

**bunch**

**set**

**string**

**list**

**bunch**

uncontained

unindexed

**set**

**string**

**list**

**bunch**

uncontained

unindexed

**set**

contained

unindexed

**string**

**list**

<b>bunch</b>	uncontained	unindexed
--------------	-------------	-----------

<b>set</b>	contained	unindexed
------------	-----------	-----------

<b>string</b>	uncontained	indexed
---------------	-------------	---------

**list**

<b>bunch</b>	uncontained	unindexed
<b>set</b>	contained	unindexed
<b>string</b>	uncontained	indexed
<b>list</b>	contained	indexed

# String Theory

Strings are indexed sequences.

# String Theory

Strings are indexed sequences.

*nil*

the empty string

# String Theory

Strings are indexed sequences.

*nil*

the empty string

3

a one-item string

# String Theory

Strings are indexed sequences.

*nil*

the empty string

3

a one-item string

3; 5; 7; 9

a four-item string

# String Theory

Strings are indexed sequences.

*nil*

the empty string

3

a one-item string

3; 5; 7; 9

a four-item string

$\leftrightarrow(3; 5; 7; 9) = 4$

string length operator

# String Theory

Strings are indexed sequences.

*nil*

the empty string

3

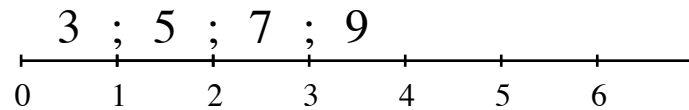
a one-item string

3; 5; 7; 9

a four-item string

$\leftrightarrow(3; 5; 7; 9) = 4$

string length operator



# String Theory

Strings are indexed sequences.

*nil*

the empty string

3

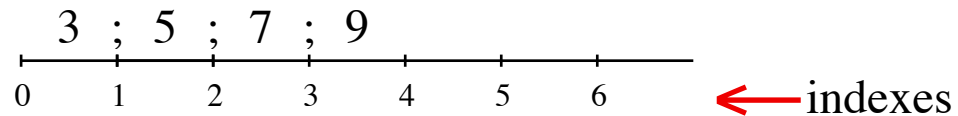
a one-item string

3; 5; 7; 9

a four-item string

$\leftrightarrow(3; 5; 7; 9) = 4$

string length operator



# String Theory

Strings are indexed sequences.

*nil*

the empty string

3

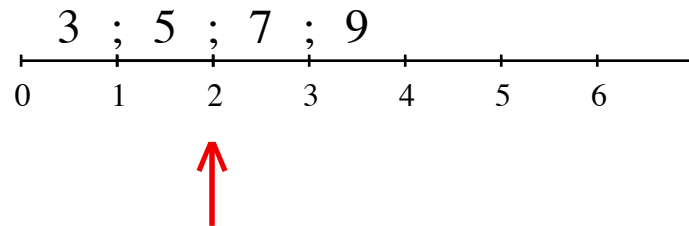
a one-item string

3; 5; 7; 9

a four-item string

$\leftrightarrow(3; 5; 7; 9) = 4$

string length operator



# String Theory

Strings are indexed sequences.

*nil*

the empty string

3

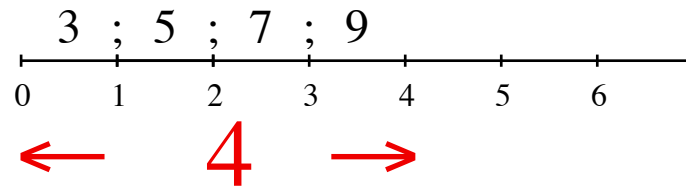
a one-item string

3; 5; 7; 9

a four-item string

$\leftrightarrow(3; 5; 7; 9) = 4$

string length operator



# String Theory

Strings are indexed sequences.

*nil*

the empty string

3

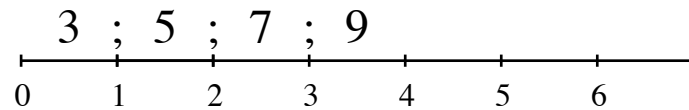
a one-item string

3; 5; 7; 9

a four-item string

$\leftrightarrow(3; 5; 7; 9) = 4$

string length operator

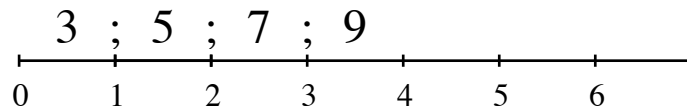


$(3; 5; 7; 9)_2 = 7$

# String Theory

Strings are indexed sequences.

<i>nil</i>	the empty string
3	a one-item string
3; 5; 7; 9	a four-item string
$\leftrightarrow(3; 5; 7; 9) = 4$	string length operator



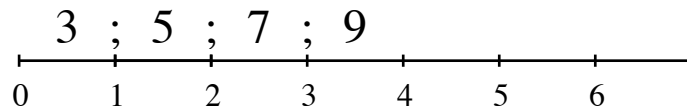
$$(3; 5; 7; 9)_2 = 7$$

**At index  $n$  , the number of items processed is  $n$   
the next item to be processed is item  $n$**

# String Theory

Strings are indexed sequences.

