Quantifiers

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It is defined from a two-operand symmetric associative operator.

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$\forall p$ is defined from $\wedge$ "for all" $\forall\langle r{:}\ rat\cdot\ r\geq 0\rangle$ $= \bot$

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$\forall p$ is defined from $\wedge$	"for all"	$\forall\langle r: rat\cdot r\geq 0\rangle$	$= \bot$
$\exists p$ is defined from $\vee$	"there exists"	$\exists\langle n: nat\cdot n=0\rangle$	$= \top$

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$\exists p$ is defined from $\vee$	"there exists"	$\exists\langle n\text{: } nat\cdot\ n=0\rangle$	$=\ \top$
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$\exists p$ is defined from $\vee$	"there exists"	$\exists\langle n: nat\cdot n=0\rangle$	$= \top$
Σf is defined from $+$	"sum of"	$\Sigma\langle n: nat+1\cdot 1/2^n\rangle$	$= 1$

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Σf is defined from $+$	"sum of"	$\Sigma\langle n\!: nat{+}1 \cdot 1/2^n\rangle$	$= 1$
Πf is defined from $\times$	"product of"	$\Pi\langle n\!: nat{+}1 \cdot (4{\times}n^2)/(4{\times}n^2 - 1)\rangle$	$= \pi/2$

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$\exists p$ is defined from $\vee$	"there exists"	$\exists\langle n\colon nat\cdot n=0\rangle$	$=\ \top$
Σf is defined from $+$	"sum of"	$\Sigma\langle n\colon nat{+}1\cdot 1/2^n\rangle$	$=\ 1$
Πf is defined from $\times$	"product of"	$\Pi\langle n\colon nat{+}1\cdot (4\times n^2)/(4\times n^2-1)\rangle$	$=\ \pi/2$
$\Uparrow f$ is defined from $\uparrow$	"maximum of"		

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Σf is defined from $+$	"sum of"	$\Sigma\langle n: nat+1\cdot\ 1/2^n\rangle$	$= 1$
Πf is defined from $\times$	"product of"	$\Pi\langle n: nat+1\cdot\ (4\times n^2)/(4\times n^2-1)\rangle$	$= \pi/2$
$\Uparrow f$ is defined from $\uparrow$	"maximum of"	$\Uparrow\langle x: rat\cdot\ x\times(4-x)\rangle$	$= 4$

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Σf is defined from $+$	"sum of"	$\Sigma\langle n{:}\ nat+1\cdot\ 1/2^n\rangle$	$= 1$
Πf is defined from $\times$	"product of"	$\Pi\langle n{:}\ nat+1\cdot\ (4\times n^2)/(4\times n^2 - 1)\rangle$	$= \pi/2$
$\Uparrow f$ is defined from $\uparrow$	"maximum of"	$\Uparrow\langle x{:}\ rat\cdot\ x\times(4-x)\rangle$	$= 4$
$\Downarrow f$ is defined from $\downarrow$	"minimum of"		

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Σf is defined from $+$	"sum of"	$\Sigma\langle n{:}\ nat{+}1\cdot\ 1/2^n\rangle$	$=\ 1$
Πf is defined from $\times$	"product of"	$\Pi\langle n{:}\ nat{+}1\cdot\ (4\times n^2)/(4\times n^2-1)\rangle$	$=\ \pi/2$
$\Uparrow f$ is defined from $\uparrow$	"maximum of"	$\Uparrow\langle x{:}\ rat\cdot\ x\times(4-x)\rangle$	$=\ 4$
$\Downarrow f$ is defined from $\downarrow$	"minimum of"	$\Downarrow\langle n{:}\ nat{+}1\cdot\ 1/n\rangle$	$=\ 0$

Quantifiers

abbreviations

$\forall r{:}\ rat\cdot\ r\ge 0$	abbreviates	$\forall\langle r{:}\ rat\cdot\ r\ge 0\rangle$
$\Sigma n{:}\ nat{+}1\cdot\ 1/2^n$	abbreviates	$\Sigma\langle n{:}\ nat{+}1\cdot\ 1/2^n\rangle$

