

**bunch**

**set**

**string**

**list**

**bunch**

uncontained

unindexed

**set**

**string**

**list**

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unindexed

**set**

contained

unindexed

**string**

**list**

**bunch**            uncontained            unindexed

**set**                    contained                unindexed

**string**            uncontained            indexed

**list**

<b>bunch</b>	uncontained	unindexed
<b>set</b>	contained	unindexed
<b>string</b>	uncontained	indexed
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# Bunch Theory

Bunches can be used to represent collections.

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1, 3, 7

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1, 3, 7       $\top$ ,  $\perp$ , 5, “a”

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1, 3, 7       $\top, \perp, 5, \text{“a”}$       2

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Any number, character, binary, or set is an **elementary bunch**, or **element**.

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$A, B$       union

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$A \text{ ‘ } B$       intersection

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$A, B$	union
$A \text{ ‘ } B$	intersection
$A \overline{\text{ ‘ } } B$	removal

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$A \cap B$	intersection
$A \setminus B$	removal
$\#A$	size, cardinality (number)

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$\#A$	size, cardinality (number)
$A : B$	inclusion (binary)

# Bunch Theory

# Bunch Theory

$$1, 3, 7 = 3, 1, 7, 1$$

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$$1, 3, 7 = 3, 1, 7, 1$$

$$\phi(2) = 1$$

$$\phi(1, 3, 7) = 3$$

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$$2: 0, 2, 5, 9$$

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$$2: 2$$

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$$\phi(1, 3, 7) = 3 = \phi(3, 1, 7, 1)$$

$$2: 0, 2, 5, 9$$

$$2: 2$$

$$2, 9: 0, 2, 5, 9$$

# Bunch Theory

## axioms

$x: y = x=y$	elementary axiom
$x: A, B = x: A \vee x: B$	union axiom
$x: A' B = x: A \wedge x: B$	intersection axiom
$x: A, \bar{B} = x: A \wedge \neg x: B$	removal axiom
$A, A = A$	idempotence
$A, B = B, A$	symmetry
$A, (B, C) = (A, B), C$	associativity
$A' A = A$	idempotence
$A' B = B' A$	symmetry
$A' (B' C) = (A' B)' C$	associativity
$A, B: C = A: C \wedge B: C$	antidistributivity
$A: B' C = A: B \wedge A: C$	distributivity

# Bunch Theory

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intersection axiom

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removal axiom

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idempotence

$$A, B = B, A$$

symmetry

$$A, (B, C) = (A, B), C$$

associativity

$$A \text{ ' } A = A$$

idempotence

$$A \text{ ' } B = B \text{ ' } A$$

symmetry

$$A \text{ ' } (B \text{ ' } C) = (A \text{ ' } B) \text{ ' } C$$

associativity

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removal axiom

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idempotence

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symmetry

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idempotence

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symmetry

$$A \text{ ' } (B \text{ ' } C) = (A \text{ ' } B) \text{ ' } C$$

associativity

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antidistributivity

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associativity

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idempotence

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$$A \text{' } (B \text{' } C) = (A \text{' } B) \text{' } C$$

associativity

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antidistributivity

$$A: B \text{' } C = A: B \wedge A: C$$

distributivity

# Bunch Theory

**axioms**

# Bunch Theory

## axioms

$A: A, B$	generalization
$A \dot{B}: A$	specialization
$A: A$	reflexivity
$A: B \wedge B: A = A=B$	antisymmetry
$A: B \wedge B: C \Rightarrow A: C$	transitivity
$\wp x = 1$	size
$\wp(A, B) + \wp(A \dot{B}) = \wp A + \wp B$	size
$\neg x: A \Rightarrow \wp(A \dot{x}) = 0$	size
$A: B \Rightarrow \wp A \leq \wp B$	size

# Bunch Theory

## axioms

$A: A, B$

generalization

$A \dot{B}: A$

specialization

→  $A: A$

reflexivity

→  $A: B \wedge B: A = A=B$

antisymmetry

→  $A: B \wedge B: C \Rightarrow A: C$

transitivity

$\wp x = 1$

size

$\wp(A, B) + \wp(A \dot{B}) = \wp A + \wp B$

size

$\neg x: A \Rightarrow \wp(A \dot{x}) = 0$

size

$A: B \Rightarrow \wp A \leq \wp B$

size

# Bunch Theory

# Bunch Theory

## laws

$A, (A' B) = A$	absorption
$A' (A, B) = A$	absorption
$A: B \Rightarrow C, A: C, B$	monotonicity
$A: B \Rightarrow C' A: C' B$	monotonicity
$A: B = A, B = B = A = A' B$	inclusion
$A, (B, C) = (A, B), (A, C)$	distributivity
$A, (B' C) = (A, B)' (A, C)$	distributivity
$A' (B, C) = (A' B), (A' C)$	distributivity
$A' (B' C) = (A' B)' (A' C)$	distributivity
$A: B \wedge C: D \Rightarrow A, C: B, D$	conflation
$A: B \wedge C: D \Rightarrow A' C: B' D$	conflation

