

# Data Transformation

user's variables  $u$

implementer's variables  $v$

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user's variables  $u$

implementer's variables  $v$

new implementer's variables  $w$

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**data transformer**  $D$  relates  $v$  and  $w$  such that  $\forall w \cdot \exists v \cdot D$

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**data transformer**  $D$  relates  $v$  and  $w$  such that  $\forall w \cdot \exists v \cdot D$

specification  $S$  is transformed to  $\forall v \cdot D \Rightarrow \exists v' \cdot D' \wedge S$

# Data Transformation

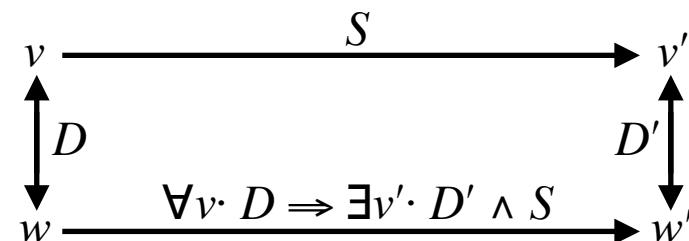
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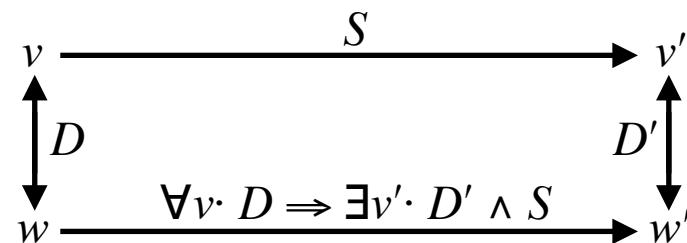
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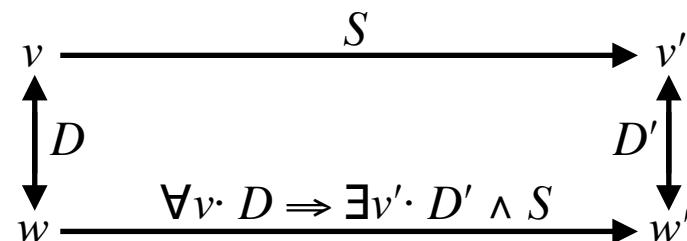
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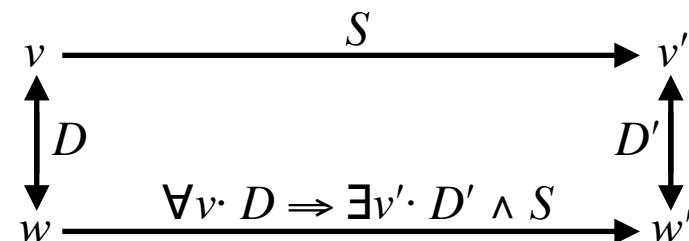
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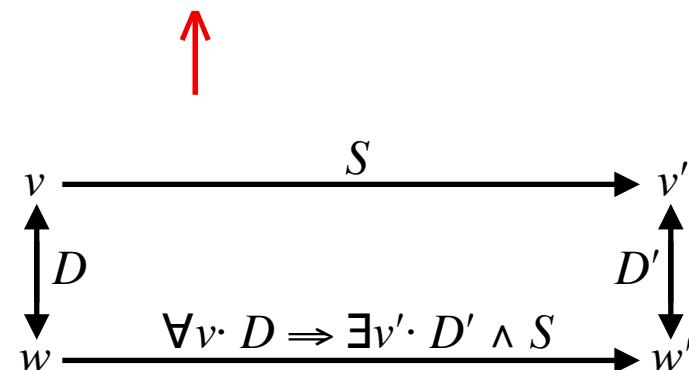
