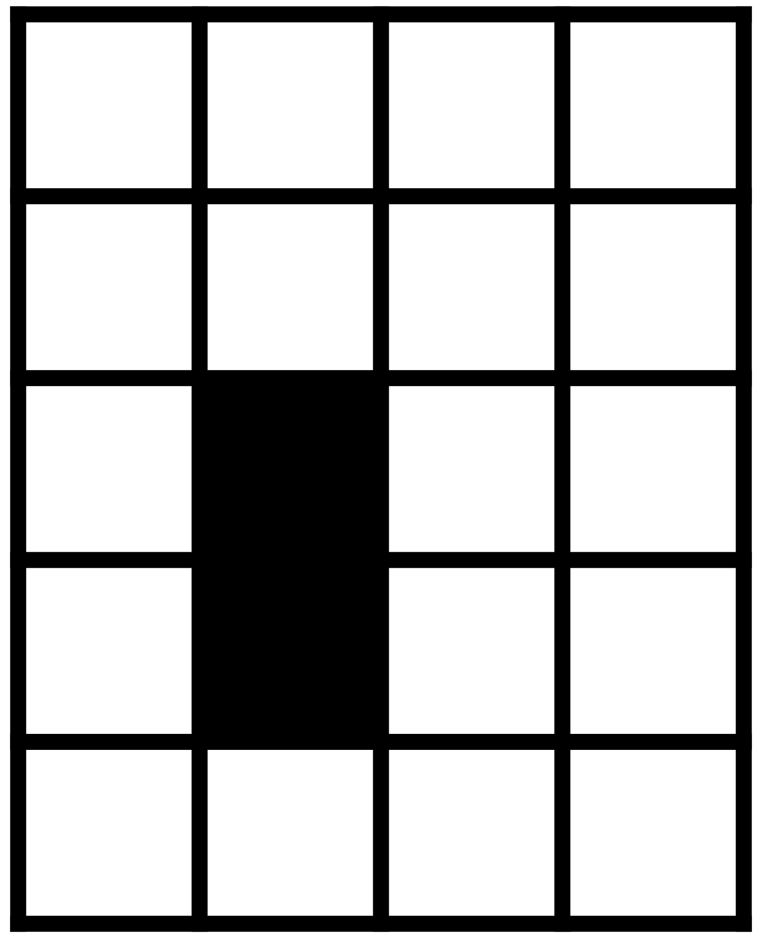

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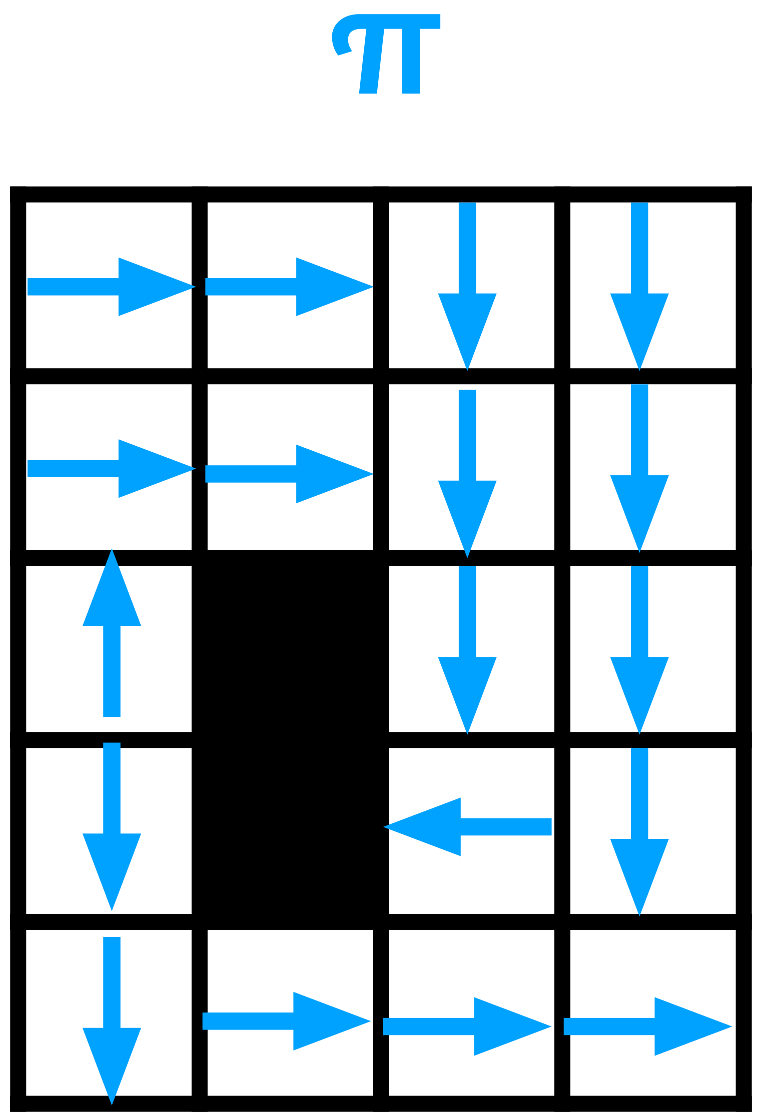