<i>nil</i>	the empty string
3	a one-item string
3; 5; 7; 9	a four-item string
$\leftrightarrow(3; 5; 7; 9) = 4$	string length operator



$$(3; 5; 7; 9)_2 = 7$$

$$(3; 5; 7; 9)_{2; 1; 2} = 7; 5; 7$$

# Zero

# Zero

John Allen Paulos:

*Innumeracy: Mathematical Illiteracy and its Consequences*, Hill and Wang, 1988

*Beyond Numeracy*, Knopf, 1991

# Zero

John Allen Paulos:

*Innumeracy: Mathematical Illiteracy and its Consequences*, Hill and Wang, 1988

*Beyond Numeracy*, Knopf, 1991

0.10¢

# Zero

John Allen Paulos:

*Innumeracy: Mathematical Illiteracy and its Consequences*, Hill and Wang, 1988

*Beyond Numeracy*, Knopf, 1991

0.10¢    \$1.02.9

# Zero

John Allen Paulos:

*Innumeracy: Mathematical Illiteracy and its Consequences*, Hill and Wang, 1988

*Beyond Numeracy*, Knopf, 1991

0.10¢    \$1.02.9

There are a number of things to discuss. (But not zero things to discuss.)

# Zero

John Allen Paulos:

*Innumeracy: Mathematical Illiteracy and its Consequences*, Hill and Wang, 1988

*Beyond Numeracy*, Knopf, 1991

0.10¢    \$1.02.9

There are a number of things to discuss. (But not zero things to discuss.)

Subtract line A from line B; if there is no difference, write “nil”.

# Zero

John Allen Paulos:

*Innumeracy: Mathematical Illiteracy and its Consequences*, Hill and Wang, 1988

*Beyond Numeracy*, Knopf, 1991

0.10¢    \$1.02.9

There are a number of things to discuss. (But not zero things to discuss.)

Subtract line A from line B; if there is no difference, write “nil”.

keyboard, telephone: 1 2 3 4 5 6 7 8 9 0

If you need more space, use the "Comments" section on page 28.

**STEP 7**

How many persons who have a usual home somewhere else in Canada stayed here overnight between June 3 and 4, 1991?

None

OR

02  ◀ Number of persons

**STEP 8**

Does anyone in this household OPERATE a farm, ranch or other agricultural holding?

03  No

04  Yes

*Other agricultural holdings include, for example: feedlots; greenhouses; mushroom houses; nurseries; fur farms; and beekeeping, sod, berry and maple syrup operations.*

**STEP 9**

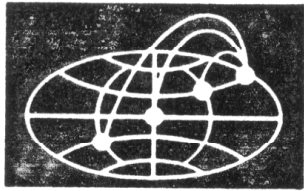
Turn the page and copy the names from Step 5 into the spaces across the top of the page.

Then continue with the questionnaire.

**Note:**

If there are **more than six persons** in this household, enter the first six on this questionnaire and continue on a second questionnaire. If you do not have a second questionnaire, note this in the "Comments" section on page 28. A census representative will contact you.

**36 Overseas Codes**



# Long Distance Calls

## Codes for Dialing Overseas

**For station-to-station calls; DIAL/PRESS:**  
 011 + Country Code + Routing Code + Local Number

**For automated Calling Card™ calls; PRESS:**  
 01 + Country Code + Routing Code + Local Number  
 + (after tone) your 14 digit North American card number

**For person-to-person and other types of calls; DIAL/PRESS:**  
 01 + Country Code + Routing Code + Local Number

**For countries or cities not listed; DIAL/PRESS:**  
 "0" (zero) and ask the operator for the routing codes.

**Legend:** \* Routing codes not required  
 TD Approximate time difference  
 -- in hours from Eastern Standard time  
 NA Time difference not applicable

These overseas codes were in effect at the time of printing this directory.

COUNTRY & COUNTRY CODE	ROUTING CODE	TD	COUNTRY & COUNTRY CODE	ROUTING CODE	TD	COUNTRY & COUNTRY CODE	ROUTING CODE	TD
Cameroon 237	*	+6	Cuba 53	*	NA	Ethiopia 251		+8
			Except Havana	7		Addis Ababa	1	
Chile 56		+2	Cyprus 357		+7	Asmara	4	
Santiago	2		Larnaca	4		Dire Dawa	5	
Valparaiso	32		Limassol	5		Faroe Islands 298	*	+6
Viña Del Mar	32		Nicosia	2		Fiji Islands 679	*	+17
China 86		+13	Czech And Slovak 42		+6	Finland 358		+7
Beijing (Peking)	1		Bratislava	7		Helsinki	0	
Chengdu	28		Brno	5		Tampere	31	
Shanghai	21		Prague	2				
Christmas Island 672		+12						

1993  
1996 same

**34 Overseas Codes**



**Overseas Calls**

Codes for frequently called countries

For station-to-station calls: dial 011 + the country code + the routing code + the local number.

For person-to-person and other types of calls: dial 01 + the country code + the routing code + the local number.

For Automated Billing Service calls: dial 01 + the country code + the routing code + the local number, and (after the tone) the Automated Billing Service will tell you what steps to take to complete your call.

Dial 0 (zero):  
 • to obtain a number you don't know.  
 • to obtain credit for unsatisfactory calls, e.g. when you reach a wrong number.

Legend: \* Routing codes not required.  
 TD Approximate time difference in hours from Eastern Standard time  
 = Time difference not applicable.

These overseas codes were in effect at the time of printing this directory.