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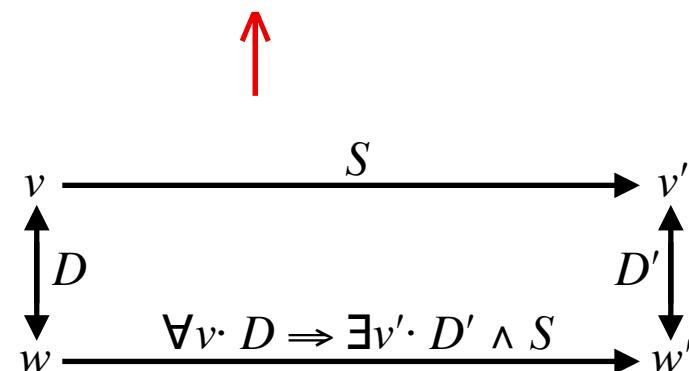
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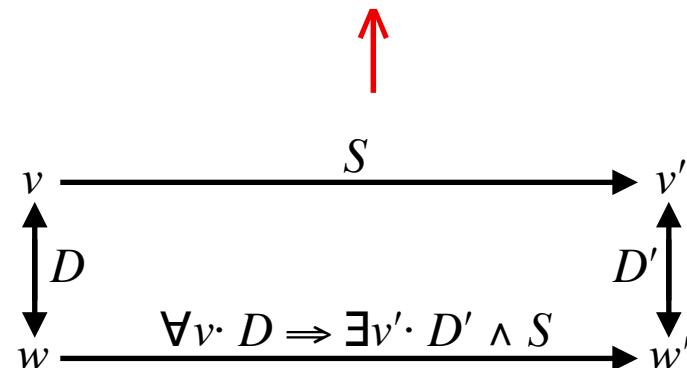
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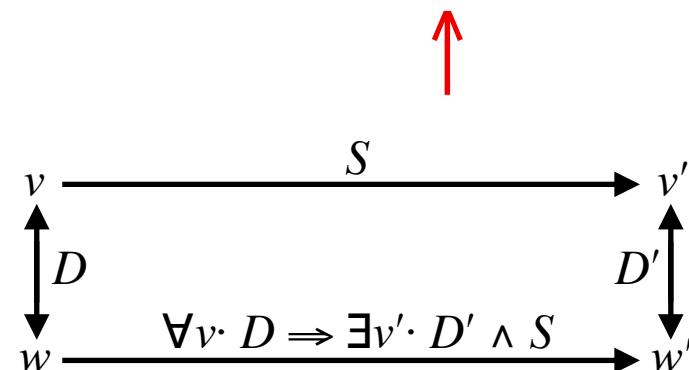
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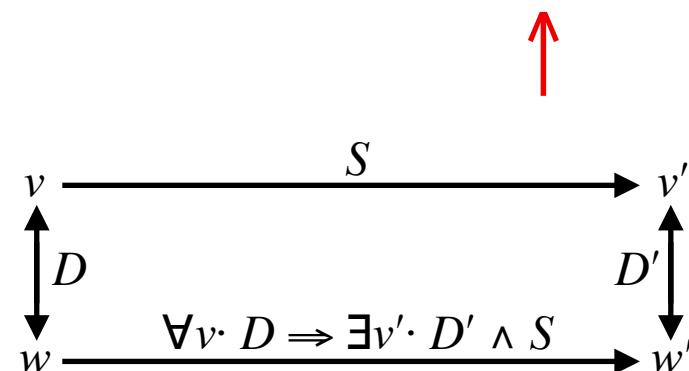
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# Data Transformation

## example

user's variable  $u: bin$

implementer's variable  $v: nat$

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user's variable  $u: bin$

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operations

$zero = v := 0$

$increase = v := v + 1$

$inquire = u := even v$

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operations

$zero = v := 0$

$increase = v := v + 1$

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new implementer's variable  $w: bin$

data transformer  $w = even v$

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$\forall v \cdot D \Rightarrow \exists v' \cdot D' \wedge zero$

