CSC321: Introduction to Neural Networks and Machine Learning

Lecture 22: Transforming autoencoders for learning the right representation of shapes

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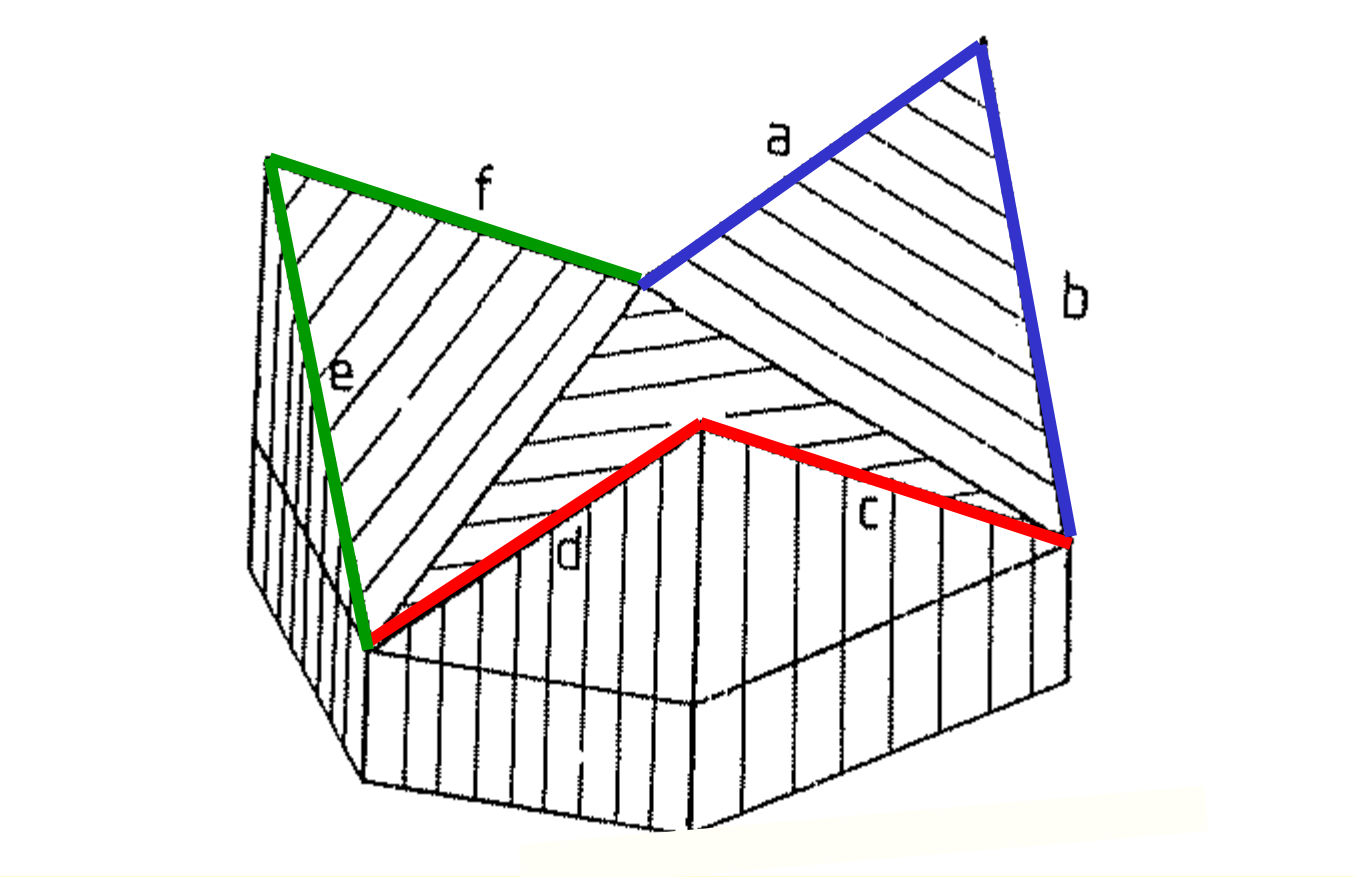
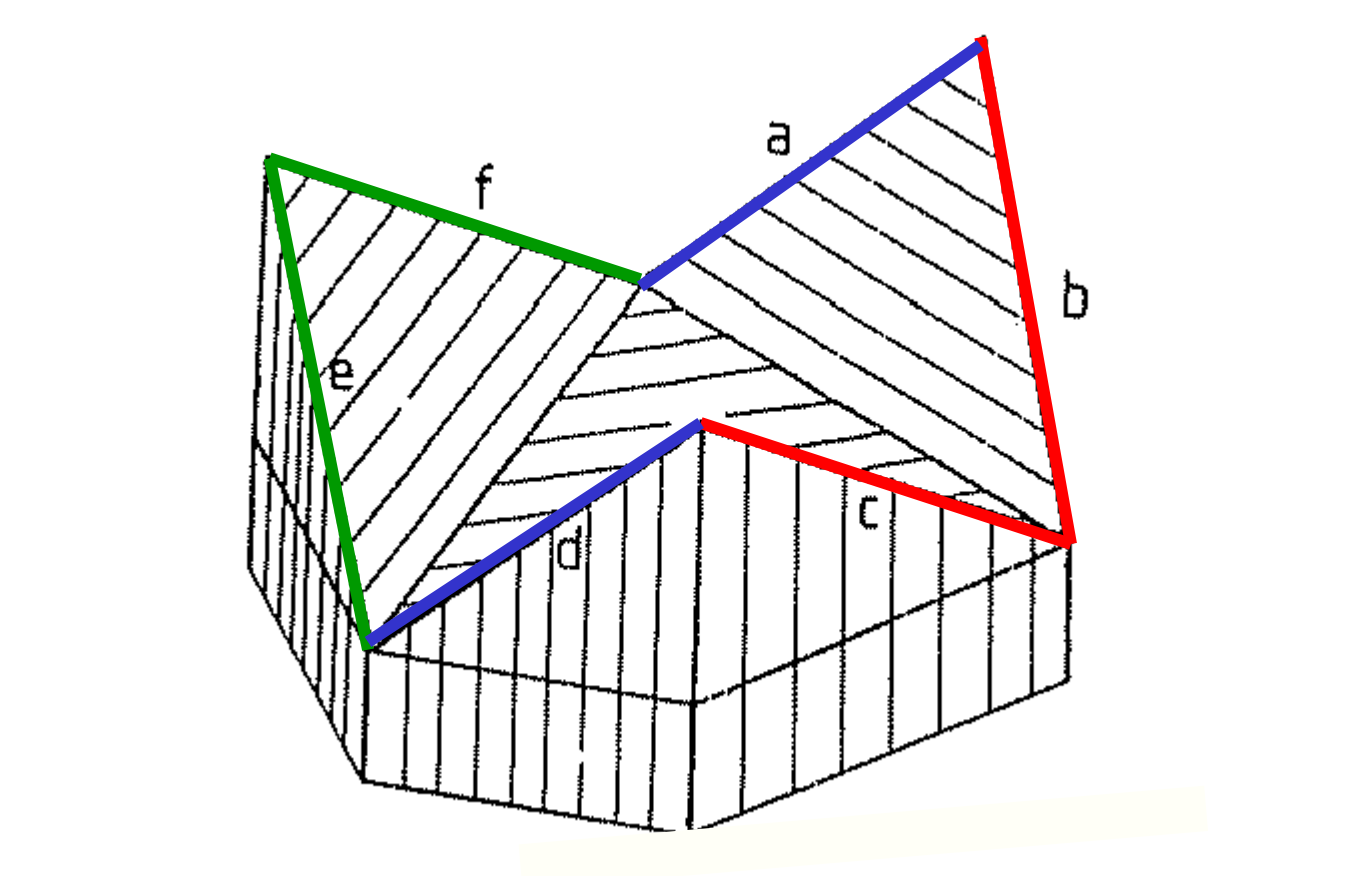
What is the right representation of images?

- Computer vision is inverse graphics, so the higher levels should look like the representations used in graphics.
  - Graphics programs use matrices to represent spatial relationships.
  - Graphics programs do not use sigmoid belief nets to generate images.
- There is a lot of psychological evidence that people use hierarchical structural descriptions to represent images.

