This assignment is due at the **start** of your lecture on Friday, 14 Feb. 2020.

1. [10 marks]

   Assume that $f : \mathbb{D} \to \mathbb{R}$ is twice continuously differentiable for all $x \in \mathbb{D}$, where the domain $\mathbb{D}$ of f is an open, convex subset of $\mathbb{R}^n$. Show that, its Hessian matrix, $\nabla^2 f(x)$, is symmetric positive-semi-definite for all $x \in \mathbb{D}$ if and only if f is a convex function on $\mathbb{D}$.

   Moreover, show that, if its Hessian matrix, $\nabla^2 f(x)$, is symmetric positive-definite for all $x \in \mathbb{D}$, then f is a strictly convex function on $\mathbb{D}$.

   Show that the converse of this last statement is not true. That is, there is a strictly convex function on an open, convex domain $\mathbb{D}$ such that its Hessian matrix, $\nabla^2 f(x)$, is not symmetric positive-definite for all $x \in \mathbb{D}$.

2. [5 marks]

   Do question 2.1 on page 27 of your textbook.

3. [5 marks]

   Do question 2.8 on page 28 of your textbook.

4. [10 marks]

   Do question 2.11 on page 29 of your textbook.

   You can take the result of question 2.10 as given; you don't have to include a proof of it. However, for your own interest, you might try to verify this result.

5. [10 marks]

   Do question 2.12 on page 29 of your textbook.

   Take the phrase "at the solution x^*" in the question to mean that x^* is a local minimizer of $f(x)$.

   If A is a nonsingular matrix, it's condition number is $\text{cond}(A) = \|A\| \cdot \|A^{-1}\|$; take $\text{cond}(A) = \infty$ if A is singular. It's best to use the 2-norm in this question. In this case, if A is symmetric positive-semi-definite, then

   $$\text{cond}_2(A) = \|A\|_2 \cdot \|A^{-1}\|_2 = \frac{\lambda_{\text{max}}}{\lambda_{\text{min}}}$$

   where λ_{max} is the largest eigenvalue of A, λ_{min} is the smallest eigenvalue of A and both $\lambda_{\text{max}} \geq 0$ and $\lambda_{\text{min}} \geq 0$. However, you will see when you do the question that $\lambda_{\text{max}} > 0$. If $\lambda_{\text{min}} = 0$, then A is singular and, as noted above, in this case you can take $\text{cond}(A) = \infty$.

6. [5 marks]

   Do question 2.16 on page 29 of your textbook.