Quantifiers

abbreviations

$\forall r: rat\cdot\ r\ge 0$	abbreviates	$\forall\langle r: rat\cdot\ r\ge 0\rangle$
$\Sigma n: nat{+}1\cdot\ 1/2^n$	abbreviates	$\Sigma\langle n: nat{+}1\cdot\ 1/2^n\rangle$
$\forall x, y: rat\cdot\ x = y{+}1\ \Rightarrow\ x > y$	abbreviates	$\forall x: rat\cdot\ \forall y: rat\cdot\ x = y{+}1\ \Rightarrow\ x > y$
$\Sigma n, m: 0,..10\cdot\ n{\times}m$	abbreviates	$\Sigma n: 0,..10\cdot\ \Sigma m: 0,..10\cdot\ n{\times}m$

$\forall v\colon null\cdot\ b\ =\ \top$

$\forall v\colon x\cdot\ b\ =\ \langle v\colon x\cdot\ b\rangle\, x$

$\forall v\colon A,B\cdot\ b\ =\ (\forall v\colon A\cdot\ b) \wedge (\forall v\colon B\cdot\ b)$

$\exists v\colon null\cdot\ b\ =\ \bot$

$\exists v\colon x\cdot\ b\ =\ \langle v\colon x\cdot\ b\rangle\, x$

$\exists v\colon A,B\cdot\ b\ =\ (\exists v\colon A\cdot\ b) \vee (\exists v\colon B\cdot\ b)$

$\Sigma v\colon null\cdot\ n\ =\ 0$

$\Sigma v\colon x\cdot\ n\ =\ \langle v\colon x\cdot\ n\rangle\, x$

$(\Sigma v\colon A,B\cdot\ n) + (\Sigma v\colon A`B\cdot\ n)\ =\ (\Sigma v\colon A\cdot\ n) + (\Sigma v\colon B\cdot\ n)$

$\Pi v\colon null\cdot\ n\ =\ 1$

$\Pi v\colon x\cdot\ n\ =\ \langle v\colon x\cdot\ n\rangle\, x$

$(\Pi v\colon A,B\cdot\ n) \times (\Pi v\colon A`B\cdot\ n)\ =\ (\Pi v\colon A\cdot\ n) \times (\Pi v\colon B\cdot\ n)$

$\Uparrow v\colon null\cdot\ b\ =\ -\infty$

$\Uparrow v\colon x\cdot\ b\ =\ \langle v\colon x\cdot\ b\rangle\, x$

$\Uparrow v\colon A,B\cdot\ b\ =\ (\Uparrow v\colon A\cdot\ b) \uparrow (\Uparrow v\colon B\cdot\ b)$

$\Downarrow v\colon null\cdot\ b\ =\ \infty$

$\Downarrow v\colon x\cdot\ b\ =\ \langle v\colon x\cdot\ b\rangle\, x$

$\Downarrow v\colon A,B\cdot\ b\ =\ (\Downarrow v\colon A\cdot\ b) \downarrow (\Downarrow v\colon B\cdot\ b)$

$$\forall v: null \cdot b \;=\; \top \quad \leftarrow$$

$$\forall v: x \cdot b \;=\; \langle v: x \cdot b \rangle\, x$$

$$\forall v: A{,}B \cdot b \;=\; (\forall v: A \cdot b) \wedge (\forall v: B \cdot b)$$

$$\exists v: null \cdot b \;=\; \bot \quad \leftarrow$$

$$\exists v: x \cdot b \;=\; \langle v: x \cdot b \rangle\, x$$

$$\exists v: A{,}B \cdot b \;=\; (\exists v: A \cdot b) \vee (\exists v: B \cdot b)$$

$$\Sigma v: null \cdot n \;=\; 0 \quad \leftarrow$$

$$\Sigma v: x \cdot n \;=\; \langle v: x \cdot n \rangle\, x$$

$$(\Sigma v: A{,}B \cdot n) + (\Sigma v: A`B \cdot n) \;=\; (\Sigma v: A \cdot n) + (\Sigma v: B \cdot n)$$

$$\Pi v: null \cdot n \;=\; 1 \quad \leftarrow$$

$$\Pi v: x \cdot n \;=\; \langle v: x \cdot n \rangle\, x$$

$$(\Pi v: A{,}B \cdot n) \times (\Pi v: A`B \cdot n) \;=\; (\Pi v: A \cdot n) \times (\Pi v: B \cdot n)$$

$$\Uparrow v: null \cdot b \;=\; -\infty \quad \leftarrow$$

$$\Uparrow v: x \cdot b \;=\; \langle v: x \cdot b \rangle\, x$$

$$\Uparrow v: A{,}B \cdot b \;=\; (\Uparrow v: A \cdot b) \uparrow (\Uparrow v: B \cdot b)$$