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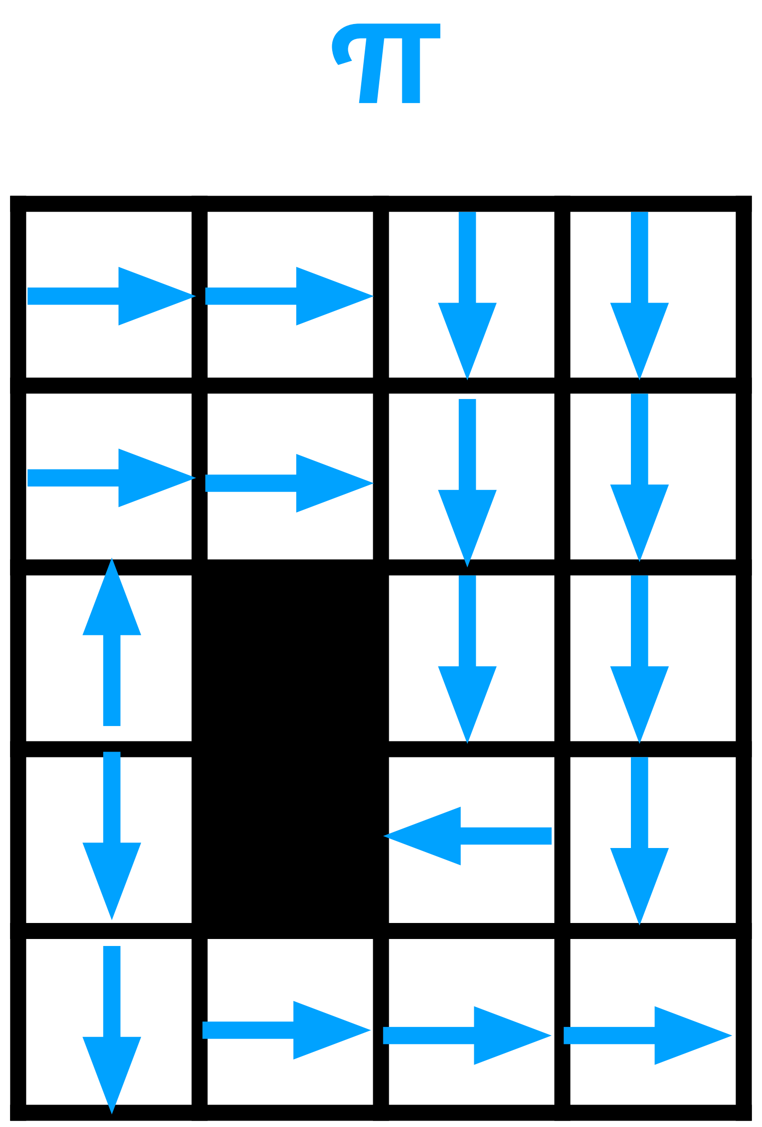
- A policy: π : S $\rightarrow$ A
- **Goal:** find the optimal policy



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- **Find the optimal policy of an MDP**

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- The optimal policy is the one with highest expected utility: $EU(\pi^*) \geq EU(\pi) \ \ \forall \pi$

Solving an MDP

- Three well-known techniques:
  1. Value iteration
  2. Policy Iteration
  3. Linear Programming

Value Function

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$$V(s_t) := \text{expected sum of rewards of being in s}$$

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- The policy is implicit
  - Once converged: $\boldsymbol{\pi}^*(\mathbf{s}) = \arg\max_{\mathbf{a}} \left\{ \mathbf{R}(\mathbf{s}) + \gamma \sum_{\mathbf{s}'} \mathbf{P}(\mathbf{s}' \mid \mathbf{s}, \mathbf{a}) \mathbf{V}^*(\mathbf{s}') \right\} \; \forall \mathbf{s}$

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Policy Evaluation

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Policy Update

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- Demo of the PI algorithm in a deterministic environment

http://www.cs.toronto.edu/~lcharlin/courses/60629/reinforcejs/gridworld_dp.html