COUNTRY & ROUTING CODE	TD	COUNTRY & ROUTING CODE	TD	COUNTRY & ROUTING CODE	TD	COUNTRY & ROUTING CODE	TD
<b>Algeria 213</b> Alger 2 Constantine 4 Oran 6	+6	<b>Brazil 55</b> Belo Horizonte 31 Brasilia 61 Porto Alegre 51 Recife 81 Rio de Janeiro 21 Salvador 71 São Paulo 11	+2	<b>Denmark 45</b> *	+6	<b>Greece 30</b> Athens 1 Iraklion Kritis 81 Kavalla 51 Lárisa 41 Pátraí 61 Thessaloniki 31 Volos 421	+7
<b>Argentina 54</b> Buenos Aires 1 Córdoba 51 Rosario 41	+2	<b>Chile 56</b> Santiago 2 Valparaíso 32	+1	<b>Egypt 20</b> Alexandria 3 Cairo 2 Port Said 66	+7	<b>Guatemala 502</b> Escuintla 9 Guatemala City 2 Quetzaltenango 9	-1
<b>Australia 61</b> Adelaide 8 Brisbane 7 Canberra 6 Melbourne 2	+16	<b>China 86</b> Beijing (Peking) 1 Shanghai 21	+13	<b>El Salvador 503</b> *	-1	<b>Haiti 509</b> *	=
				<b>Finland 358</b> Helsinki 0 Tampere 31 Turku (Åbo) 21	+7		

1997 28  
also 1998 49

## Legend for Country and City Code Table

0 No time difference

★ City codes not required.

TD Approximate time difference in hours from Eastern Standard time

## FREQUENTLY CALLED PLACES

COUNTRY & CITY CODE	TD	COUNTRY & CITY CODE	TD	COUNTRY & CITY CODE	TD	COUNTRY & CITY CODE	TD
<b>Algeria 213</b> Alger 2 Constantine 4	+6	<b>Bangladesh 880</b> Chittagong 31 Khulna 41	+11	<b>Chile 56</b> Santiago 2 Valparaíso 32	+1	<b>Costa Rica 506</b> *	-1
<b>Argentina 54</b> Buenos Aires 1 Córdoba 51	+2	<b>Belgium 32</b> Antwerpen 3 Brussels 2 Gent 9	+6	<b>China 86</b> Beijing (Peking) 10 Shanghai 21 Tianjim (Tientsin) 22	+13	<b>Croatia 385</b>	+6
<b>Australia 61</b> Adelaide 8 Canberra 6 Melbourne 3 Sydney 2	+16	<b>Bosnia- Herzegovina 387</b>	+6	<b>C.I.S. 7</b> (Commonwealth of Independent States) Moscow 095 St. Petersburg 812	+8	<b>Cyprus 357</b> Larnaca 4 Limassol 5	+7
<b>Austria 43</b> Innsbruck 512 Salzburg 662 Vienna 1	+6	<b>Brazil 55</b> Brasilia 61 Pôrto Alegre 51 Rio de Janeiro 21 São Paulo 11	+2	<b>Colombia 57</b> Bogotá 1 Medellín 4	0	<b>Czech &amp; Slovak 42</b> Bratislava 7 Prague 2	+6
						<b>Denmark 45</b> *	+6

For countries or cities *NOT* listed dial "0" (zero) and ask the operator for the appropriate codes. These overseas codes were in effect at the time of printing this directory.

\*Trade mark of Teleglobe Canada Inc.

# Zero

Measuring must start at 0.

# Zero

Measuring must start at 0. Counting is measuring.

# Zero

Measuring must start at 0. Counting is measuring.

An octave is an interval of 8. What interval is 2 octaves?

# Zero

Measuring must start at 0. Counting is measuring.

An octave is an interval of 8. What interval is 2 octaves? It's 15.

# Zero

Measuring must start at 0. Counting is measuring.

An octave is an interval of 8. What interval is 2 octaves? It's 15.

How many years from July 1 in year X to July 1 in year Y?

# Zero

Measuring must start at 0. Counting is measuring.

An octave is an interval of 8. What interval is 2 octaves? It's 15.

How many years from July 1 in year X to July 1 in year Y?  $Y-X$  years?

# Zero

Measuring must start at 0. Counting is measuring.

An octave is an interval of 8. What interval is 2 octaves? It's 15.

How many years from July 1 in year X to July 1 in year Y?  $Y-X$  years?

Fortran 1955 loop body had to be executed at least once.

# Zero

Measuring must start at 0. Counting is measuring.

An octave is an interval of 8. What interval is 2 octaves? It's 15.

How many years from July 1 in year X to July 1 in year Y?  $Y-X$  years?

Fortran 1955 loop body had to be executed at least once.

```
count := 0.
```

```
while there's another one
```

```
do
```

```
    count := count + 1
```

```
od
```

# Zero

Measuring must start at 0. Counting is measuring.

An octave is an interval of 8. What interval is 2 octaves? It's 15.

How many years from July 1 in year X to July 1 in year Y?  $Y-X$  years?

Fortran 1955 loop body had to be executed at least once.

```
count := 0.
```

```
while there's another one
```

```
do
```

```
    count := count + 1
```

```
od
```

Algol 1958, PL/I, Pascal: array must have at least 1 element.