An arrangement of 6 rods



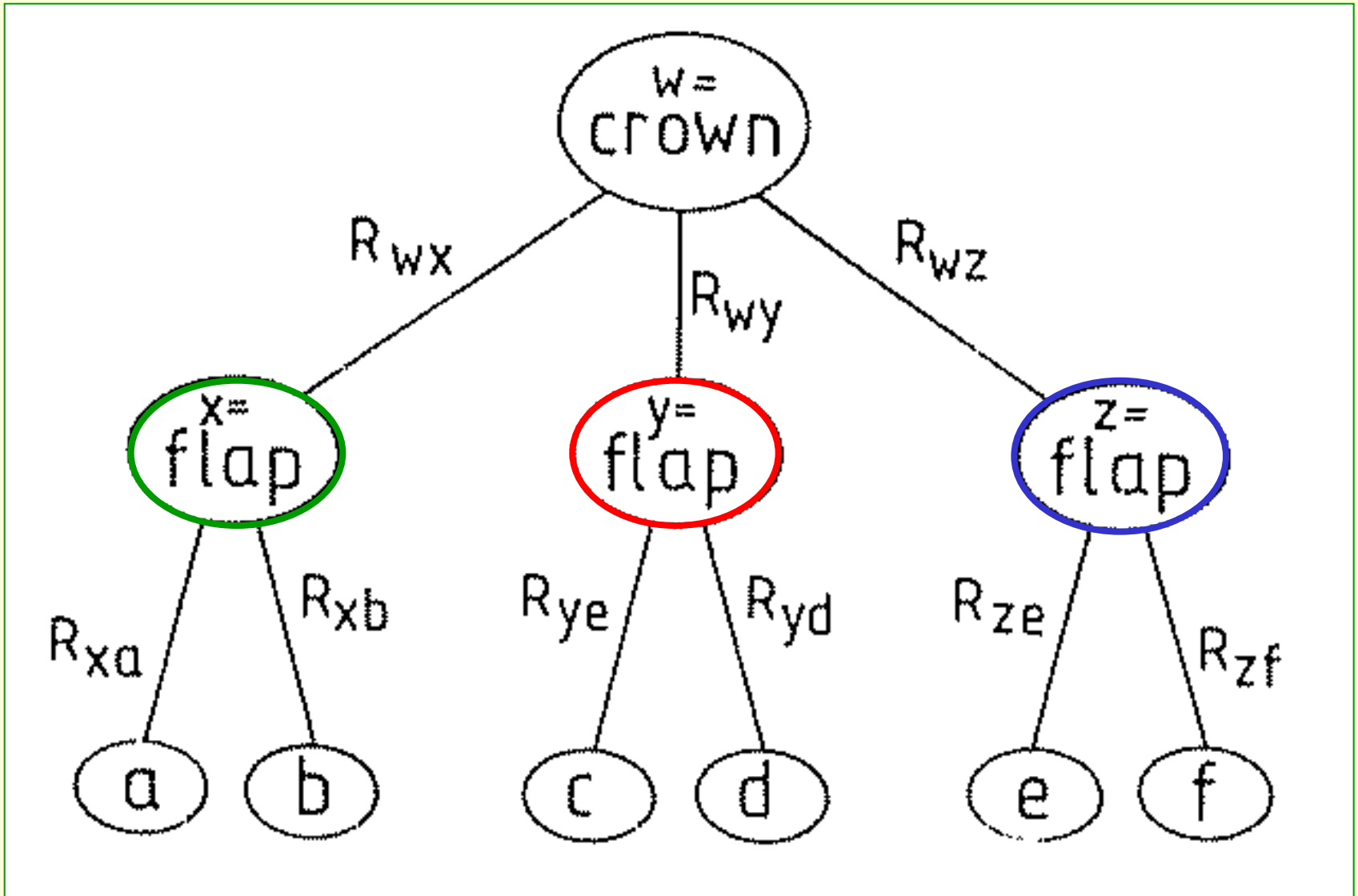
A different percept of the 6 rods



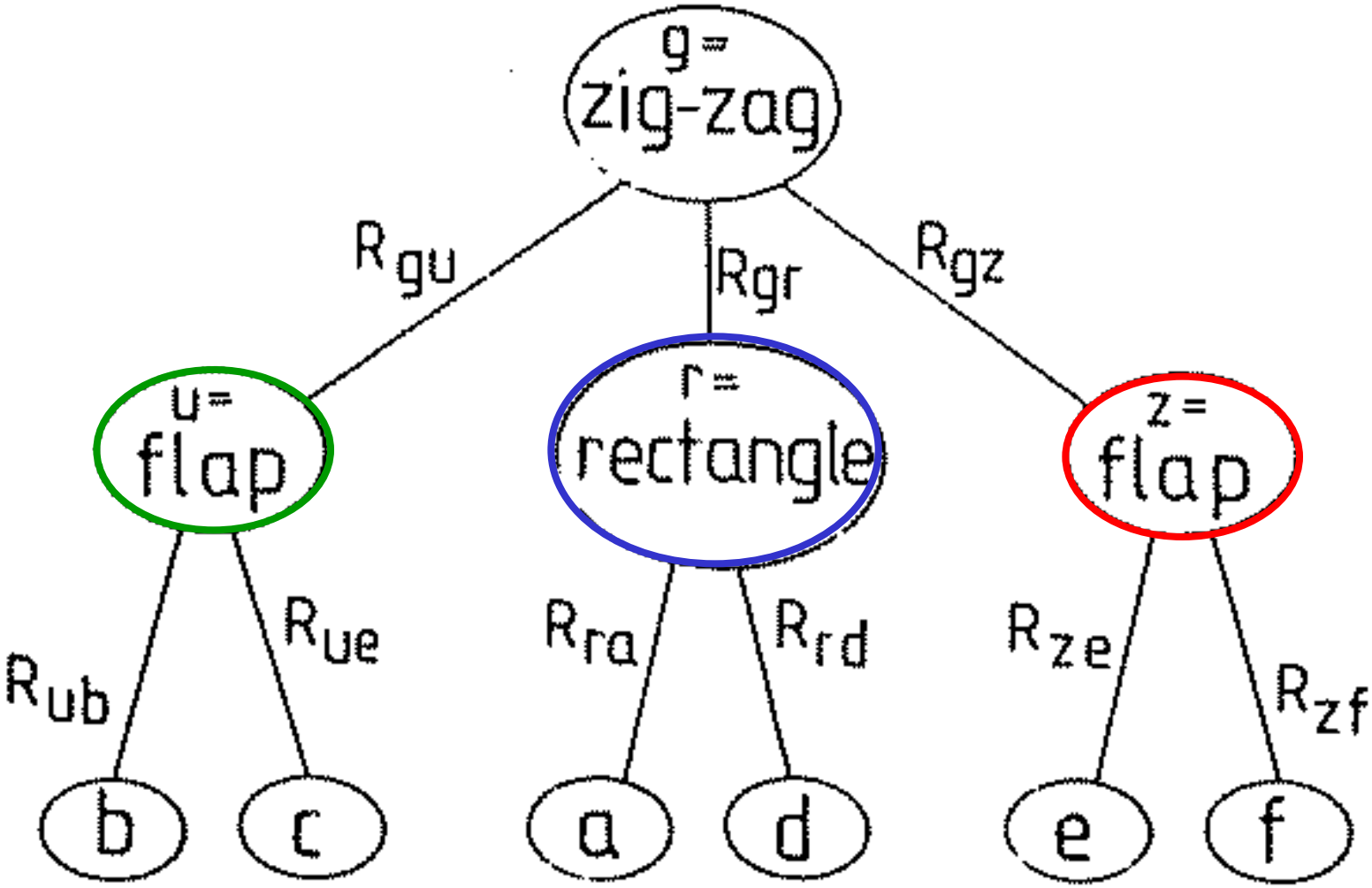
Alternative representations

- The very same arrangement of rods can be represented in quite different ways.
  - Its not like the Necker cube where the alternative percepts disagree on depth.
  - The alternative percepts do not disagree, but they make different facts obvious.
  - In the zig-zag representation it is obvious that there is one pair of parallel edges. In the crown representation there are no obvious pairs of parallel edges because the edges do not align with the natural intrinsic frame of any of the parts.

A structural description of the “crown” formed by the six rods



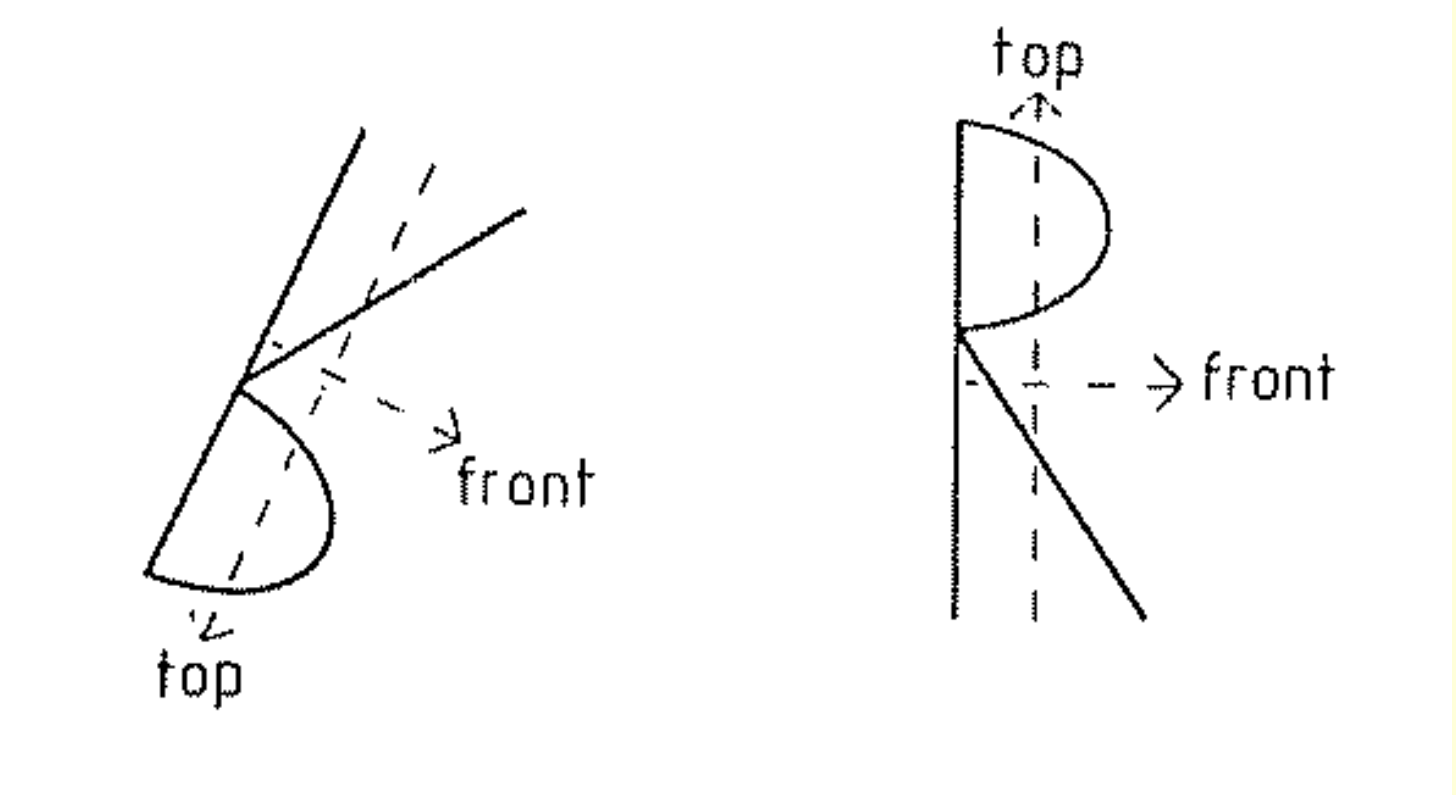
A structural description of the “zig-zag”