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$$\Downarrow v: x \cdot b \;=\; \langle v: x \cdot b \rangle\, x$$

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$\forall v\colon null\cdot\ b \ = \ \top$

$\forall v\colon x\cdot\ b \ = \ \langle v\colon x\cdot\ b\rangle\ x$

$\forall v\colon A{,}B\cdot\ b \ = \ (\forall v\colon A\cdot\ b) \wedge (\forall v\colon B\cdot\ b)$

$\exists v\colon null\cdot\ b \ = \ \bot$

$\exists v\colon x\cdot\ b \ = \ \langle v\colon x\cdot\ b\rangle\ x$

$\exists v\colon A{,}B\cdot\ b \ = \ (\exists v\colon A\cdot\ b) \vee (\exists v\colon B\cdot\ b)$

$\Sigma v\colon null\cdot\ n \ = \ 0$ ⟵ because $x+0 = x$

$\Sigma v\colon x\cdot\ n \ = \ \langle v\colon x\cdot\ n\rangle\ x$

$(\Sigma v\colon A{,}B\cdot\ n) + (\Sigma v\colon A\text{‘}B\cdot\ n) \ = \ (\Sigma v\colon A\cdot\ n) + (\Sigma v\colon B\cdot\ n)$

$\Pi v\colon null\cdot\ n \ = \ 1$

$\Pi v\colon x\cdot\ n \ = \ \langle v\colon x\cdot\ n\rangle\ x$

$(\Pi v\colon A{,}B\cdot\ n) \times (\Pi v\colon A\text{‘}B\cdot\ n) \ = \ (\Pi v\colon A\cdot\ n) \times (\Pi v\colon B\cdot\ n)$

$\Uparrow v\colon null\cdot\ b \ = \ -\infty$

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$\Downarrow v\colon null\cdot\ b \ = \ \infty$

$\Downarrow v\colon x\cdot\ b \ = \ \langle v\colon x\cdot\ b\rangle\ x$

$\Downarrow v\colon A{,}B\cdot\ b \ = \ (\Downarrow v\colon A\cdot\ b) \downarrow (\Downarrow v\colon B\cdot\ b)$

$\forall v\text{: } null\cdot\ b \ = \ \top$

$\exists v\text{: } null\cdot\ b \ = \ \bot$

$\forall v\text{: } x\cdot\ b \ = \ \langle v\text{: } x\cdot\ b\rangle\ x$

$\exists v\text{: } x\cdot\ b \ = \ \langle v\text{: } x\cdot\ b\rangle\ x$

$\forall v\text{: } A,B\cdot\ b \ = \ (\forall v\text{: } A\cdot\ b) \wedge (\forall v\text{: } B\cdot\ b)$

$\exists v\text{: } A,B\cdot\ b \ = \ (\exists v\text{: } A\cdot\ b) \vee (\exists v\text{: } B\cdot\ b)$

$\Sigma v\text{: } null\cdot\ n \ = \ 0$

$\Sigma v\text{: } x\cdot\ n \ = \ \langle v\text{: } x\cdot\ n\rangle\ x$

$(\Sigma v\text{: } A,B\cdot\ n) + (\Sigma v\text{: } A\text{‘}B\cdot\ n) \ = \ (\Sigma v\text{: } A\cdot\ n) + (\Sigma v\text{: } B\cdot\ n)$

$\Pi v\text{: } null\cdot\ n \ = \ 1$ ⟵ because $x{\times}1 = x$

$\Pi v\text{: } x\cdot\ n \ = \ \langle v\text{: } x\cdot\ n\rangle\ x$

$(\Pi v\text{: } A,B\cdot\ n) \times (\Pi v\text{: } A\text{‘}B\cdot\ n) \ = \ (\Pi v\text{: } A\cdot\ n) \times (\Pi v\text{: } B\cdot\ n)$