# Zero

# Zero

first: preceding all others in time, order, or importance

# Zero

first: preceding all others in time, order, or importance

last: following all others in time, order, or importance

# Zero

first: preceding all others in time, order, or importance

last: following all others in time, order, or importance

second: following the first

# Zero

first: preceding all others in time, order, or importance 1st

last: following all others in time, order, or importance

second: following the first 2nd

# Zero

first: preceding all others in time, order, or importance

1<sup>st</sup>

0<sup>st</sup>

last: following all others in time, order, or importance

second: following the first

~~2<sup>nd</sup>~~

1<sup>nd</sup>

# Zero

first: preceding all others in time, order, or importance

~~1st~~

0st

last: following all others in time, order, or importance

second: following the first

~~2nd~~

~~1nd~~

third year of life = what age?

# Zero

first: preceding all others in time, order, or importance

~~1st~~

0st

last: following all others in time, order, or importance

second: following the first

~~2nd~~

~~1nd~~

third year of life = age 2

# Zero

first: preceding all others in time, order, or importance

~~1st~~

0st

last: following all others in time, order, or importance

second: following the first

~~2nd~~

~~1nd~~

third year of life = age 2

the eleventh hour = the latest possible time

# Zero

first: preceding all others in time, order, or importance      ~~1st~~      0st

last: following all others in time, order, or importance

second: following the first      ~~2nd~~      ~~1nd~~

third year of life = age 2

the eleventh hour = the latest possible time      11 to 12 o'clock?

# Zero

first: preceding all others in time, order, or importance

~~1st~~

0st

last: following all others in time, order, or importance

second: following the first

~~2nd~~

~~1nd~~

third year of life = age 2

the eleventh hour = the latest possible time

11 to 12 o'clock?

10 to 11 o'clock?

# Zero

first: preceding all others in time, order, or importance

~~1~~st

0st

last: following all others in time, order, or importance

second: following the first

~~2~~nd

~~1~~nd

~~third~~ year of life = age 2

the ~~eleventh~~ hour = the latest possible time    11 to 12 o'clock?    10 to 11 o'clock?

# Zero

first: preceding all others in time, order, or importance

~~1~~st

0st

last: following all others in time, order, or importance

second: following the first

~~2~~nd

~~1~~nd

~~third~~ year of life = age 2

the ~~eleventh~~ hour = the latest possible time    11 to 12 o'clock?    10 to 11 o'clock?

~~fifteenth~~ item = item 15? item 14?

# Zero

first: preceding all others in time, order, or importance      ~~1st~~      0st

last: following all others in time, order, or importance

second: following the first      ~~2nd~~      1nd

~~third~~ year of life = age 2

the ~~eleventh~~ hour = the latest possible time      11 to 12 o'clock?      10 to 11 o'clock?

~~fifteenth~~ item = item 15? item 14?

~~zeroth~~ item = item 0

# Zero

first: preceding all others in time, order, or importance

~~1~~st

0st

last: following all others in time, order, or importance

second: following the first

~~2~~nd

~~1~~nd

~~third~~ year of life = age 2

the ~~eleventh~~ hour = the latest possible time    11 to 12 o'clock?    10 to 11 o'clock?

~~fifteenth~~ item = item 15? item 14?

~~zereth~~ item = item 0 or **first item**

# Zero

first: preceding all others in time, order, or importance

~~1~~st

0st

last: following all others in time, order, or importance

second: following the first

~~2~~nd

~~1~~nd

~~third~~ year of life = age 2

the ~~eleventh~~ hour = the latest possible time    11 to 12 o'clock?    10 to 11 o'clock?

~~fifteenth~~ item = item 15? item 14?

~~zereth~~ item = item 0 or **first item**

twenty-first 21st century

# Zero

first: preceding all others in time, order, or importance

~~1st~~

0st

last: following all others in time, order, or importance

second: following the first

~~2nd~~

~~1nd~~

~~third~~ year of life = age 2

the ~~eleventh~~ hour = the latest possible time    11 to 12 o'clock?    10 to 11 o'clock?

~~fifteenth~~ item = item 15? item 14?

~~zereth~~ item = item 0 or **first item**

~~twenty first~~ 21st century = century 20

# String Theory

Strings are indexed sequences.

# String Theory

Strings are indexed sequences.

3; 6; 4; 1 < 3; 7; 2

order

# String Theory

Strings are indexed sequences.

3; 6; 4; 1 < 3; 7; 2      order

3; 6; 4 < 3; 6; 4; 1      order

# String Theory

Strings are indexed sequences.

3; 6; 4; 1 < 3; 7; 2

order

3; 6; 4 < 3; 6; 4; 1

order

$x;..y$

“ $x$  to  $y$ ” for  $x \leq y$

# String Theory

Strings are indexed sequences.

$3; 6; 4; 1 < 3; 7; 2$

order

$3; 6; 4 < 3; 6; 4; 1$

order

$x;..y$

“ $x$  to  $y$ ” for  $x \leq y$

$\leftrightarrow(x;..y) = y-x$

length

# String Theory

Strings are indexed sequences.

$3; 6; 4; 1 < 3; 7; 2$

order

$3; 6; 4 < 3; 6; 4; 1$

order

$x;..y$

“ $x$  to  $y$ ” for  $x \leq y$

$\leftrightarrow(x;..y) = y-x$

length

$(x;..y) ; (y;..z) = x;..z$

join

# String Theory

Strings are indexed sequences.

