CSC2535: Computation in Neural Networks

Lecture 13: Representing things with neurons

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Localist representations

- The simplest way to represent things with neural networks is to dedicate one neuron to each thing.
  - Easy to understand.
  - Easy to code by hand
    - Often used to represent inputs to a net
  - Easy to learn
    - This is what mixture models do.
    - Each cluster corresponds to one neuron
  - Easy to associate with other representations or responses.
- But localist models are very inefficient whenever the data has componential structure.

Examples of componential structure

- Big, yellow, Volkswagen
  - Do we have a neuron for this combination
    - Is the BYV neuron set aside in advance?
    - Is it created on the fly?
    - How is it related to the neurons for big and yellow and Volkswagen?
- Consider a visual scene
  - It contains many different objects
  - Each object has many properties like shape, color, size, motion.
  - Objects have spatial relationships to each other.

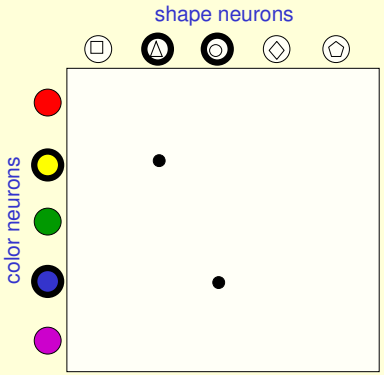
Using simultaneity to bind things together

Represent conjunctions by activating all the constituents at the same time.

- This doesn't require connections between the constituents.
- But what if we want to represent yellow triangle and blue circle at the same time?

Maybe this explains the serial nature of consciousness.

- And maybe it doesn't!



Using space to bind things together

- Conventional computers can bind things together by putting them into neighboring memory locations.
  - This works nicely in vision. Surfaces are generally opaque, so we only get to see one thing at each location in the visual field.
    - If we use topographic maps for different properties, we can assume that properties at the same location belong to the same thing.

The definition of "distributed representation"

- Each neuron must represent something
  - so it's a local representation of whatever this something is.
- "Distributed representation" means a many-to-many relationship between two types of representation (such as concepts and neurons).
  - Each concept is represented by many neurons
  - Each neuron participates in the representation of many concepts

Coarse coding

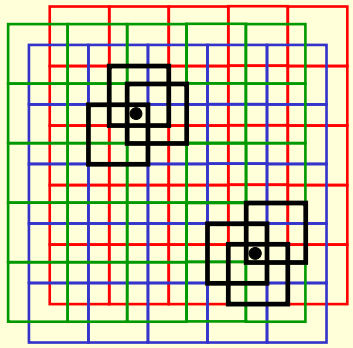
- Using one neuron per entity is inefficient.
  - An efficient code would have each neuron active half the time.
    - This might be inefficient for other purposes (like associating responses with representations).
- Can we get accurate representations by using lots of inaccurate neurons?
  - If we can it would be very robust against hardware failure.

Coarse coding

Use three overlapping arrays of large cells to get an array of fine cells

- If a point falls in a fine cell, code it by activating 3 coarse cells.

- This is more efficient than using a neuron for each fine cell.
  - It loses by needing 3 arrays
  - It wins by a factor of 3x3 per array
  - Overall it wins by a factor of 3



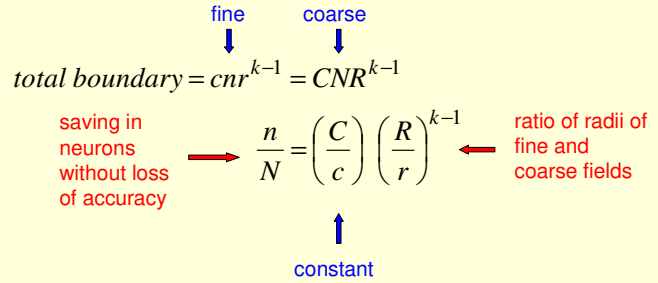
How efficient is coarse coding?