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$$\forall v \cdot D \Rightarrow \exists v' \cdot D' \wedge \text{zero}$$

$$= \forall v \cdot w = \text{even } v \Rightarrow \exists v' \cdot w' = \text{even } v' \wedge (v := 0)$$

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$$= \forall v \cdot w = \text{even } v \Rightarrow \exists v' \cdot w' = \text{even } v' \wedge u' = u \wedge v' = 0$$

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$$\begin{aligned} & \forall v \cdot D \Rightarrow \exists v' \cdot D' \wedge \text{zero} \\ = & \quad \forall v \cdot w = \text{even } v \Rightarrow \exists v' \cdot w' = \text{even } v' \wedge (v := 0) \\ = & \quad \forall v \cdot w = \text{even } v \Rightarrow \exists v' \cdot w' = \text{even } v' \wedge u' = u \wedge v' = 0 \quad \text{1-pt} \\ & \qquad \qquad \qquad \uparrow \qquad \qquad \qquad \uparrow \end{aligned}$$

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$$\begin{aligned} & \forall v: \text{domain} \cdot (\text{substitute } fv \text{ for } r \text{ in } b) \\ = & \quad \forall r: f \text{ domain} \cdot b \end{aligned}$$

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# Data Transformation

$\forall v \cdot D \Rightarrow \exists v' \cdot D' \wedge \text{increase}$

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$\forall v \cdot D \Rightarrow \exists v' \cdot D' \wedge \text{increase}$

=  $\forall v \cdot w = \text{even } v \Rightarrow \exists v' \cdot w' = \text{even } v' \wedge (v := v + 1)$

# Data Transformation

$\forall v \cdot D \Rightarrow \exists v' \cdot D' \wedge \text{increase}$

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=  $\forall v \cdot w = \text{even } v \Rightarrow \exists v' \cdot w' = \text{even } v' \wedge u' = u \wedge v' = v + 1$

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# Data Transformation

$\forall v \cdot D \Rightarrow \exists v' \cdot D' \wedge inquire$

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# Data Transformation

## example

user's variable  $u: bin$

implementer's variable  $v: bin$

operations

$set = v := \top$

$flip = v := \neg v$

$ask = u := v$

new implementer's variable  $w: nat$

data transformer  $v = even w$

# Data Transformation

$$\begin{aligned} & \forall v \cdot D \Rightarrow \exists v' \cdot D' \wedge \text{set} \\ = & \quad \forall v \cdot v = even w \Rightarrow \exists v' \cdot v' = even w' \wedge (v := \top) \\ = & \quad even w' \wedge u' = u \\ \Leftarrow & \quad w := 0 \end{aligned}$$

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$$\begin{aligned} & \forall v \cdot D \Rightarrow \exists v' \cdot D' \wedge \text{flip} \\ = & \quad \forall v \cdot v = \text{even } w \Rightarrow \exists v' \cdot v' = \text{even } w' \wedge (v := \neg v) \\ = & \quad \text{even } w' \neq \text{even } w \wedge u' = u \\ \Leftarrow & \quad w := w + 1 \end{aligned}$$

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$$\begin{aligned} & \forall v \cdot D \Rightarrow \exists v' \cdot D' \wedge \text{ask} \\ = & \quad \forall v \cdot v = \text{even } w \Rightarrow \exists v' \cdot v' = \text{even } w' \wedge (u := v) \\ = & \quad \text{even } w' = \text{even } w = u' \\ \Leftarrow & \quad u := \text{even } w \end{aligned}$$

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# Security Switch

A security switch has three binary user's variables  $a$ ,  $b$ , and  $c$ . The users assign values to  $a$  and  $b$  as input to the switch. The switch's output is assigned to  $c$ . The output changes when both inputs have changed. More precisely, the output changes when both inputs differ from what they were the previous time the output changed. The idea is that one user might flip their input indicating a desire for the output to change, but the output does not change until the other user flips their input indicating agreement that the output should change. If the first user changes back before the second user changes, the output does not change.