$3; 6; 4; 1 < 3; 7; 2$

order

$3; 6; 4 < 3; 6; 4; 1$

order

$x;..y$

“ $x$  to  $y$ ” for  $x \leq y$

$\Leftrightarrow(x;..y) = y-x$

length

$(x;..y) ; (y;..z) = x;..z$

join

“Don't say ““no””.”

text

# String Theory

Strings are indexed sequences.

3; 6; 4; 1 < 3; 7; 2

order

3; 6; 4 < 3; 6; 4; 1

order

$x;..y$

“ $x$  to  $y$ ” for  $x \leq y$

$\leftrightarrow(x;..y) = y-x$

length

$(x;..y) ; (y;..z) = x;..z$

join

“Don't say “no”.”

text

= “D”; “o”; “n”; “ ”; “t”; “ ”; “s”; “a”; “y”; “ ”; “”; “n”; “o”; “ ”; “.”

# String Theory

Strings are indexed sequences.

$3; 6; 4; 1 < 3; 7; 2$  order

$3; 6; 4 < 3; 6; 4; 1$  order

$x;..y$  “ $x$  to  $y$ ” for  $x \leq y$

$\leftrightarrow(x;..y) = y-x$  length

$(x;..y) ; (y;..z) = x;..z$  join

“Don't say “no”.” text

= “D”; “o”; “n”; “ ”; “t”; “ ”; “s”; “a”; “y”; “ ”; “ ”; “n”; “o”; “ ”; “.”

“abcdefghij”<sub>3;..6</sub> = “def” subtext

# String Theory

Strings are indexed sequences.

$3; 6; 4; 1 < 3; 7; 2$

order

$3; 6; 4 < 3; 6; 4; 1$

order

$x;..y$

“ $x$  to  $y$ ” for  $x \leq y$

$\Leftrightarrow(x;..y) = y-x$

length

$(x;..y) ; (y;..z) = x;..z$

join

“Don't say “no”.”

text

= “D”; “o”; “n”; “ ”; “t”; “ ”; “s”; “a”; “y”; “ ”; “ ”; “n”; “o”; “ ”; “ ”

“abcdefg hij”<sub>3;..6</sub> = “def”

subtext

$nat; 1; (0,..10)$

distribution

# String Theory

Strings are indexed sequences.

$3; 6; 4; 1 < 3; 7; 2$

order

$3; 6; 4 < 3; 6; 4; 1$

order

$x;..y$

“ $x$  to  $y$ ” for  $x \leq y$

$\Leftrightarrow(x;..y) = y-x$

length

$(x;..y) ; (y;..z) = x;..z$

join

“Don't say “no”.”

text

= “D”; “o”; “n”; “ ”; “t”; “ ”; “s”; “a”; “y”; “ ”; “ ”; “n”; “o”; “ ”; “ ”

“abcdefghij”<sub>3;..6</sub> = “def”

subtext

0; 1; 2: *nat*; 1; (0,..10)

distribution

# String Theory

Strings are indexed sequences.

$3; 6; 4; 1 < 3; 7; 2$

order

$3; 6; 4 < 3; 6; 4; 1$

order

$x;..y$

“ $x$  to  $y$ ” for  $x \leq y$

$\Leftrightarrow(x;..y) = y-x$

length

$(x;..y) ; (y;..z) = x;..z$

join

“Don't say “no”.”

text

= “D”; “o”; “n”; “”; “t”; “ ”; “s”; “a”; “y”; “ ”; “”; “n”; “o”; “”; “.”

“abcdefghij”<sub>3;..6</sub> = “def”

subtext

$0; 1; 2: \text{nat}; 1; (0,..10)$

distribution

$3*(4; 5) = 4; 5; 4; 5; 4; 5$

repetition

# String Theory

Strings are indexed sequences.

$3; 6; 4; 1 < 3; 7; 2$

order

$3; 6; 4 < 3; 6; 4; 1$

order

$x;..y$

“ $x$  to  $y$ ” for  $x \leq y$

$\Leftrightarrow(x;..y) = y-x$

length

$(x;..y) ; (y;..z) = x;..z$

join

“Don't say “no”.”

text

= “D”; “o”; “n”; “ ”; “t”; “ ”; “s”; “a”; “y”; “ ”; “ ”; “n”; “o”; “ ”; “ ”

“ $abcde\text{fghij}$ ”<sub>3;..6</sub> = “def”

subtext

$0; 1; 2: \text{nat}; 1; (0,..10)$

distribution

$3*(4; 5) = 4; 5; 4; 5; 4; 5$

repetition

$*3 = \text{nil}, 3, 3;3, 3;3;3, \dots$

repetition

# String Theory

Strings are indexed sequences.

$3; 6; 4; 1 < 3; 7; 2$

order

$3; 6; 4 < 3; 6; 4; 1$

order

$x;..y$

“ $x$  to  $y$ ” for  $x \leq y$

$\Leftrightarrow(x;..y) = y-x$

length

$(x;..y) ; (y;..z) = x;..z$

join

“Don't say “no”.”

text

= “D”; “o”; “n”; “”; “t”; “ ”; “s”; “a”; “y”; “ ”; “”; “n”; “o”; “”; “.”