# Bunch Theory

## laws

$$A,(A'B) = A$$

absorption

$$A'(A,B) = A$$

absorption

$$\rightarrow A: B \Rightarrow C, A: C, B$$

monotonicity

$$\rightarrow A: B \Rightarrow C'A: C'B$$

monotonicity

$$A: B = A, B = B = A = A'B$$

inclusion

$$A,(B,C) = (A,B),(A,C)$$

distributivity

$$A,(B'C) = (A,B)'(A,C)$$

distributivity

$$A'(B,C) = (A'B), (A'C)$$

distributivity

$$A'(B'C) = (A'B)'(A'C)$$

distributivity

$$A: B \wedge C: D \Rightarrow A, C: B, D$$

conflation

$$A: B \wedge C: D \Rightarrow A'C: B'D$$

conflation

# Bunch Theory

# Bunch Theory

<i>null</i>			the empty bunch
<i>bin</i>	=	$\top, \perp$	the binary values
<i>nat</i>	=	$0, 1, 2, \dots$	the natural numbers
<i>int</i>	=	$\dots, -2, -1, 0, 1, 2, \dots$	the integer numbers
<i>rat</i>	=	$\dots, -1, 0, 2/3, \dots$	the rational numbers
<i>real</i>	=	$\dots, 2^{1/2}, \dots$	the real numbers
<i>xnat</i>	=	$0, 1, 2, \dots, \infty$	the extended natural numbers
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<i>char</i>	=	$\dots, \text{"a"}, \text{"A"}, \dots$	the character values


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# Bunch Theory

$x,..y$

# Bunch Theory

$x..y$       “ $x$  to  $y$ ”

# Bunch Theory

$x..y$  “ $x$  to  $y$ ” for  $x \leq y$

# Bunch Theory

$x,..y$

# Bunch Theory

$$i: x, ..y \quad = \quad x \leq i < y$$

# Bunch Theory

$$i: x,..y = x \leq i < y$$

$$\phi(x,..y) = y-x$$

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$$i: x,..y = x \leq i < y$$

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$$0,..3 = 0, 1, 2$$

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$$0,..2 = 0, 1$$

$$0,..1 = 0$$

$$0,..0 = \textit{null}$$

# Bunch Theory

$$i: x,..y = x \leq i < y$$

$$\#(x,..y) = y-x$$

$$0,..3 = 0, 1, 2$$

$$0,..2 = 0, 1$$

$$0,..1 = 0$$

$$0,..0 = \textit{null}$$

$$0,..∞ = \textit{nat}$$

# Bunch Theory

**distribution**

# Bunch Theory

**distribution**

$$-(1, 3, 7) = -1, -3, -7$$

# Bunch Theory

## distribution

$$-(1, 3, 7) = -1, -3, -7$$

$$(1, 2) + (10, 20) = 11, 12, 21, 22$$

# Bunch Theory

## distribution

$$-(1, 3, 7) = -1, -3, -7$$

$$(1, 2) + (10, 20) = 11, 12, 21, 22$$

$$(1, 2) + 10 = 11, 12$$

# Bunch Theory

## distribution

$$-(1, 3, 7) = -1, -3, -7$$

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$$(1, 2) + 10 = 11, 12$$

$$1 + 10 = 11$$

# Bunch Theory

## distribution

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$$(1, 2) + 10 = 11, 12$$

$$1 + 10 = 11$$

$$\textit{null} + 10 =$$

# Bunch Theory

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$$(1, 2) + 10 = 11, 12$$

$$1 + 10 = 11$$

$$\mathit{null} + 10 = \mathit{null}$$

$$\mathit{nat} + 2 = 2, 3, 4, 5, 6, \dots$$

# Bunch Theory

## distribution

$$-(1, 3, 7) = -1, -3, -7$$

$$(1, 2) + (10, 20) = 11, 12, 21, 22$$

$$(1, 2) + 10 = 11, 12$$

$$1 + 10 = 11$$

$$\mathit{null} + 10 = \mathit{null}$$

$$\mathit{nat} + 2 = 2, 3, 4, 5, 6, \dots$$

$$\mathit{nat} \times 2 = 0, 2, 4, 6, 8, \dots$$

# Bunch Theory

## distribution

$$-(1, 3, 7) = -1, -3, -7$$

$$(1, 2) + (10, 20) = 11, 12, 21, 22$$

$$(1, 2) + 10 = 11, 12$$

$$1 + 10 = 11$$

$$\mathit{null} + 10 = \mathit{null}$$

$$\mathit{nat} + 2 = 2, 3, 4, 5, 6, \dots$$

$$\mathit{nat} \times 2 = 0, 2, 4, 6, 8, \dots$$

$$\mathit{nat}^2 = 0, 1, 4, 9, 16, \dots$$

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# Set Theory

provides nested structure (things within things)

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$\{A\}$

“set containing  $A$ ”

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size, cardinality

$\mathcal{P}(0, 1) = \{\text{null}\}, \{0\}, \{1\}, \{0, 1\}$

power

# Set Theory

## axioms

$$\{\sim S\} = S$$

$$\sim\{A\} = A$$

$$\{A\} \neq A$$

$$\$\{A\} = \emptyset A$$

$$A \in \{B\} = A: B$$

$$\{A\} \subseteq \{B\} = A: B$$

$$\{A\}: \not\subseteq B = A: B$$

$$\{A\} \cup \{B\} = \{A, B\}$$

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