Analog operations on structural descriptions

- We can imagine the “petals” of the crown all folding upwards and inwards.
  - What is happening in our heads when we imagine this continuous transformation?
  - Why are the easily imagined transformations quite different for the two different structural descriptions?
- One particular continuous transformation called mental rotation was intensively studied by Roger Shepard and other psychologists in the 1970’s
  - Mental rotation really is continuous: When we are halfway through performing a mental rotation we have a mental representation at the intermediate orientation.

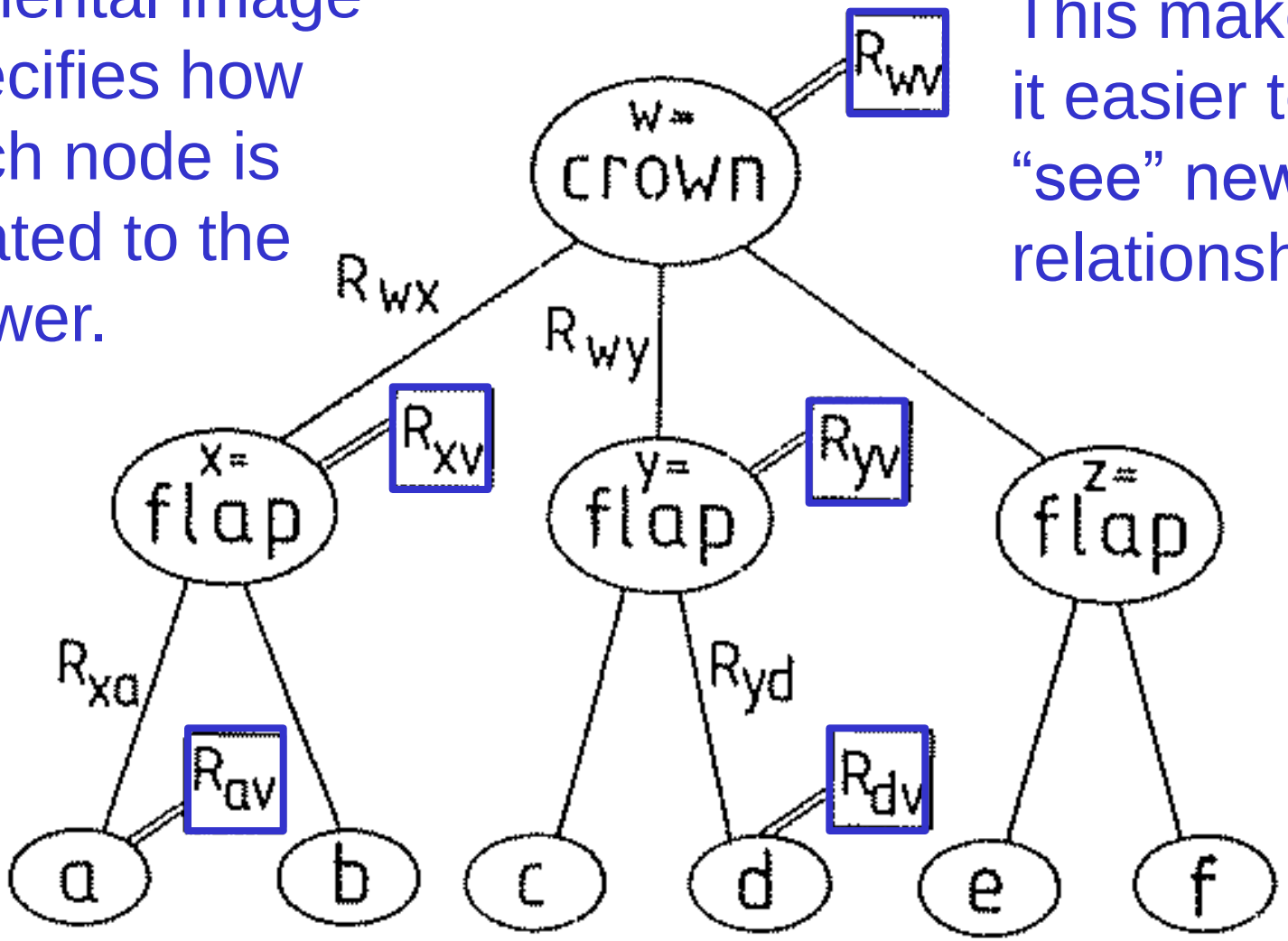
Mental rotation is not for recognition



We perform mental rotation to decide if the tilted R has the correct handedness, not to recognize that it is an R.

A mental image of the crown

A mental image specifies how each node is related to the viewer.



This makes it easier to "see" new relationships

A psychological theory of the right representation of images

- The representation should be a tree-structured structural description.
  - Knowledge of the viewpoint-invariant relationship between a part and a whole should be stored as a weight matrix.
  - Knowledge of the varying relationship of each node to the viewer should be in the neural activities.
- Mental imagery accesses stored knowledge of spatial relationships by propagating viewpoint information over a structural description.
  - This explains why imagery is needed for the geographic direction task and the dog's ears task.

The representation used by the neural nets that work best for recognition

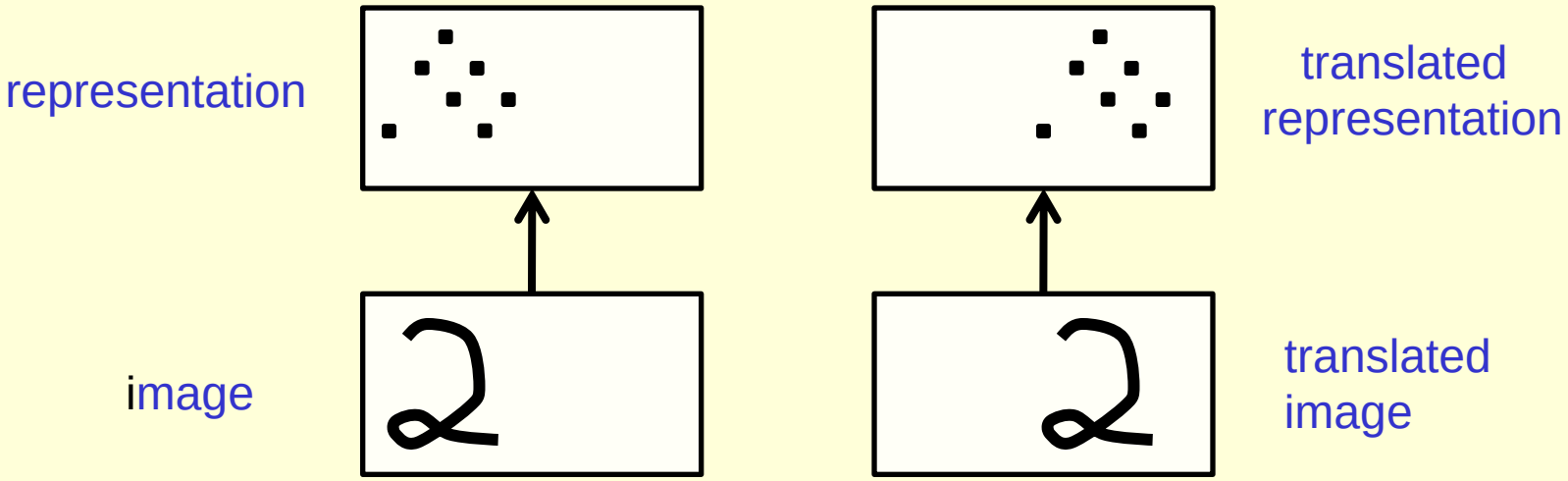
- This is nothing like a structural description.
  - It uses multiple layers of feature detectors that have local receptive fields (and maybe some weight sharing).
  - The feature extraction layers are interleaved with sub-sampling layers that throw away information about position in order to achieve some translation invariance.
- This architecture is doomed because it loses the precise spatial relationships between higher-level parts such as a nose and a mouth.