$\Uparrow v\text{: } null\cdot\ b \ = \ -\infty$

$\Downarrow v\text{: } null\cdot\ b \ = \ \infty$

$\Uparrow v\text{: } x\cdot\ b \ = \ \langle v\text{: } x\cdot\ b\rangle\ x$

$\Downarrow v\text{: } x\cdot\ b \ = \ \langle v\text{: } x\cdot\ b\rangle\ x$

$\Uparrow v\text{: } A,B\cdot\ b \ = \ (\Uparrow v\text{: } A\cdot\ b) \uparrow (\Uparrow v\text{: } B\cdot\ b)$

$\Downarrow v\text{: } A,B\cdot\ b \ = \ (\Downarrow v\text{: } A\cdot\ b) \downarrow (\Downarrow v\text{: } B\cdot\ b)$

$\forall v\colon \mathit{null}\cdot\ b\ =\ \top$ $\qquad$ $\exists v\colon \mathit{null}\cdot\ b\ =\ \bot$ $\longleftarrow$ because $x \lor \bot = x$

$\forall v\colon x\cdot\ b\ =\ \langle v\colon x\cdot\ b\rangle\, x$ $\qquad$ $\exists v\colon x\cdot\ b\ =\ \langle v\colon x\cdot\ b\rangle\, x$

$\forall v\colon A{,}B\cdot\ b\ =\ (\forall v\colon A\cdot\ b) \land (\forall v\colon B\cdot\ b)$ $\qquad$ $\exists v\colon A{,}B\cdot\ b\ =\ (\exists v\colon A\cdot\ b) \lor (\exists v\colon B\cdot\ b)$

$\Sigma v\colon \mathit{null}\cdot\ n\ =\ 0$

$\Sigma v\colon x\cdot\ n\ =\ \langle v\colon x\cdot\ n\rangle\, x$

$(\Sigma v\colon A{,}B\cdot\ n) + (\Sigma v\colon A\text{‘}B\cdot\ n)\ =\ (\Sigma v\colon A\cdot\ n) + (\Sigma v\colon B\cdot\ n)$

$\Pi v\colon \mathit{null}\cdot\ n\ =\ 1$

$\Pi v\colon x\cdot\ n\ =\ \langle v\colon x\cdot\ n\rangle\, x$

$(\Pi v\colon A{,}B\cdot\ n) \times (\Pi v\colon A\text{‘}B\cdot\ n)\ =\ (\Pi v\colon A\cdot\ n) \times (\Pi v\colon B\cdot\ n)$

$\Uparrow v\colon \mathit{null}\cdot\ b\ =\ -\infty$ $\qquad$ $\Downarrow v\colon \mathit{null}\cdot\ b\ =\ \infty$

$\Uparrow v\colon x\cdot\ b\ =\ \langle v\colon x\cdot\ b\rangle\, x$ $\qquad$ $\Downarrow v\colon x\cdot\ b\ =\ \langle v\colon x\cdot\ b\rangle\, x$

$\Uparrow v\colon A{,}B\cdot\ b\ =\ (\Uparrow v\colon A\cdot\ b) \uparrow (\Uparrow v\colon B\cdot\ b)$ $\qquad$ $\Downarrow v\colon A{,}B\cdot\ b\ =\ (\Downarrow v\colon A\cdot\ b) \downarrow (\Downarrow v\colon B\cdot\ b)$

$\forall v\colon null\cdot\ b\ =\ \top$ ← because $x\land\top = x$

$\forall v\colon x\cdot\ b\ =\ \langle v\colon x\cdot\ b\rangle\, x$

$\forall v\colon A{,}B\cdot\ b\ =\ (\forall v\colon A\cdot\ b)\land(\forall v\colon B\cdot\ b)$

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$\Uparrow v\colon null\cdot\ b \ = \ -\infty$ ← because $x\uparrow-\infty = x$

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$\exists v\colon A,B \cdot b \;=\; (\exists v\colon A \cdot b) \vee (\exists v\colon B \cdot b)$

$\Sigma v\colon null \cdot n \;=\; 0$

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$\Downarrow v\colon null \cdot b \;=\; \infty$ ⟵ because $x \downarrow \infty = x$

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$\forall v\colon A,B\cdot\ b\ =\ (\forall v\colon A\cdot\ b)\wedge(\forall v\colon B\cdot\ b)$ ⟵

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$(\Pi v\colon A,B\cdot\ n)\times(\Pi v\colon A`B\cdot\ n)\ =\ (\Pi v\colon A\cdot\ n)\times(\Pi v\colon B\cdot\ n)$

