CSC321: 2011
Introduction to Neural Networks and Machine Learning

Lecture 12: Combining models

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Combining networks

- When the amount of training data is limited, we need to avoid overfitting.
  - Averaging the predictions of many different networks is a good way to do this.
  - It works best if the networks are as different as possible.
- If the data is really a mixture of several different “regimes” it is helpful to identify these regimes and use a separate, simple model for each regime.
  - We want to use the desired outputs to help cluster cases into regimes. Just clustering the inputs is not as efficient.

Combining networks reduces variance

- We want to compare two expected squared errors
  - Method 1: Pick one of the predictors at random
  - Method 2: Use the average of the predictors, $\bar{y}$

$$\bar{y} \quad = \quad < y_i >_i \quad = \quad \frac{1}{N}\sum_{i=1}^{N} y_i$$

$$< (d - y_i)^2 >_i = < \big((d - \bar{y}) - (y_i - \bar{y})\big)^2 >_i$$

$$= < (d - \bar{y})^2 + (y_i - \bar{y})^2 - 2(d - \bar{y})(y_i - \bar{y}) >_i$$

$$= < (d - \bar{y})^2 >_i + < (y_i - \bar{y})^2 >_i \ldots$$

$$- 2(d - \bar{y}) < (y_i - \bar{y}) >_i$$

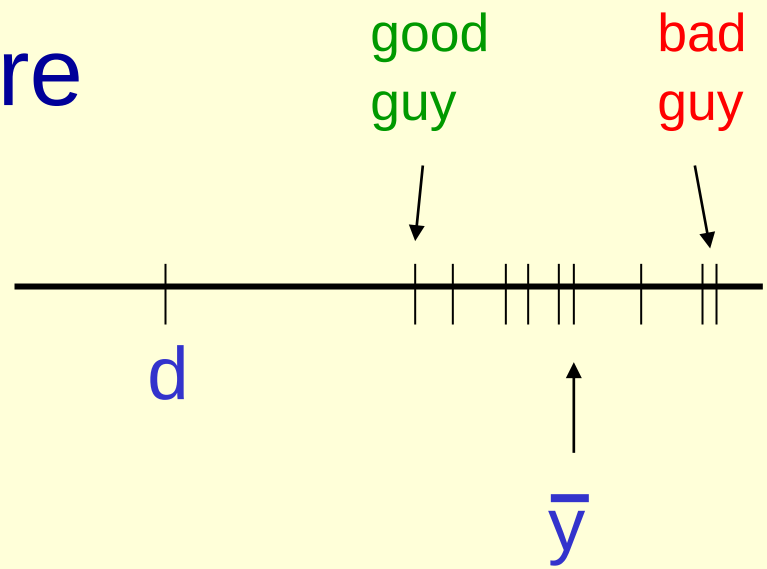
This term vanishes

A picture

- The predictors that are further than average from d make bigger than average squared errors.

- The predictors that are nearer than average to d make smaller then average squared errors.

- The first effect dominates because squares work like that.
  - Don't try averaging if you want to synchronize a bunch of clocks !



$$(a+\varepsilon)^2 + (a-\varepsilon)^2 = 2a^2 + 2e^2$$

How the combined predictor compares with the individual predictors

- On any one test case, some individual predictors will be better than the combined predictor.
  - But different individuals will be better on different cases.
- If the individual predictors disagree a lot, the combined predictor is typically better than all of the individual predictors when we average over test cases.
  - So how do we make the individual predictors disagree? (without making them much worse individually).

Ways to make predictors differ

- Rely on the learning algorithm getting stuck in a different local optimum on each run.
  - A dubious hack unworthy of a true computer scientist (but definitely worth a try).
- Use lots of different kinds of models:
  - Different architectures
  - Different learning algorithms.

Making predictors differ by using different training data for each model

Bagging

Resample (with replacement) from the training set: a,b,c,d,e -> a c c d d

Boosting

Fit models one at a time. Re-weight each training case by how badly it is predicted by the models already fitted.

- This makes efficient use of computer time because it does not bother to “back-fit” models that were fitted earlier.

Boosting slides

- Boosting was invented and developed by Freund and Shapire.
- They made nice slides.
- Ignore slides 5 & 6. Stop at slide 14.