Equivariance vs Invariance

- Sub-sampling tries to make the neural activities invariant for small changes in viewpoint.
  - This is a silly goal, motivated by the fact that the final label needs to be viewpoint-invariant.
- Its better to aim for equivariance: Changes in viewpoint lead to corresponding changes in neural activities.
  - In the perceptual system, its the weights that code viewpoint-invariant knowledge, not the neural activities.

Equivariance

- Without the sub-sampling, convolutional neural nets give "place-coded" equivariance for discrete translations.



- A small amount of translational invariance can be achieved at each layer by using local averaging or maxing.

Two types of equivariance

- In “place-coded” equivariance, a discrete change in a property of a visual entity leads to a discrete change in which neurons are used for encoding that visual entity.
  - This is what happens in convolutional nets.
- In “rate-coded” equivariance, a real-valued change in a property of a visual entity leads to a real-valued change in the output of some of the neurons used for coding that visual entity, but there is no change in which neurons are used.
  - Our visual systems may use both types.

A way to achieve rate-coded equivariance

Use a “capsule” that uses quite a lot of internal computation (using non-linear “recognition units”) and encapsulates the results of this computation into a low dimensional output:

$$x = \sum_j u_j \Phi_j(\text{image}); \quad y = \sum_j v_j \Phi_j(\text{image});$$

$$p = \sigma\left(\sum_j w_j \Phi_j(\text{image})\right)$$

probability the visual entity is present

learned weights

learned non-linear recognition units

The real difference between rate-coded equivariance and convolutional nets.

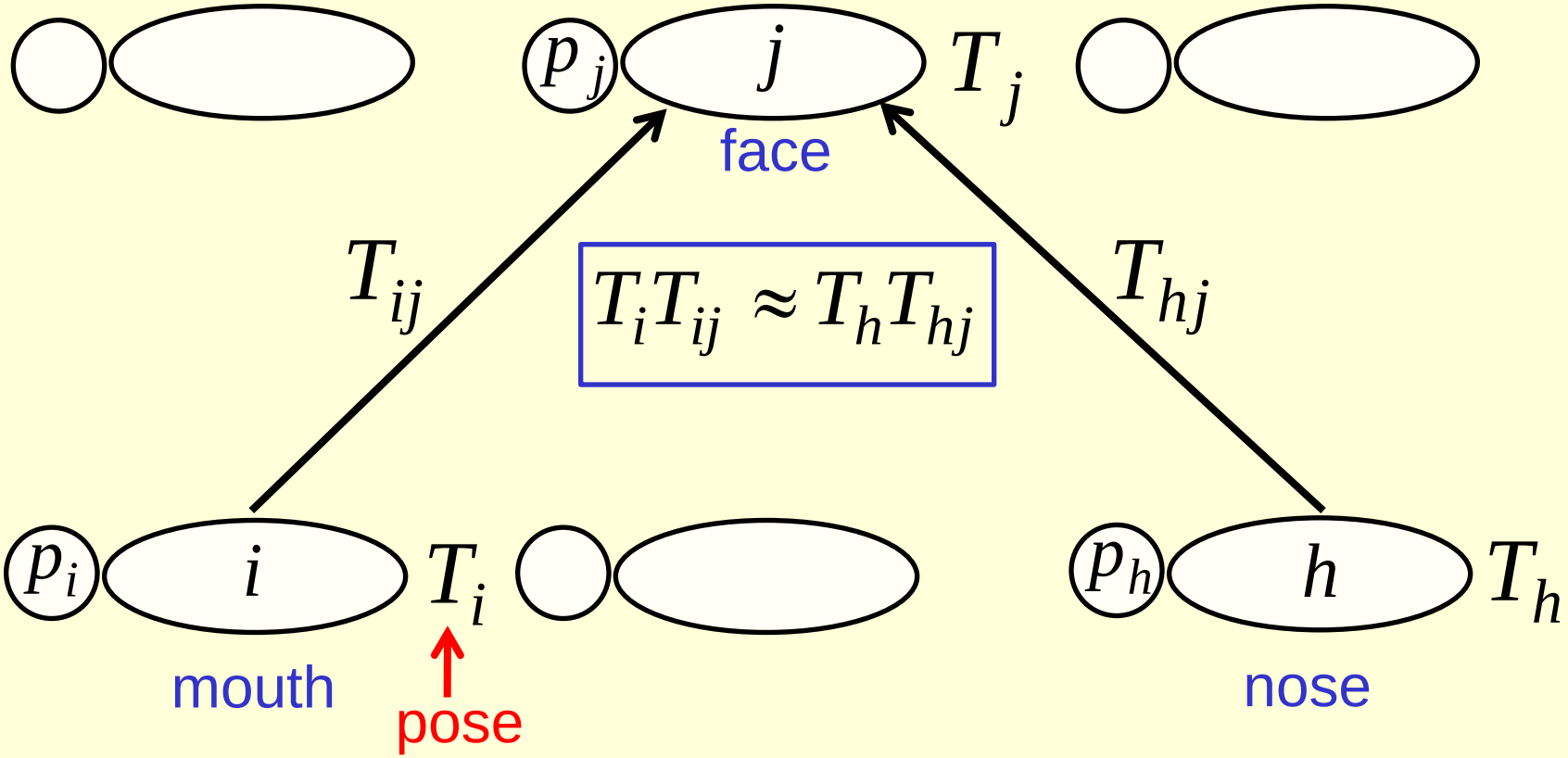
- Sub-sampling compresses the outputs of a pool of convolutional units into the activity level of the most active unit.
  - It may also use the location of the winner.
- A capsule encapsulates all of the information provided by the recognition units into two kinds of information:
  - The first is the probability that the visual entity represented by the capsule is present.
  - The second is a set of real-valued outputs that represent the pose of the entity (and possibly other properties to do with deformation, lighting etc.)

A crucial property of the pose outputs

- They allow spatial transformations to be modeled by linear operations.
  - This makes it easy to learn a hierarchy of visual entities.

Two layers in a hierarchy of capsules

- A higher level visual entity is present if several parts can agree on their predictions for its pose.



A simple way to learn the lowest level capsules

- Use pairs of images that are related by a known coordinate transformation (e.g. a small translation of the image).

Step 1: Compute the capsule outputs for the first image.

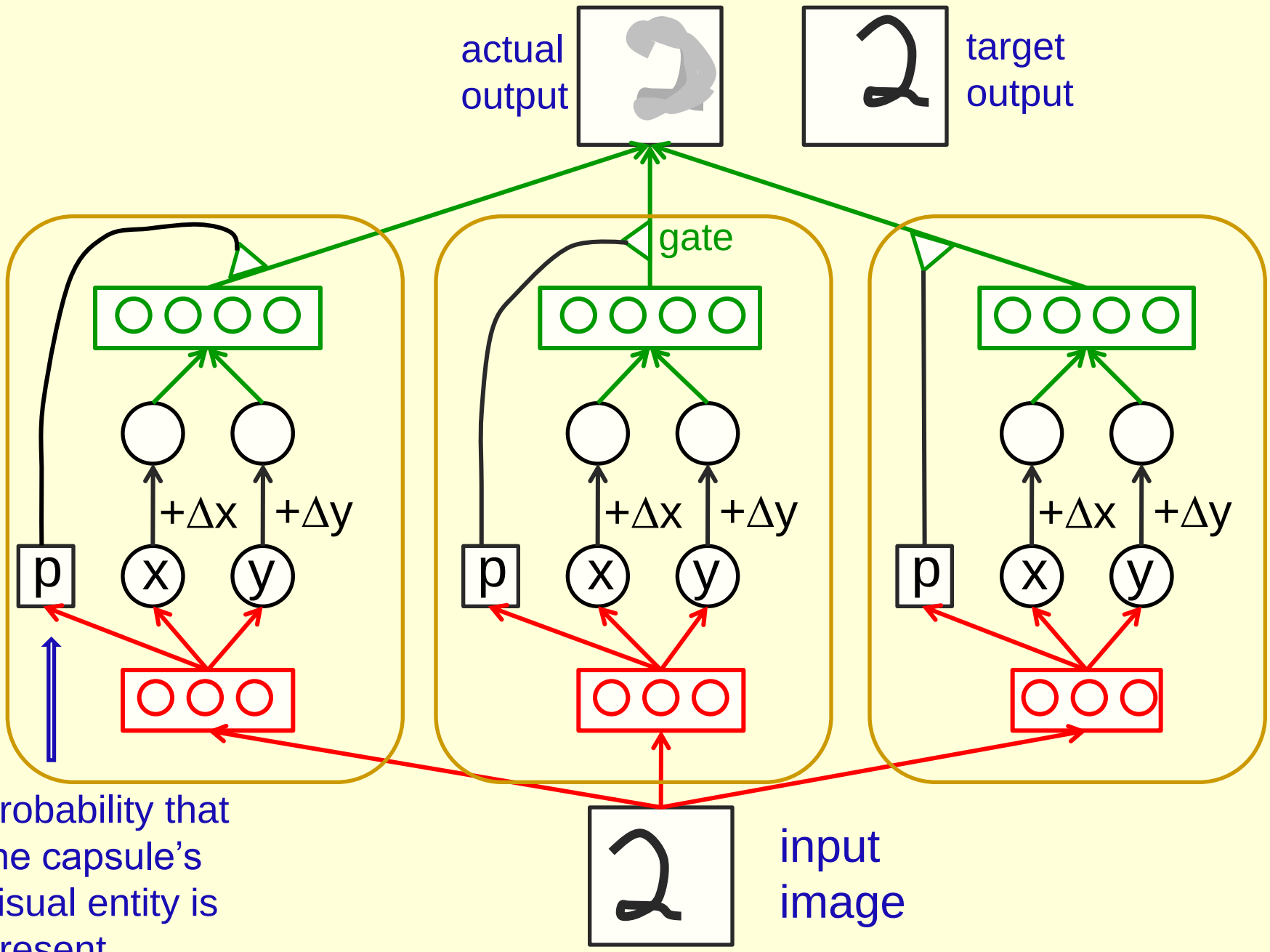
  – Each capsule uses its own set of "recognition" hidden units to extract the x and y coordinates of the visual entity it represents (and also the probability of existence)