$\Uparrow v\colon null\cdot\ b\ =\ -\infty$

$\Uparrow v\colon x\cdot\ b\ =\ \langle v\colon x\cdot\ b\rangle\ x$

$\Uparrow v\colon A,B\cdot\ b\ =\ (\Uparrow v\colon A\cdot\ b)\uparrow(\Uparrow v\colon B\cdot\ b)$

$\Downarrow v\colon null\cdot\ b\ =\ \infty$

$\Downarrow v\colon x\cdot\ b\ =\ \langle v\colon x\cdot\ b\rangle\ x$

$\Downarrow v\colon A,B\cdot\ b\ =\ (\Downarrow v\colon A\cdot\ b)\downarrow(\Downarrow v\colon B\cdot\ b)$

$\forall v\colon null\cdot\ b \;=\; \top$

$\forall v\colon x\cdot\ b \;=\; \langle v\colon x\cdot\ b\rangle\ x$

$\forall v\colon A{,}B\cdot\ b \;=\; (\forall v\colon A\cdot\ b) \wedge (\forall v\colon B\cdot\ b)$

$\exists v\colon null\cdot\ b \;=\; \bot$

$\exists v\colon x\cdot\ b \;=\; \langle v\colon x\cdot\ b\rangle\ x$

$\exists v\colon A{,}B\cdot\ b \;=\; (\exists v\colon A\cdot\ b) \vee (\exists v\colon B\cdot\ b)$

$\Sigma v\colon null\cdot\ n \;=\; 0$

$\Sigma v\colon x\cdot\ n \;=\; \langle v\colon x\cdot\ n\rangle\ x$

$(\Sigma v\colon A{,}B\cdot\ n) + (\Sigma v\colon A\text{`}B\cdot\ n) \;=\; (\Sigma v\colon A\cdot\ n) + (\Sigma v\colon B\cdot\ n)$ ←

$\Pi v\colon null\cdot\ n \;=\; 1$

$\Pi v\colon x\cdot\ n \;=\; \langle v\colon x\cdot\ n\rangle\ x$

$(\Pi v\colon A{,}B\cdot\ n) \times (\Pi v\colon A\text{`}B\cdot\ n) \;=\; (\Pi v\colon A\cdot\ n) \times (\Pi v\colon B\cdot\ n)$ ←

$\Uparrow v\colon null\cdot\ b \;=\; -\infty$

$\Uparrow v\colon x\cdot\ b \;=\; \langle v\colon x\cdot\ b\rangle\ x$

$\Uparrow v\colon A{,}B\cdot\ b \;=\; (\Uparrow v\colon A\cdot\ b) \uparrow (\Uparrow v\colon B\cdot\ b)$

$\Downarrow v\colon null\cdot\ b \;=\; \infty$

$\Downarrow v\colon x\cdot\ b \;=\; \langle v\colon x\cdot\ b\rangle\ x$

$\Downarrow v\colon A{,}B\cdot\ b \;=\; (\Downarrow v\colon A\cdot\ b) \downarrow (\Downarrow v\colon B\cdot\ b)$

Solution Quantifier

$\S p$ is the (bunch of) solutions of predicate p

$\S v: null\cdot b$ =

$\S v: x\cdot b$ =

$\S v: A,B\cdot b$ =

Solution Quantifier

$\S p$ is the (bunch of) solutions of predicate p

$\S v \colon null \cdot b \;=\; null$

$\S v \colon x \cdot b \;=$

$\S v \colon A,B \cdot b \;=$

Solution Quantifier

$\S p$ is the (bunch of) solutions of predicate p

$\S v\text{: } null\cdot\ b \;=\; null$

$\S v\text{: } x\cdot\ b \;=\; \textbf{if}\ \langle v\text{: } x\cdot\ b\rangle\ x\ \textbf{then}\ x\ \textbf{else}\ null\ \textbf{fi}$

$\S v\text{: } A{,}B\cdot\ b \;=$

Solution Quantifier

$\S p$ is the (bunch of) solutions of predicate p

$\S v\colon null\cdot\ b \;=\; null$

$\S v\colon x\cdot\ b \;=\; \textbf{if}\ \langle v\colon x\cdot\ b\rangle\ x\ \textbf{then}\ x\ \textbf{else}\ null\ \textbf{fi}$