“abcdefghij”<sub>3;..6</sub> = “def”

subtext

$0; 1; 2: \text{nat}; 1; (0,..10)$

distribution

$3*(4; 5) = 4; 5; 4; 5; 4; 5$

repetition

$*3 = \text{nil}, 3, 3;3, 3;3;3, \dots$

repetition

$(3; 5; 9) \triangleleft 2 \triangleright 8 = 3; 5; 8$

modification

# List Theory

# List Theory

[0; 1; 2]

a string in a container

# List Theory

[0; 1; 2]

a string in a container

[*nat*; 1; (0,..10)]

distribution

# List Theory

[0; 1; 2]

a string in a container

[0; 1; 2]: [*nat*; 1; (0,..10)]

distribution

# List Theory

[0; 1; 2]

a string in a container

[0; 1; 2]: [*nat*; 1; (0,..10)]: [ $3**nat*$ ]

distribution

# List Theory

[0; 1; 2] a string in a container  
[0; 1; 2]: [*nat*; 1; (0,..10)]: [*3\**nat**]: [*\**nat**] distribution

# List Theory

$[0; 1; 2]$  a string in a container

$[0; 1; 2]: [nat; 1; (0,..10)]: [3*nat]: [*nat]$  distribution

$$-(a, b) = -a, -b$$

negation of bunch = bunch of negations

# List Theory

$[0; 1; 2]$  a string in a container

$[0; 1; 2]: [nat; 1; (0,..10)]: [3*nat]: [*nat]$  distribution

$$-(a, b) = -a, -b$$

negation of bunch = bunch of negations

$$(a+b) \times (c+d) = a \times c + a \times d + b \times c + b \times d$$

product of sums = sum of products

# List Theory

$[0; 1; 2]$  a string in a container

$[0; 1; 2]: [nat; 1; (0,..10)]: [3*nat]: [*nat]$  distribution

$$-(a, b) = -a, -b$$

negation of bunch = bunch of negations

$$(a+b) \times (c+d) = a \times c + a \times d + b \times c + b \times d$$

product of sums = sum of products

$$(a \vee b) \wedge (c \vee d) = a \wedge c \vee a \wedge d \vee b \wedge c \vee b \wedge d$$

conjunction of disjunctions = disjunction of conjunctions

# List Theory

$[0; 1; 2]$  a string in a container

$[0; 1; 2]: [nat; 1; (0,..10)]: [3*nat]: [*nat]$  distribution

$$-(a, b) = -a, -b$$

negation of bunch = bunch of negations

$$(a+b) \times (c+d) = a \times c + a \times d + b \times c + b \times d$$

product of sums = sum of products

$$(a \vee b) \wedge (c \vee d) = a \wedge c \vee a \wedge d \vee b \wedge c \vee b \wedge d$$

conjunction of disjunctions = disjunction of conjunctions

$$[(2, 3); (4, 5)] = [2; 4], [2; 5], [3; 4], [3; 5]$$

list of bunches = bunch of lists

# List Theory

[0; 1; 2] a string in a container  
[0; 1; 2]: [*nat*; 1; (0,..10)]: [*3\**nat**]: [*\**nat**] distribution

# List Theory

$[0; 1; 2]$	a string in a container
$[0; 1; 2]: [nat; 1; (0,..10)]: [3*nat]: [*nat]$	distribution
$\sim[3; [5; 7]; 4] = 3; [5; 7]; 4$	content

# List Theory

$[0; 1; 2]$	a string in a container
$[0; 1; 2]: [nat; 1; (0,..10)]: [3*nat]: [*nat]$	distribution
$\sim[3; [5; 7]; 4] = 3; [5; 7]; 4$	content
$\#[3; [5; 7]; 4] = 3$	length

# List Theory

$[0; 1; 2]$	a string in a container
$[0; 1; 2]: [nat; 1; (0,..10)]: [3*nat]: [*nat]$	distribution
$\sim[3; [5; 7]; 4] = 3; [5; 7]; 4$	content
$\#[3; [5; 7]; 4] = 3$	length
$[3; [5; 7]; 4] 2 = 4$	index

# List Theory

$[0; 1; 2]$	a string in a container
$[0; 1; 2]: [nat; 1; (0,..10)]: [3*nat]: [*nat]$	distribution
$\sim[3; [5; 7]; 4] = 3; [5; 7]; 4$	content
$\#[3; [5; 7]; 4] = 3$	length
$[3; [5; 7]; 4] 2 = 4$	index
$\square[3; [5; 7]; 4] = 0,..3$	domain

# List Theory

$[0; 1; 2]$	a string in a container
$[0; 1; 2]: [nat; 1; (0,..10)]: [3*nat]: [*nat]$	distribution
$\sim[3; [5; 7]; 4] = 3; [5; 7]; 4$	content
$\#[3; [5; 7]; 4] = 3$	length
$[3; [5; 7]; 4] 2 = 4$	index
$\square[3; [5; 7]; 4] = 0,..3$	domain
$[3; 5; 7; 4] [2; 1; 2] = [7; 5; 7]$	composition

# List Theory

$[0; 1; 2]$	a string in a container
$[0; 1; 2]: [nat; 1; (0,..10)]: [3*nat]: [*nat]$	distribution
$\sim[3; [5; 7]; 4] = 3; [5; 7]; 4$	content
$\#[3; [5; 7]; 4] = 3$	length
$[3; [5; 7]; 4] 2 = 4$	index
$\square[3; [5; 7]; 4] = 0,..3$	domain
$[3; 5; 7; 4] [2; 1; 2] = [7; 5; 7]$	composition
$[3; 5; 7; 4]; [2; 1; 2] = [3; 5; 7; 4; 2; 1; 2]$	join