Step 2: Apply the transformation to the outputs of each capsule

  - Just add Δx to each x output and Δy to each y output

Step 3: Predict the transformed image from the transformed outputs of the capsules

  – Each capsule uses its own set of "generative" hidden units to compute its contribution to the prediction.


actual
output
target
output
gate
+Δx
+Δy
p
x
y
probability that
the capsule's
visual entity is
present
input
image

Why it has to work

- When the net is trained with back-propagation, the only way it can get the transformations right is by using x and y in a way that is consistent with the way we are using Δx and Δy.

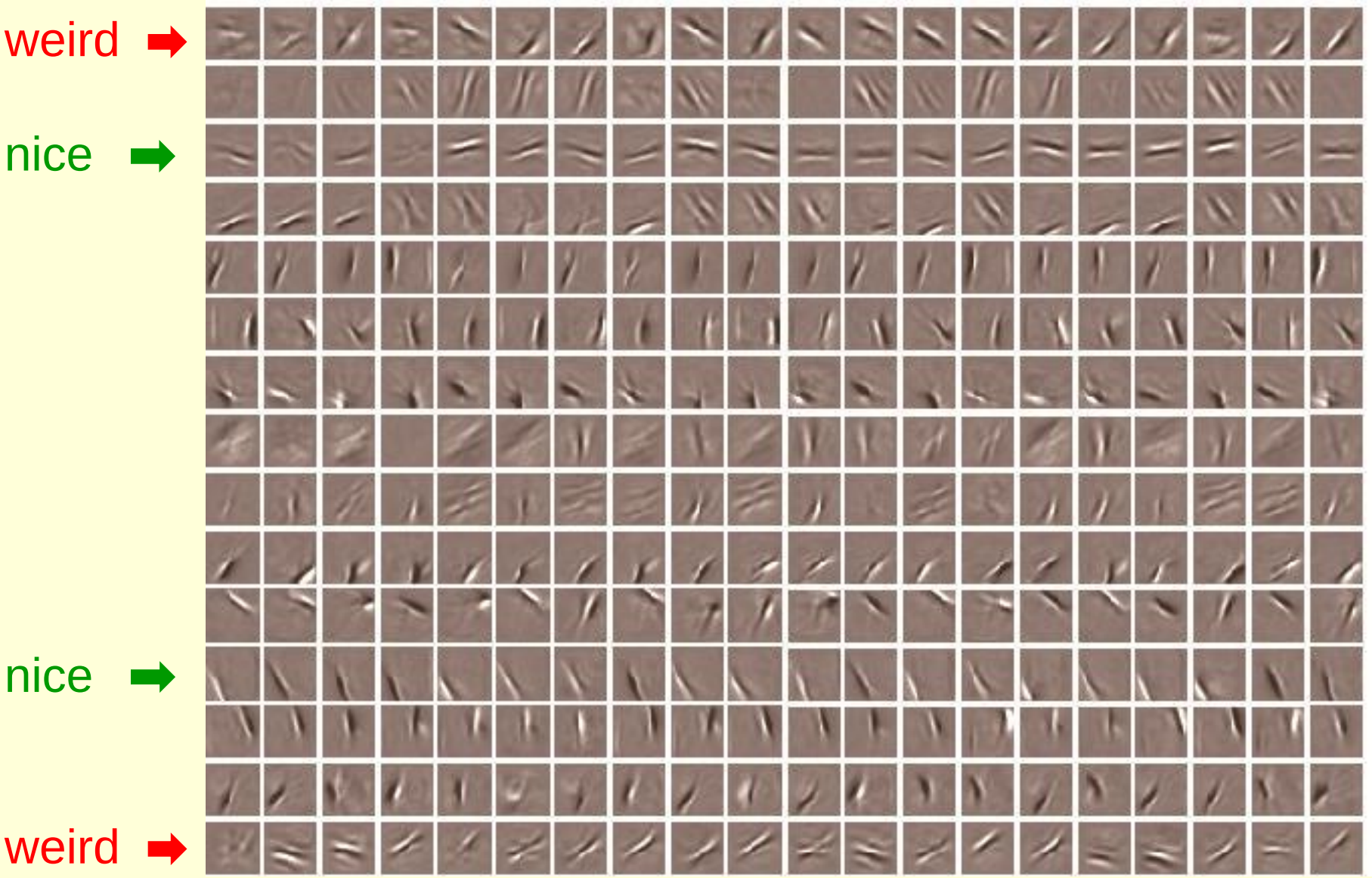
- This allows us to force the capules to extract the coordinates of visual entities without having to decide what the entities are or where they are.

How many capsules do we need?

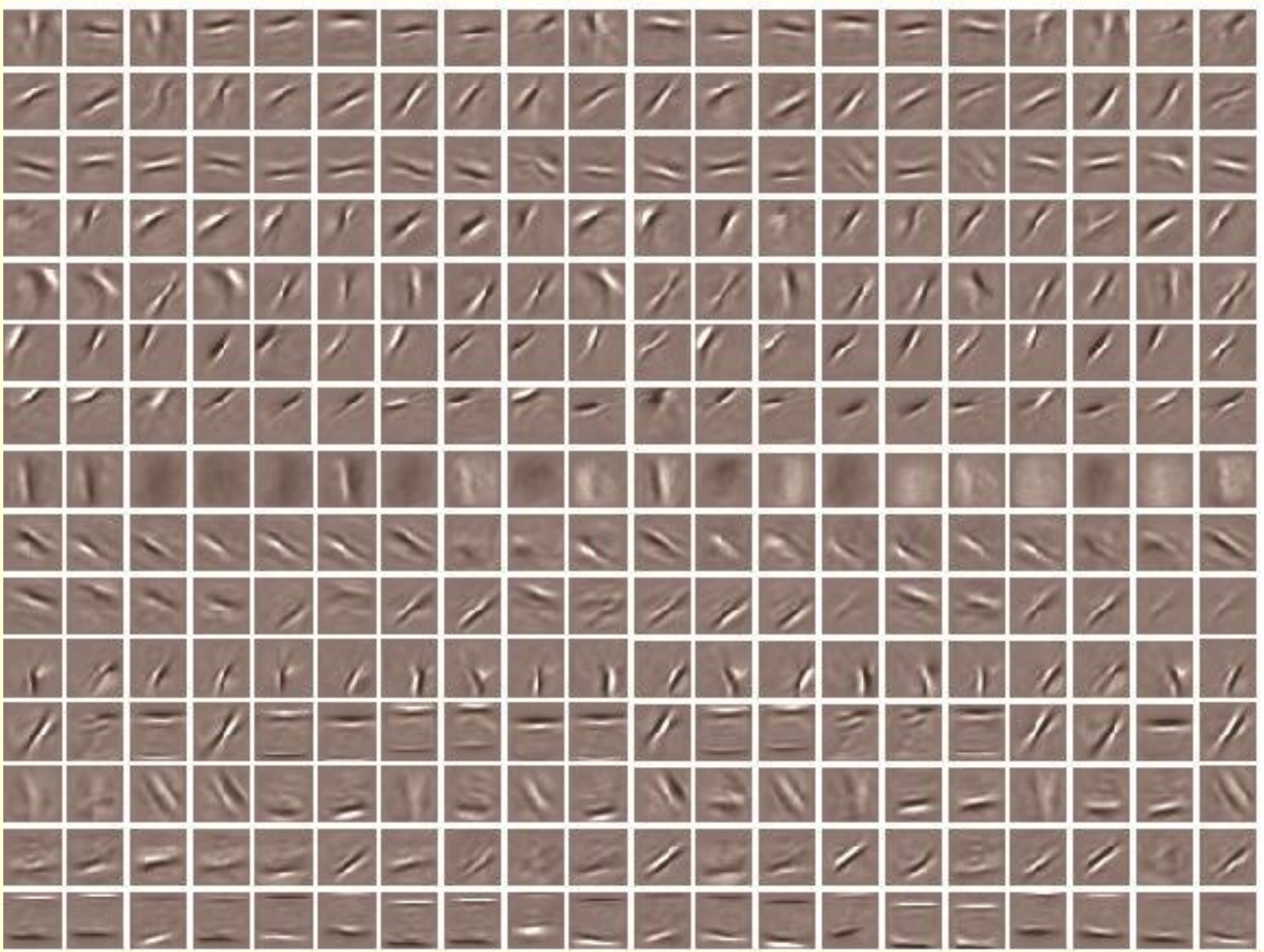
- Surprisingly few.
  - Each capsule is worth a large number of standard logistic dumb features (like 100)
  - 30 capsules is more than enough for representing an MNIST digit image.