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## binary implementer's variables

$A$  records the state of input  $a$  at last output change

$B$  records the state of input  $b$  at last output change

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## operations

```
a:= $\neg$ a. if  $a \neq A \wedge b \neq B$  then  $c := \neg c$ .  $A := a$ .  $B := b$  else ok fi
```

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$a := \neg a$ . **if**  $a \neq A \wedge b \neq B$  **then**  $c := \neg c$ .  $A := a$ .  $B := b$  **else**  $ok$  **fi**



# Security Switch

A security switch has three binary user's variables  $a$ ,  $b$ , and  $c$ . The users assign values to  $a$  and  $b$  as input to the switch. The switch's output is assigned to  $c$ . The output changes when both inputs have changed. More precisely, the output changes when both inputs differ from what they were the previous time the output changed. The idea is that one user might flip their input indicating a desire for the output to change, but the output does not change until the other user flips their input indicating agreement that the output should change. If the first user changes back before the second user changes, the output does not change.

## operations

$a := \neg a$ . **if**  $a \neq A \wedge b \neq B$  **then**  $c := \neg c$ .  $A := a$ .  $B := b$  **else**  $ok$  **fi**

$b := \neg b$ . **if**  $a \neq A \wedge b \neq B$  **then**  $c := \neg c$ .  $A := a$ .  $B := b$  **else**  $ok$  **fi**

# Security Switch

replace old implementer's variables  $A$  and  $B$  with nothing!

## operations

$a := \neg a.$  **if**  $a \neq A \wedge b \neq B$  **then**  $c := \neg c.$   $A := a.$   $B := b$  **else**  $ok$  **fi**

$b := \neg b.$  **if**  $a \neq A \wedge b \neq B$  **then**  $c := \neg c.$   $A := a.$   $B := b$  **else**  $ok$  **fi**

# Security Switch

replace old implementer's variables  $A$  and  $B$  with nothing!

## data transformer

$$A=B=c$$

## operations

$a := \neg a.$  **if**  $a \neq A \wedge b \neq B$  **then**  $c := \neg c.$   $A := a.$   $B := b$  **else**  $ok$  **fi**

$b := \neg b.$  **if**  $a \neq A \wedge b \neq B$  **then**  $c := \neg c.$   $A := a.$   $B := b$  **else**  $ok$  **fi**

# Security Switch

replace old implementer's variables  $A$  and  $B$  with nothing!

## data transformer

$$A=B=c$$

## proof

$\forall new \cdot \exists old \cdot \text{transformer}$

## operations

$a := \neg a. \text{ if } a \neq A \wedge b \neq B \text{ then } c := \neg c. \ A := a. \ B := b \text{ else } ok \text{ fi}$

$b := \neg b. \text{ if } a \neq A \wedge b \neq B \text{ then } c := \neg c. \ A := a. \ B := b \text{ else } ok \text{ fi}$

# Security Switch

replace old implementer's variables  $A$  and  $B$  with nothing!

## data transformer

$$A=B=c$$

## proof

$$\exists A, B \cdot A=B=c$$

## operations

$a := \neg a$ . **if**  $a \neq A \wedge b \neq B$  **then**  $c := \neg c$ .  $A := a$ .  $B := b$  **else**  $ok$  **fi**

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# Security Switch

replace old implementer's variables  $A$  and  $B$  with nothing!