$\S v\colon A{,}B\cdot\ b \;=\; (\S v\colon A\cdot\ b), (\S v\colon B\cdot\ b)$

Solution Quantifier

$\S p$ is the (bunch of) solutions of predicate p

$\S v\text{: } null\cdot\ b \ = \ null$

$\S v\text{: } x\cdot\ b \ = \ \textbf{if}\ \langle v\text{: } x\cdot\ b\rangle\ x\ \textbf{then}\ x\ \textbf{else}\ null\ \textbf{fi}$

$\S v\text{: } A,B\cdot\ b \ = \ (\S v\text{: } A\cdot\ b), (\S v\text{: } B\cdot\ b)$

$\S i\text{: } int\cdot\ i^2{=}4$

Solution Quantifier

$\S p$ is the (bunch of) solutions of predicate p

$\S v\colon null\cdot\ b \ = \ null$

$\S v\colon x\cdot\ b \ = \ \textbf{if}\ \langle v\colon x\cdot\ b\rangle\ x\ \textbf{then}\ x\ \textbf{else}\ null\ \textbf{fi}$

$\S v\colon A{,}B\cdot\ b \ = \ (\S v\colon A\cdot\ b), (\S v\colon B\cdot\ b)$

$\S\, i\colon int\cdot\ i^2{=}4 \qquad = \quad -2, 2$

Solution Quantifier

$\S p$ is the (bunch of) solutions of predicate p

$\S v\colon null\cdot\ b \;=\; null$

$\S v\colon x\cdot\ b \;=\; \textbf{if}\ \langle v\colon x\cdot\ b\rangle\ x\ \textbf{then}\ x\ \textbf{else}\ null\ \textbf{fi}$

$\S v\colon A{,}B\cdot\ b \;=\; (\S v\colon A\cdot\ b), (\S v\colon B\cdot\ b)$

$\S i\colon int\cdot\ i^2{=}4 \quad = \quad -2, 2$

$\S n\colon nat\cdot\ n{<}3$

Solution Quantifier

$\S p$ is the (bunch of) solutions of predicate p

$\S v{:}\ null\cdot\ b \ = \ null$

$\S v{:}\ x\cdot\ b \ = \ \textbf{if}\ \langle v{:}\ x\cdot\ b\rangle\ x\ \textbf{then}\ x\ \textbf{else}\ null\ \textbf{fi}$

$\S v{:}\ A{,}B\cdot\ b \ = \ (\S v{:}\ A\cdot\ b),\ (\S v{:}\ B\cdot\ b)$

$\S i{:}\ int\cdot\ i^2{=}4 \quad = \quad -2,\ 2$

$\S n{:}\ nat\cdot\ n{<}3 \quad = \quad 0,..3$

Solution Quantifier

$\S p$ is the (bunch of) solutions of predicate p

$\S v{:}\ null\cdot\ b \ = \ null$

$\S v{:}\ x\cdot\ b \ = \ \textbf{if}\ \langle v{:}\ x\cdot\ b\rangle\ x\ \textbf{then}\ x\ \textbf{else}\ null\ \textbf{fi}$

$\S v{:}\ A,B\cdot\ b \ = \ (\S v{:}\ A\cdot\ b),\ (\S v{:}\ B\cdot\ b)$

$\{\S i{:}\ int\cdot\ i^2{=}4\} \ = \ \{-2, 2\}$

$\{\S n{:}\ nat\cdot\ n{<}3\} \ = \ \{0,..3\}$

Solution Quantifier

$\S p$ is the (bunch of) solutions of predicate p

$\S v\colon null\cdot\ b \ = \ null$

$\S v\colon x\cdot\ b \ = \ \textbf{if}\ \langle v\colon x\cdot\ b\rangle\ x\ \textbf{then}\ x\ \textbf{else}\ null\ \textbf{fi}$

$\S v\colon A{,}B\cdot\ b \ = \ (\S v\colon A\cdot\ b), (\S v\colon B\cdot\ b)$

$\{\ \ i\colon int\cdot\ i^2{=}4\} \qquad = \{-2, 2\}$

$\{\ \ n\colon nat\cdot\ n{<}3\} \qquad = \{0,..3\}$

An expression talks about its nonlocal variables.

$$\exists n: nat\cdot\ x = 2\times n$$

says

“ x is an even natural”