- This is very good news for the communication bandwidth that is required to higher levels of analysis.

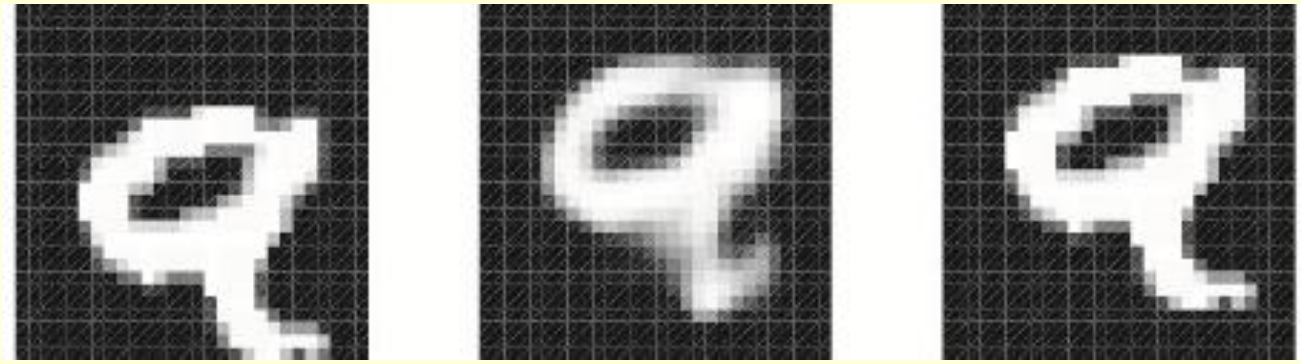
The output fields of the 20 generative hidden units in the first fifteen capsules



The output fields of the 20 generative hidden units in the second fifteen capsules



The prediction of the transformed image

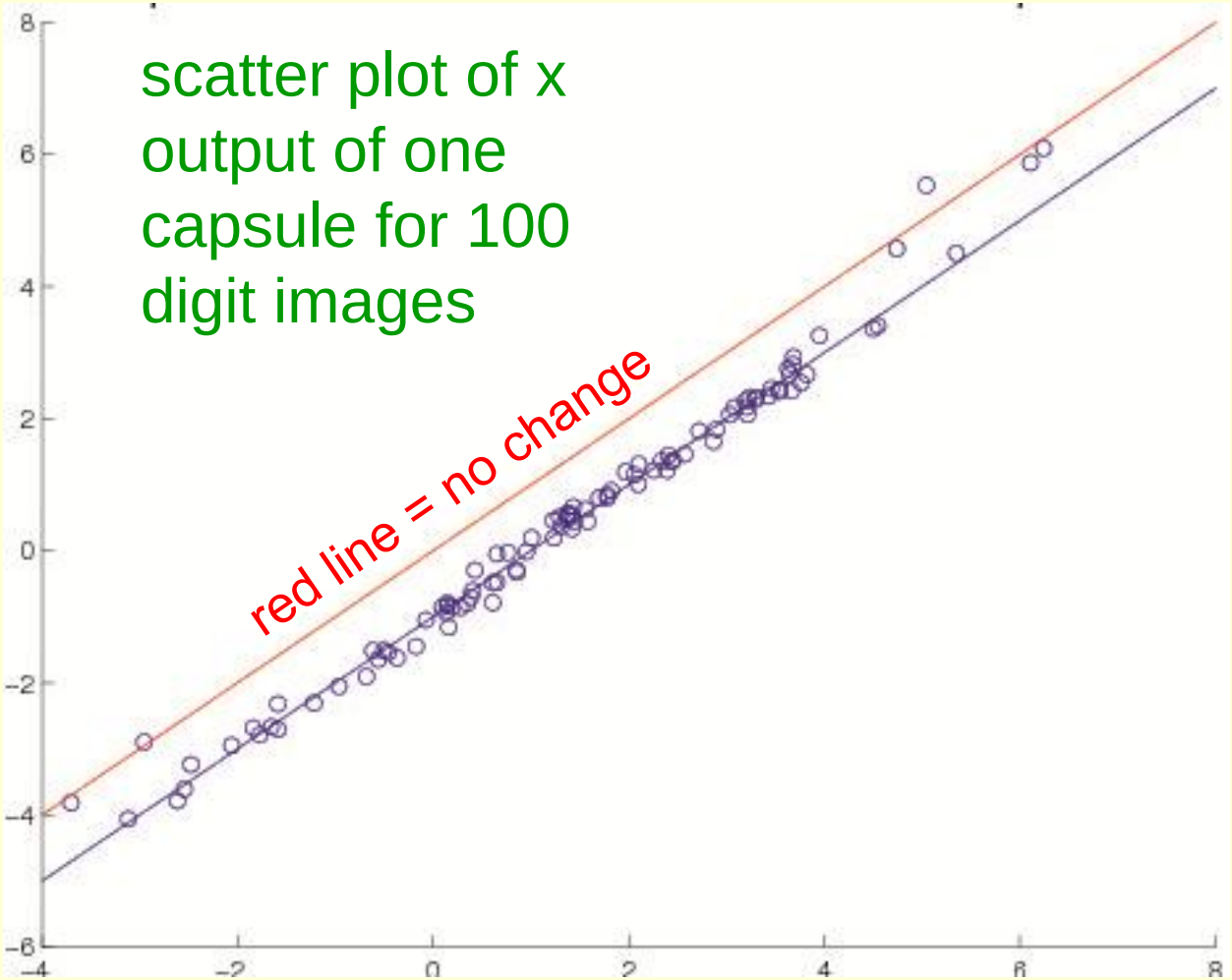


input image

predicted image

shifted image

What happens to the coordinates that a capsule outputs when we translate the input image?