## data transformer

$$A=B=c$$

## proof

$$\exists A, B \cdot A=B=c$$

generalization, using  $c$  for both  $A$  and  $B$

$\Leftarrow$

$\top$

## operations

$a := \neg a.$  **if**  $a \neq A \wedge b \neq B$  **then**  $c := \neg c.$   $A := a.$   $B := b$  **else**  $ok$  **fi**

$b := \neg b.$  **if**  $a \neq A \wedge b \neq B$  **then**  $c := \neg c.$   $A := a.$   $B := b$  **else**  $ok$  **fi**

# Security Switch

## operations

$a := \neg a$ . **if**  $a \neq A \wedge b \neq B$  **then**  $c := \neg c$ .  $A := a$ .  $B := b$  **else**  $ok$  **fi**

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# Security Switch

$\forall A, B \cdot A=B=c \Rightarrow \exists A', B' \cdot A'=B'=c' \wedge \begin{array}{l} \textbf{if } a \neq A \wedge b \neq B \textbf{ then } c := \neg c. \ A := a. \ B := b \\ \textbf{else } ok \textbf{ fi} \end{array}$

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$a := \neg a. \ \textbf{if } a \neq A \wedge b \neq B \textbf{ then } c := \neg c. \ A := a. \ B := b \textbf{ else } ok \textbf{ fi}$

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expand assignments, sequential compositions, and *ok*

=  $\forall A, B \cdot A=B=c \Rightarrow \exists A', B' \cdot A'=B'=c' \wedge \begin{array}{l} \textbf{if } a \neq A \wedge b \neq B \\ \textbf{then } a' = a \wedge b' = b \wedge c' = \neg c \wedge A' = a \wedge B' = b \\ \textbf{else } a' = a \wedge b' = b \wedge c' = c \wedge A' = A \wedge B' = B \textbf{ fi} \end{array}$

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$\forall A, B \cdot A=B=c \Rightarrow \exists A', B' \cdot A'=B'=c' \wedge \begin{array}{l} \textbf{if } a \neq A \wedge b \neq B \textbf{ then } c := \neg c. \ A := a. \ B := b \\ \textbf{else } ok \textbf{ fi} \end{array}$

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use one-point law for  $A$  and  $B$ , and for  $A'$  and  $B'$

=  $\begin{array}{l} \textbf{if } a \neq c \wedge b \neq c \textbf{ then } a' = a \wedge b' = b \wedge c' = \neg c \wedge c' = a \wedge c' = b \\ \textbf{else } a' = a \wedge b' = b \wedge c' = c \wedge c' = c \wedge c' = c \textbf{ fi} \end{array}$

# Security Switch

$\forall A, B \cdot A=B=c \Rightarrow \exists A', B' \cdot A'=B'=c' \wedge \begin{array}{l} \textbf{if } a \neq A \wedge b \neq B \textbf{ then } c := \neg c. \ A := a. \ B := b \\ \textbf{else } ok \textbf{ fi} \end{array}$

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use one-point law for  $A$  and  $B$ , and for  $A'$  and  $B'$

=  $\begin{array}{l} \textbf{if } a \neq c \wedge b \neq c \textbf{ then } a' = a \wedge b' = b \wedge c' = \neg c \wedge c' = a \wedge c' = b \\ \textbf{else } a' = a \wedge b' = b \wedge c' = c \wedge c' = c \wedge c' = c \textbf{ fi} \end{array}$

# Security Switch

$\forall A, B \cdot A=B=c \Rightarrow \exists A', B' \cdot A'=B'=c' \wedge \begin{array}{l} \textbf{if } a \neq A \wedge b \neq B \textbf{ then } c := \neg c. \ A := a. \ B := b \\ \textbf{else } ok \textbf{ fi} \end{array}$

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# Security Switch

$\forall A, B \cdot A=B=c \Rightarrow \exists A', B' \cdot A'=B'=c' \wedge \begin{array}{l} \textbf{if } a \neq A \wedge b \neq B \textbf{ then } c := \neg c. \ A := a. \ B := b \\ \textbf{else } ok \textbf{ fi} \end{array}$

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# Security Switch

$\forall A, B \cdot A=B=c \Rightarrow \exists A', B' \cdot A'=B'=c' \wedge \begin{array}{l} \textbf{if } a \neq A \wedge b \neq B \textbf{ then } c := \neg c. \ A := a. \ B := b \\ \textbf{else } ok \textbf{ fi} \end{array}$