# List Theory

$[0; 1; 2]$	a string in a container
$[0; 1; 2]: [nat; 1; (0,..10)]: [3*nat]: [*nat]$	distribution
$\sim[3; [5; 7]; 4] = 3; [5; 7]; 4$	content
$\#[3; [5; 7]; 4] = 3$	length
$[3; [5; 7]; 4] 2 = 4$	index
$\square[3; [5; 7]; 4] = 0,..3$	domain
$[3; 5; 7; 4] [2; 1; 2] = [7; 5; 7]$	composition
$[3; 5; 7; 4]; [2; 1; 2] = [3; 5; 7; 4; 2; 1; 2]$	join
$[3; 6; 4; 1] < [3; 7; 2]$	order
$[3; 6; 4] < [3; 6; 4; 1]$	order

# List Theory

$[0; 1; 2]$	a string in a container
$[0; 1; 2]: [nat; 1; (0,..10)]: [3*nat]: [*nat]$	distribution
$\sim[3; [5; 7]; 4] = 3; [5; 7]; 4$	content
$\#[3; [5; 7]; 4] = 3$	length
$[3; [5; 7]; 4] 2 = 4$	index
$\square[3; [5; 7]; 4] = 0,..3$	domain
$[3; 5; 7; 4] [2; 1; 2] = [7; 5; 7]$	composition
$[3; 5; 7; 4]; [2; 1; 2] = [3; 5; 7; 4; 2; 1; 2]$	join
$[3; 6; 4; 1] < [3; 7; 2]$	order
$[3; 6; 4] < [3; 6; 4; 1]$	order
$2 \rightarrow 22 \mid [10; ..15] = [10; 11; 22; 13; 14]$	modification

# List Theory

## modification

Let  $L = [10;..15]$

$$2 \rightarrow L \ 3 \mid 3 \rightarrow L \ 2 \mid L =$$

# List Theory

## modification

Let  $L = [10;..15] = [10; 11; 12; 13; 14]$

$$2 \rightarrow L \ 3 \mid 3 \rightarrow L \ 2 \mid L =$$

# List Theory

## modification

Let  $L = [10;..15] = [10; 11; 12; 13; 14]$

$$2 \rightarrow L 3 \mid 3 \rightarrow L 2 \mid \underline{L} =$$

# List Theory

## modification

Let  $L = [10;..15] = [10; 11; 12; 13; 14]$

$$2 \rightarrow L \ 3 \mid 3 \rightarrow L \ 2 \mid L = [10; 11; 12; 13; 14]$$

# List Theory

## modification

Let  $L = [10;..15] = [10; 11; 12; 13; 14]$

$$2 \rightarrow L 3 \mid \underline{3 \rightarrow L 2} \mid L = [10; 11; 12; 13; 14]$$



# List Theory

## modification

Let  $L = [10;..15] = [10; 11; 12; 13; 14]$

$$2 \rightarrow L 3 \mid 3 \rightarrow L 2 \mid L = [10; 11; 12; 12; 14]$$

# List Theory

## modification

Let  $L = [10;..15] = [10; 11; 12; 13; 14]$

$$\underline{2 \rightarrow L} 3 \mid 3 \rightarrow L 2 \mid L = [10; 11; 12; 12; 14]$$



# List Theory

## modification

Let  $L = [10;..15] = [10; 11; 12; 13; 14]$

$2 \rightarrow L 3 \mid 3 \rightarrow L 2 \mid L = [10; 11; 12; 12; 14]$  

# List Theory

## modification

Let  $L = [10;..15] = [10; 11; 12; 13; 14]$

$$2 \rightarrow L \ 3 \mid 3 \rightarrow L \ 2 \mid L =$$

# List Theory

## modification

Let  $L = [10;..15] = [10; 11; 12; 13; 14]$

$$2 \rightarrow L 3 \mid 3 \rightarrow L 2 \mid \underline{L} =$$

# List Theory

## modification

Let  $L = [10;..15] = [10; 11; 12; 13; 14]$

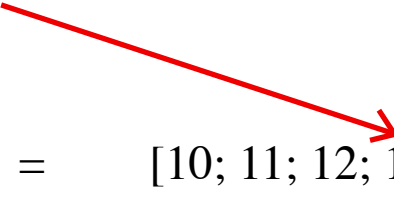
$$2 \rightarrow L \ 3 \mid 3 \rightarrow L \ 2 \mid L = [10; 11; 12; 13; 14]$$

# List Theory

## modification

Let  $L = [10;..15] = [10; 11; 12; 13; 14]$

$2 \rightarrow L 3 \mid \underline{3 \rightarrow L 2} \mid L = [10; 11; 12; 13; 14]$



# List Theory

## modification

Let  $L = [10;..15] = [10; 11; 12; 13; 14]$

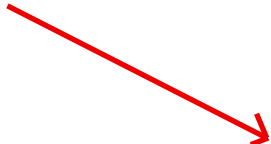
$$2 \rightarrow L \ 3 \mid 3 \rightarrow L \ 2 \mid L = [10; 11; 12; 12; 14]$$