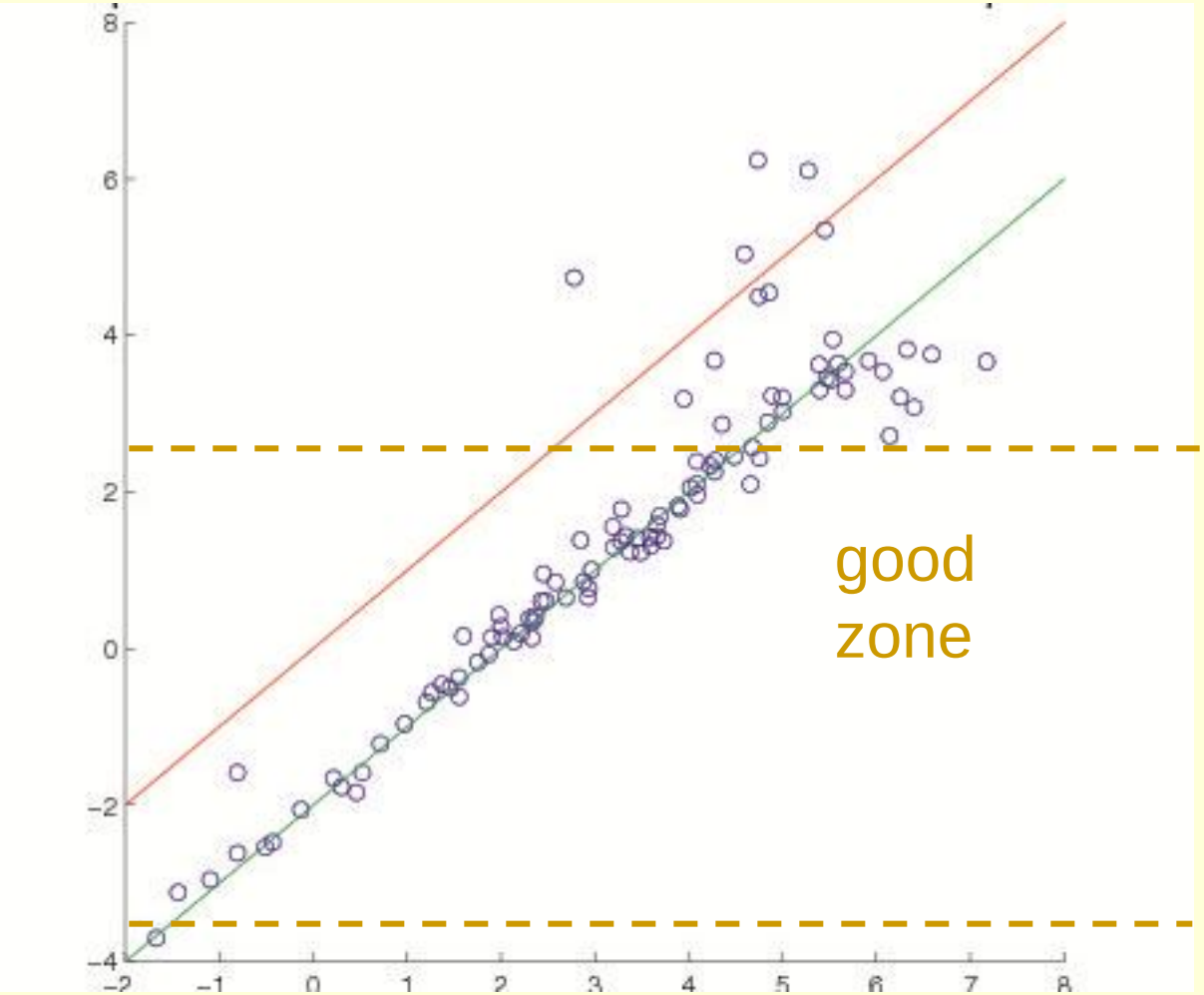


x output before shift ↑

x output after a one pixel shift →

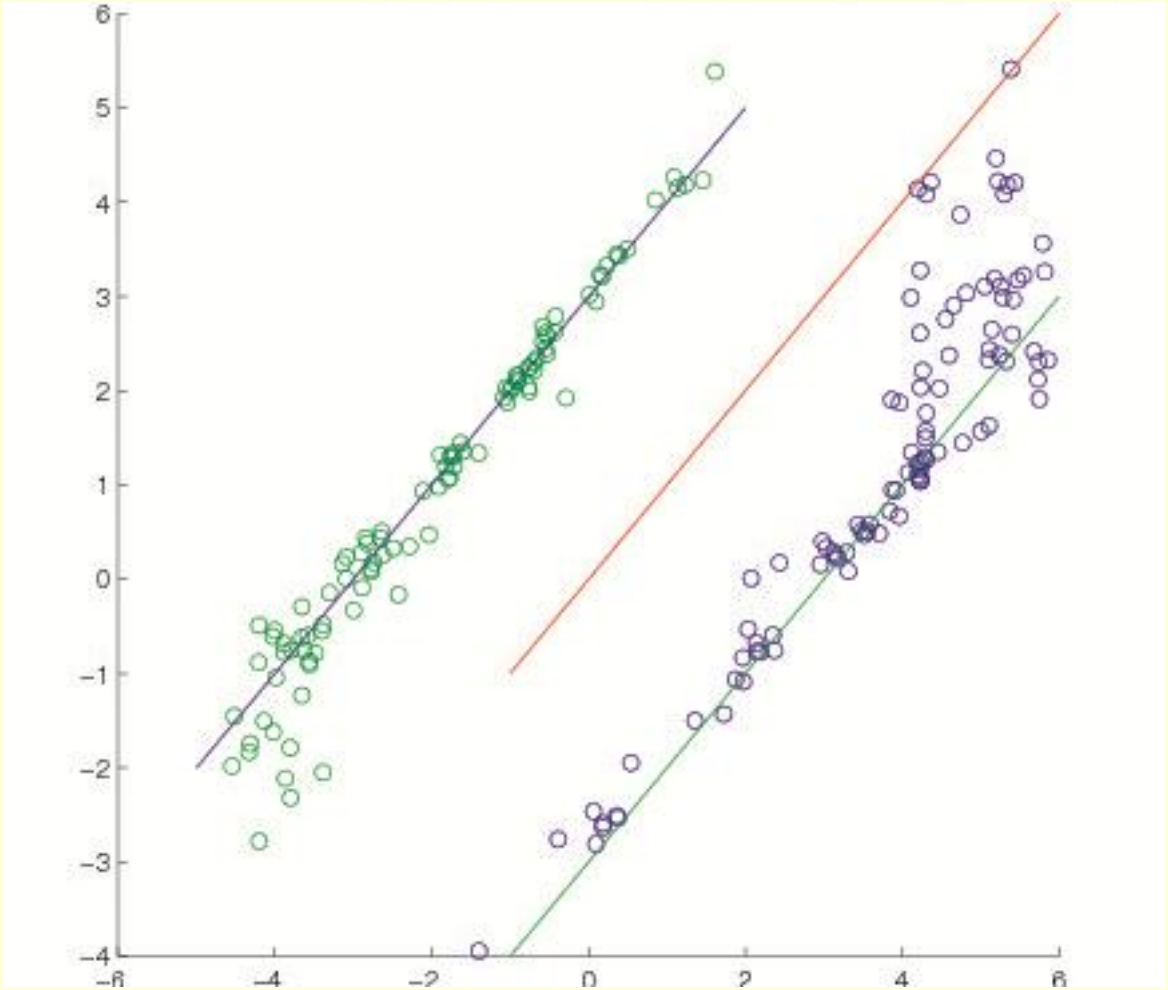
What happens to the coordinates that a capsule outputs when we translate the input image?

x output before shift



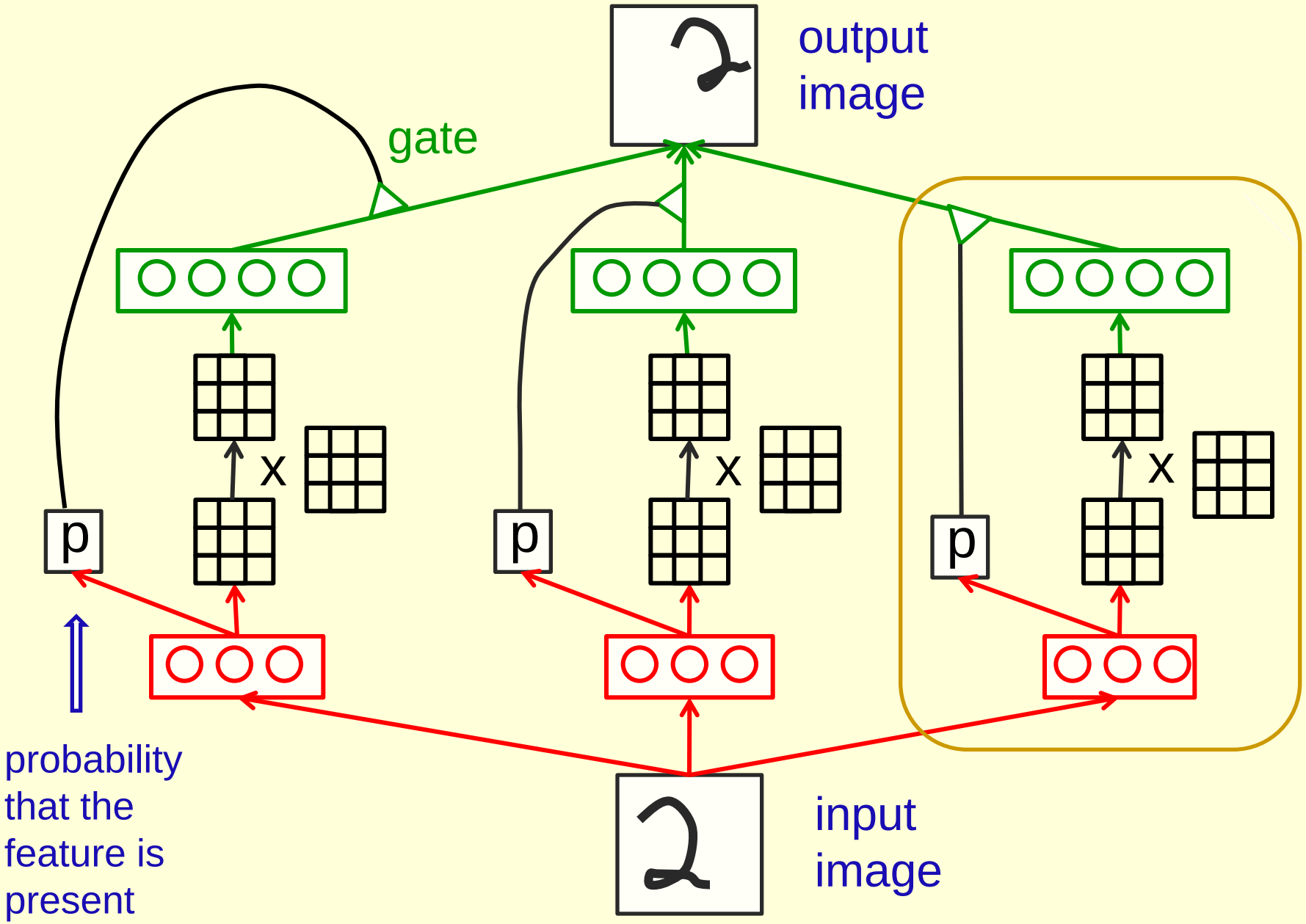
x output after a two pixel shift

What happens to the coordinates that a capsule outputs when we translate the input image?

