

This assignment is due at the start of your lecture on Tuesday, 22 October 2019.

This assignment is a continuation of Assignment 2 in which you wrote a MatLab program that uses Monte Carlo simulation to estimate the price at time $t = 0$ of a European Call Option on a stock with price S_t at time $t \in [0, T]$, where

- the initial stock price is $S_0 = \$100.00$,
- the strike price is $K = \$100.00$ (i.e., the option is *at the money*),
- the time to maturity is $T = 0.25$,
- the risk-free interest rate is $r = 0.02$, and
- the volatility is $\sigma = 0.2$.

In this assignment, you are to use the three variance reduction methods

1. antithetic variates (also called antithetic sampling),
2. control variates, and
3. stratified sampling

to price the option.

For control variates, state what your control variate is, how you determined its expectation and how you estimated the covariance between it and the option price.

For this part of the problem, pretend that you don't know the Black-Scholes formula. So, you can't use a put or a call as your control variate, since you are pretending that you don't know the associated expected value.

Hint: one possibility for a control variate is $S_T - K$, since it is not too hard to show

$$\mathbb{E}[e^{-rT}(S_T - K)] = S_0 - e^{-rT}K \quad (1)$$

If you use $S_T - K$ as your control variate, show that (1) holds.

Can you come up with a better control variate than $S_T - K$? If so, briefly explain why you think your control variate is better than $S_T - K$.

For stratified sampling, describe how you chose your strata and why you think your choice is appropriate for this problem.

For each of the three variance reduction techniques indicated above, before your MatLab program performs your main Monte Carlo simulation, it should do a pilot computation to approximate any needed parameters and to estimate how large the number of replications, N ,

should be so that you can estimate the price of the option to within $\pm \$0.01$ at a confidence level of 99%.

For each of the three variance reduction techniques indicated above, carry out the Monte Carlo simulation with the number of replications, N , estimated in your pilot computation and print the estimated price, \hat{V}_N , of the option as well as the associated confidence interval at the 99% confidence level.

Compare your estimated prices to the Black-Scholes-formula price computed by the MatLab function `blsprice`. (If you don't have access to the MatLab function `blsprice`, you can write your own using the Black-Scholes formula on page 110 of the Brandimarte textbook.)

Also, compare the number of replications, N , used by each of the variance reduction techniques indicated above to the number of replications, N , required by Monte Carlo with no variance reduction (i.e., your results from Assignment 2).