- The efficiency depends on the dimensionality
  - In one dimension coarse coding does not help
  - In 2-D the saving in neurons is proportional to the ratio of the fine radius to the coarse radius.
  - In k dimensions , by increasing the radius by a factor of R we can keep the same accuracy as with fine fields and get a saving of:

$$saving = \frac{\#\ fine\ neurons}{\#\ coarse\ neurons} = R^{k-1}$$

Coarse regions and fine regions use the same surface

- Each binary neuron defines a boundary between k-dimensional points that activate it and points that don't.
  - To get lots of small regions we need a lot of boundary.



Limitations of coarse coding

- It achieves accuracy at the cost of resolution
  - Accuracy is defined by how much a point must be moved before the representation changes.
  - Resolution is defined by how close points can be and still be distinguished in the represention.
    - Representations can overlap and still be decoded if we allow integer activities of more than 1.
- It makes it difficult to associate very different responses with similar points, because their representations overlap
  - This is useful for generalization.
- The boundary effects dominate when the fields are very big.

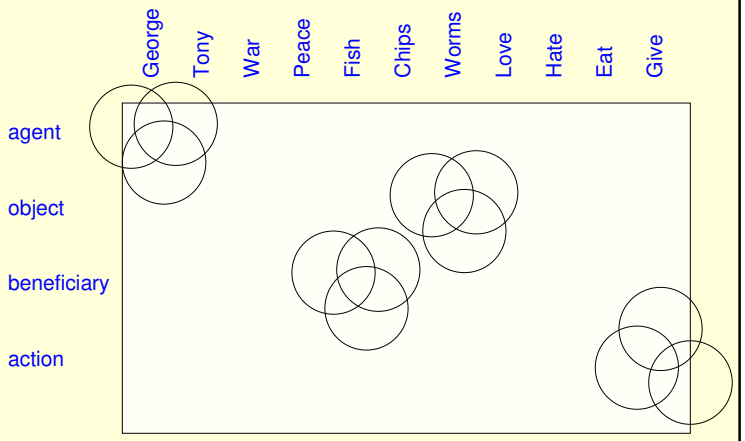
Coarse coding in the visual system

- As we get further from the retina the receptive fields of neurons get bigger and bigger and require more complicated patterns.
  - Most neuroscientists interpret this as neurons exhibiting invariance.
  - But its also just what would be needed if neurons wanted to achieve high accuracy for properties like position orientation and size.
- High accuracy is needed to decide if the parts of an object are in the right spatial relationship to each other.

Representing relational structure

- "George loves Peace"
  - How can a proposition be represented as a distributed pattern of activity?
  - How are neurons representing different propositions related to each other and to the terms in the proposition?
- We need to represent the role of each term in proposition.

A way to represent structures



The recursion problem

- Jacques was annoyed that Tony helped George
  - One proposition can be part of another proposition. How can we do this with neurons?
- One possibility is to use "reduced descriptions". In addition to having a full representation as a pattern distributed over a large number of neurons, an entity may have a much more compact representation that can be part of a larger entity.
  - It's a bit like pointers.
  - We have the full representation for the object of attention and reduced representations for its constituents.
  - This theory requires mechanisms for compressing full representations into reduced ones and expanding reduced descriptions into full ones.

Representing associations as vectors

- In most neural networks, objects and associations between objects are represented differently:
  - Objects are represented by distributed patterns of activity
  - Associations between objects are represented by distributed sets of weights
- We would like associations between objects to also be objects.
  - So we represent associations by patterns of activity.
  - An association is a vector, just like an object.