# List Theory

## modification

Let  $L = [10; ..15] = [10; 11; 12; 13; 14]$

$2 \rightarrow L 3$  |  $3 \rightarrow L 2$  |  $L = [10; 11; 12; 12; 14]$



# List Theory

## modification

Let  $L = [10;..15] = [10; 11; 12; 13; 14]$

$2 \rightarrow L 3 \mid 3 \rightarrow L 2 \mid L = [10; 11; 13; 12; 14]$  ✓

# String and List Theory

$$S_{n,m} = S_n, S_m$$

$$L(n, m) = L n, L m$$

# String and List Theory

$$S_{n,m} = S_n, S_m$$

$$S_{\{n,m\}} = \{S_n, S_m\}$$

$$S_{n;m} = S_n; S_m$$

$$S_{[n;m]} = [S_n; S_m]$$

$$L(n, m) = L n, L m$$

$$L\{n, m\} = \{L n, L m\}$$

$$L(n; m) = L n; L m$$

$$L[n; m] = [L n; L m]$$

# String and List Theory

$$S_{n,m} = S_n, S_m$$

$$S_{\{n,m\}} = \{S_n, S_m\}$$

$$S_{n;m} = S_n; S_m$$

$$S_{[n;m]} = [S_n; S_m]$$

$$L(n, m) = L n, L m$$

$$L\{n, m\} = \{L n, L m\}$$

$$L(n; m) = L n; L m$$

$$L[n; m] = [L n; L m]$$

$$S_{0, \{1, [2; 1]; 0\}}$$

$$= S_0, \{S_1, [S_2; S_1]; S_0\}$$

$$L(0, \{1, [2; 1]; 0\})$$

$$= L 0, \{L 1, [L 2; L 1]; L 0\}$$

# List Theory

## multidimensional structures

$$A = [ [6; 3; 7; 0];$$
$$[4; 9; 2; 5];$$
$$[1; 5; 8; 3] ]$$

# List Theory

## multidimensional structures

```
A = [ [6; 3; 7; 0] ;  
      [4; 9; 2; 5] ;  
      [1; 5; 8; 3] ]
```

```
A: [3*[4*nat]]
```

# List Theory

## multidimensional structures

$$A = [ [6; 3; 7; 0];$$
$$[4; 9; 2; 5];$$
$$[1; 5; 8; 3] ]$$
$$A: [3*[4*nat]]$$
$$A\ 1 = [4; 9; 2; 5]$$

# List Theory

## multidimensional structures

$$A = [ [6; 3; 7; 0];$$
$$[4; 9; 2; 5];$$
$$[1; 5; 8; 3] ]$$
$$A: [3*[4*nat]]$$
$$A\ 1 = [4; 9; 2; 5]$$
$$A\ 1\ 2 = 2$$

# List Theory

## multidimensional structures

$$A = [ [6; 3; 7; 0];$$
$$[4; 9; 2; 5];$$
$$[1; 5; 8; 3] ]$$
$$A: [3*[4*\textit{nat}]]$$
$$A\ 1 = [4; 9; 2; 5]$$
$$A\ 1\ 2 = 2$$
$$A(1, 2)$$
$$A[1, 2]$$

# List Theory

## multidimensional structures

$$A = [ [6; 3; 7; 0];$$
$$[4; 9; 2; 5];$$
$$[1; 5; 8; 3] ]$$
$$A: [3*[4*\textit{nat}]]$$
$$A\ 1 = [4; 9; 2; 5]$$
$$A\ 1\ 2 = 2$$
$$A(1, 2) = A\ 1, A\ 2$$
$$A[1, 2]$$

# List Theory

## multidimensional structures

$$A = [ [6; 3; 7; 0]; \\ [4; 9; 2; 5]; \\ [1; 5; 8; 3] ]$$
$$A: [3*[4*nat]]$$
$$A\ 1 = [4; 9; 2; 5]$$
$$A\ 1\ 2 = 2$$
$$A(1, 2) = A\ 1, A\ 2 = [4; 9; 2; 5], [1; 5; 8; 3]$$
$$A[1, 2]$$

# List Theory

## multidimensional structures

$$A = [ [6; 3; 7; 0];$$
$$[4; 9; 2; 5];$$
$$[1; 5; 8; 3] ]$$
$$A: [3*[4*nat]]$$
$$A\ 1 = [4; 9; 2; 5]$$
$$A\ 1\ 2 = 2$$
$$A(1, 2) = A\ 1, A\ 2 = [4; 9; 2; 5], [1; 5; 8; 3]$$
$$A[1, 2] = [A\ 1, A\ 2]$$

# List Theory

## multidimensional structures

$$A = [ [6; 3; 7; 0]; \\ [4; 9; 2; 5]; \\ [1; 5; 8; 3] ]$$
$$A: [3*[4*nat]]$$
$$A\ 1 = [4; 9; 2; 5]$$
$$A\ 1\ 2 = 2$$
$$A(1, 2) = A\ 1, A\ 2 = [4; 9; 2; 5], [1; 5; 8; 3]$$
$$A[1, 2] = [A\ 1, A\ 2] = [ [4; 9; 2; 5], [1; 5; 8; 3] ]$$

# List Theory

## multidimensional structures

$$A = [ [6; 3; 7; 0]; \\ [4; 9; 2; 5]; \\ [1; 5; 8; 3] ]$$
$$A: [3*[4*nat]]$$
$$A\ 1 = [4; 9; 2; 5]$$
$$A\ 1\ 2 = 2 \quad \leftarrow$$
$$A(1, 2) = A\ 1, A\ 2 = [4; 9; 2; 5], [1; 5; 8; 3]$$
$$A[1, 2] = [A\ 1, A\ 2] = [ [4; 9; 2; 5], [1; 5; 8; 3] ]$$