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=  $\begin{array}{l} \textbf{if } a \neq c \wedge b \neq c \textbf{ then } a' = a \wedge b' = b \wedge c' = \neg c \wedge c' = \neg c \wedge c' = \neg c \\ \textbf{else } a' = a \wedge b' = b \wedge c' = c \wedge c' = c \wedge c' = c \textbf{ fi} \end{array}$

# Security Switch

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expand assignments, sequential compositions, and *ok*

=  $\forall A, B \cdot A=B=c \Rightarrow \exists A', B' \cdot A'=B'=c' \wedge \begin{array}{l} \textbf{if } a \neq A \wedge b \neq B \\ \textbf{then } a' = a \wedge b' = b \wedge c' = \neg c \wedge A' = a \wedge B' = b \\ \textbf{else } a' = a \wedge b' = b \wedge c' = c \wedge A' = A \wedge B' = B \textbf{ fi} \end{array}$

use one-point law for  $A$  and  $B$ , and for  $A'$  and  $B'$

=  $\begin{array}{l} \textbf{if } a \neq c \wedge b \neq c \textbf{ then } a' = a \wedge b' = b \wedge c' = \neg c \wedge c' = a \wedge c' = b \\ \textbf{else } a' = a \wedge b' = b \wedge c' = c \wedge c' = c \wedge c' = c \textbf{ fi} \end{array}$  use context

=  $\begin{array}{l} \textbf{if } a \neq c \wedge b \neq c \textbf{ then } a' = a \wedge b' = b \wedge c' = \neg c \wedge c' = \neg c \wedge c' = \neg c \\ \textbf{else } a' = a \wedge b' = b \wedge c' = c \wedge c' = c \wedge c' = c \textbf{ fi} \end{array}$

=  $\textbf{if } a \neq c \wedge b \neq c \textbf{ then } c := \neg c \textbf{ else } ok \textbf{ fi}$

# Security Switch

$\forall A, B \cdot A=B=c \Rightarrow \exists A', B' \cdot A'=B'=c' \wedge \begin{array}{l} \textbf{if } a \neq A \wedge b \neq B \textbf{ then } c := \neg c. \ A := a. \ B := b \\ \textbf{else } ok \textbf{ fi} \end{array}$

expand assignments, sequential compositions, and *ok*

=  $\forall A, B \cdot A=B=c \Rightarrow \exists A', B' \cdot A'=B'=c' \wedge \begin{array}{l} \textbf{if } a \neq A \wedge b \neq B \\ \textbf{then } a' = a \wedge b' = b \wedge c' = \neg c \wedge A' = a \wedge B' = b \\ \textbf{else } a' = a \wedge b' = b \wedge c' = c \wedge A' = A \wedge B' = B \textbf{ fi} \end{array}$

use one-point law for  $A$  and  $B$ , and for  $A'$  and  $B'$

=  $\begin{array}{l} \textbf{if } a \neq c \wedge b \neq c \textbf{ then } a' = a \wedge b' = b \wedge c' = \neg c \wedge c' = a \wedge c' = b \\ \textbf{else } a' = a \wedge b' = b \wedge c' = c \wedge c' = c \wedge c' = c \textbf{ fi} \end{array}$  use context

=  $\begin{array}{l} \textbf{if } a \neq c \wedge b \neq c \textbf{ then } a' = a \wedge b' = b \wedge c' = \neg c \wedge c' = \neg c \wedge c' = \neg c \\ \textbf{else } a' = a \wedge b' = b \wedge c' = c \wedge c' = c \wedge c' = c \textbf{ fi} \end{array}$

=  $\textbf{if } a \neq c \wedge b \neq c \textbf{ then } c := \neg c \textbf{ else } ok \textbf{ fi}$

=  $c := (a \neq c \wedge b \neq c) \neq c$