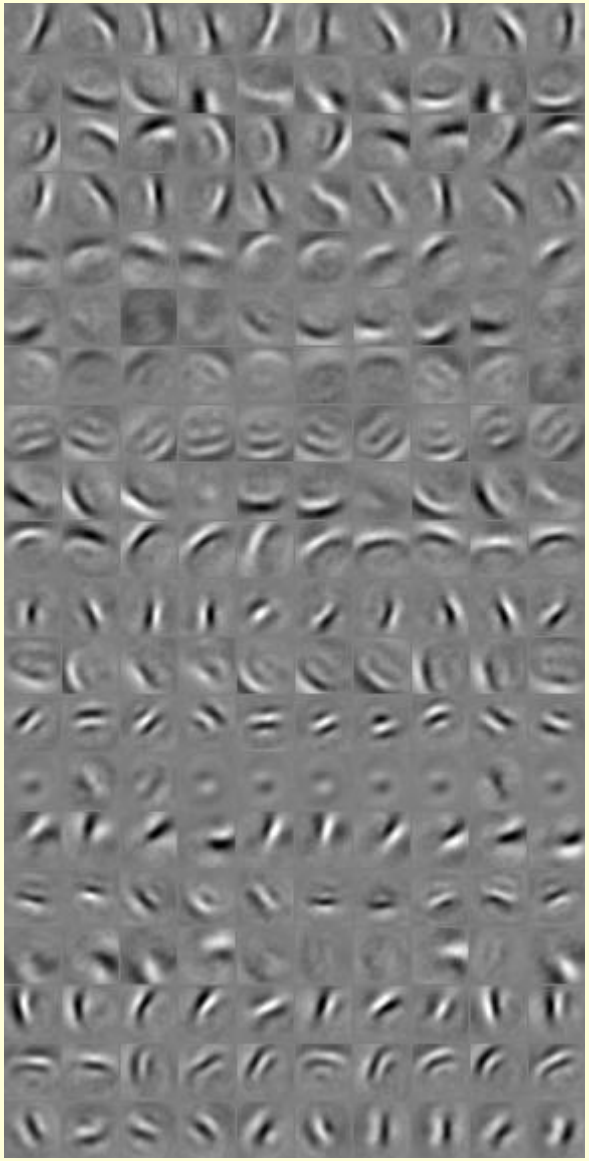


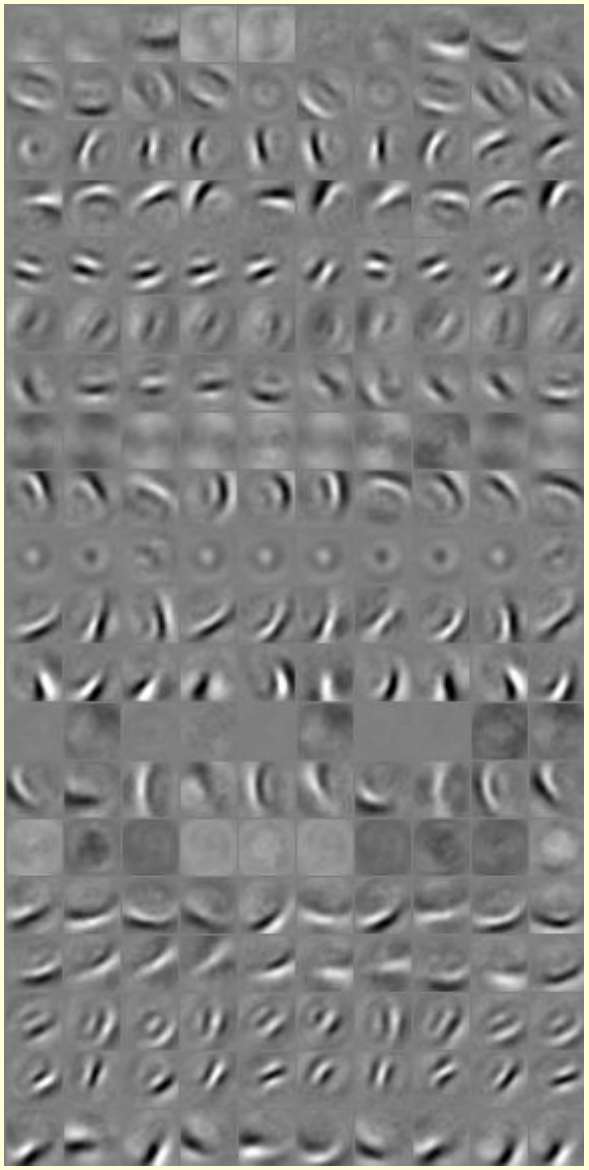
Dealing with scale and orientation (Sida Wang)

- It is easy to extend the network to deal with many more degrees of freedom.
  - Unlike a convolutional net, we do not have to grid the space with replicated filters (which is infeasible for more than a few dimensions).
- The non-linear recognition units of a capsule can be used to compute the elements of a full coordinate transformation.
  - This achieves full equivariance: As the viewpoint changes the representation changes appropriately.
- Rushing to achieve invariance is a big mistake. It makes it impossible to compute precise spatial relationships between high-level features such as noses and mouths.


output
image
gate
x
p
probability
that the
feature is
present
input
image

Reconstruction filters of 40 capsules learned on MNIST with full affine transformations (Sida Wang)

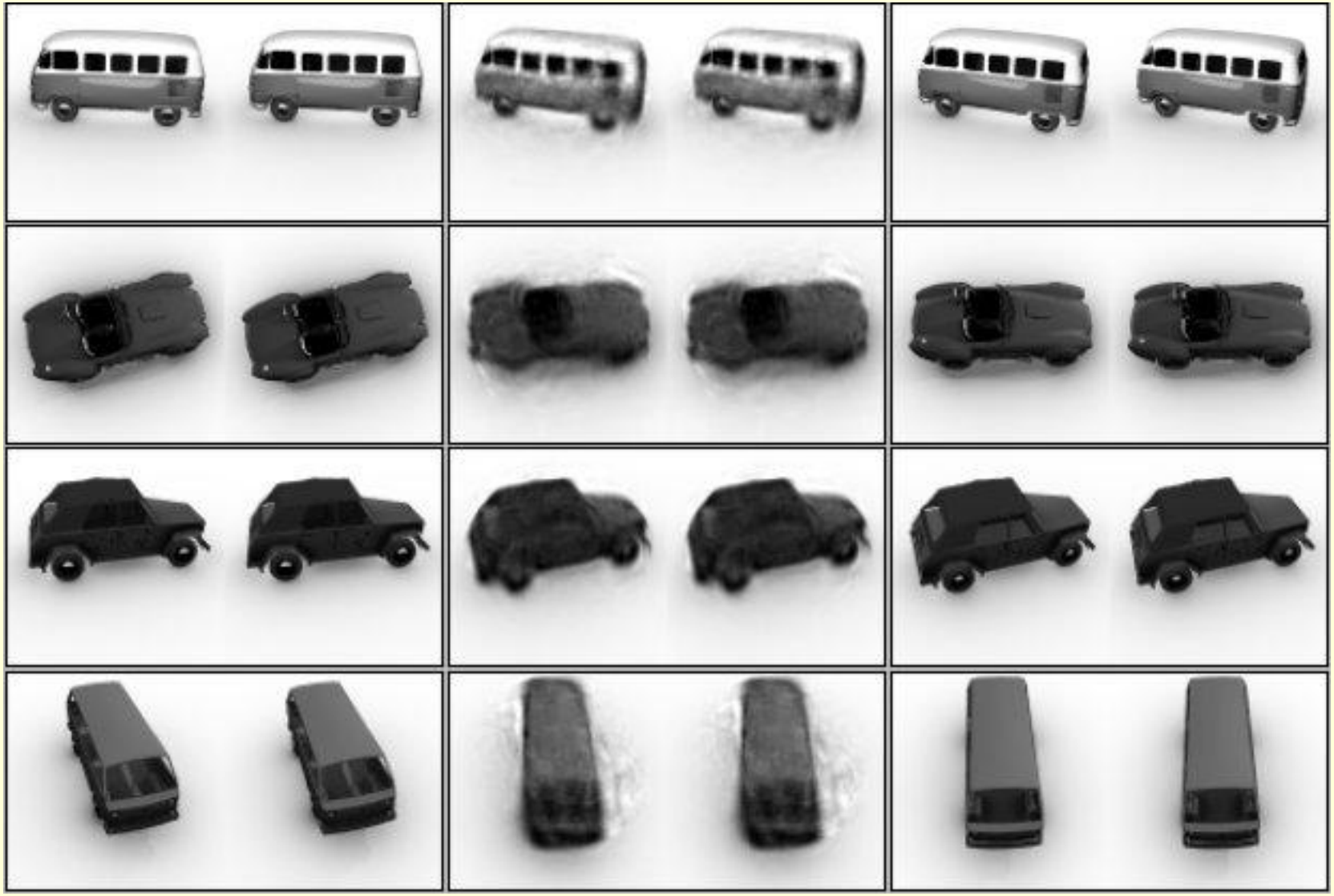




Dealing with the three–dimensional world

- Use stereo images and make the matrices 4x4
  - Using capsules, 3-D would not be much harder than 2-D if we started with 3-D pixels.
  - The loss of the depth coordinate is a separate problem from the complexity of 3-D geometry.
- At least capsules stand a chance of dealing with the 3-D geometry properly.

An initial attempt to deal with 3-D viewpoint properly (Alex Krizhevsky)



Its not so good on previously unseen objects