Circular convolution

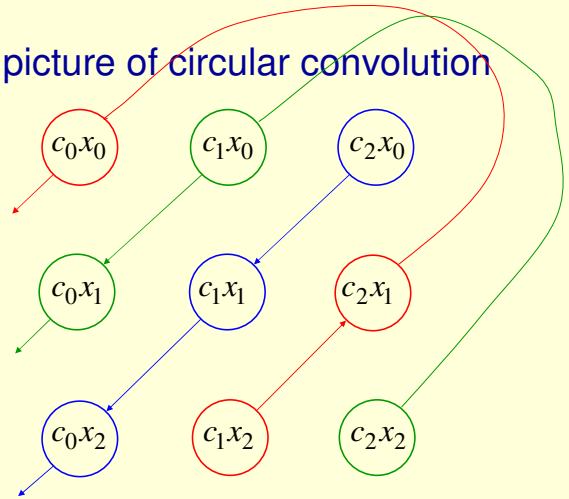
- Circular convolution is a way of creating a new vector, t, that represents the association of the vectors c and x.
  - t is the same length as c or x
  - t is a compressed version of the outer product of c and x
  - T can be computed quickly in O(n log n) using FFT
- Circular correlation is a way of using c as a cue to approximately recover x from t.
  - It is a different way of compressing the outer product.

$$t_j = \sum_{k=0}^{n-1} c_k x_{j-k}$$

$$y_j = \sum_{k=0}^{n-1} c_k t_{k+j}$$

scalar product with shift of j

A picture of circular convolution



Circular correlation is compression along the other diagonals

Constraints required for decoding

- Circular correlation only decodes circular convolution if the elements of each vector are distributed in the right way:
  - They must be independently distributed
  - They must have mean 0.
  - They must have variance of 1/n
    - i.e. they must have expected length of 1.
- Obviously vectors cannot have independent features when they encode meaningful stuff.
  - So the decoding will be imperfect.

Storage capacity of convolution memories

- The memory only contains n numbers.
  - So it cannot store even one association of two n-component vectors accurately.
    - This does not matter if the vectors are big and we use a clean-up memory.
- Multiple associations are stored by just adding the vectors together.
  - The sum of two vectors is remarkably close to both of them compared with its distance from other vectors.
  - When we try to decode one of the associations, the others just create extra random noise.
    - This makes it even more important to have a clean-up memory.

The clean-up memory

- Every atomic vector and every association is stored in the clean-up memory.
  - The memory can take a degraded vector and return the closest stored vector, plus a goodness of fit.
  - It needs to be a matrix memory (or something similar) that can store many different vectors accurately.
- Each time a cue is used to decode a representation, the clean-up memory is used to clean up the very degraded output of the circular correlation operation.

Representing structures

- A structure is a label plus a set of roles
  - Like a verb
  - The vectors representing similar roles in different structures can be similar.
  - We can implement all this in a very literal way!

$$\mathbf{t}_{prop13} = \mathbf{l}_{see} + \mathbf{r}_{agent} \otimes \mathbf{f}_{jane} + \mathbf{r}_{object} \otimes \mathbf{f}_{fido}$$

A particular proposition

structure label

circular convolution

Representing sequences using chunking

- Consider the representation of “abcdefgh”.
  - First create chunks for subsequences

$$\mathbf{s}_{abc} = \mathbf{a} + \mathbf{a} \otimes \mathbf{b} + \mathbf{a} \otimes \mathbf{b} \otimes \mathbf{c}$$
$$\mathbf{s}_{de} = \mathbf{d} + \mathbf{d} \otimes \mathbf{e}$$
$$\mathbf{s}_{fgh} = \mathbf{f} + \mathbf{f} \otimes \mathbf{g} + \mathbf{f} \otimes \mathbf{g} \otimes \mathbf{h}$$

  - Then add the chunks together

$$\mathbf{s}_{abcdefgh} = \mathbf{s}_{abc} + \mathbf{s}_{abc} \otimes \mathbf{s}_{de} + \mathbf{s}_{abc} \otimes \mathbf{s}_{de} \otimes \mathbf{s}_{fgh}$$