Recursive Program Definition

Recursive Specification Definition

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zap = **if** x=0 **then** y:= 0 **else** x:= x–1 . t:= t+1 . zap **fi**

Recursive Specification Definition

zap = **if** x=0 **then** y:= 0 **else** x:= x–1. t:= t+1. *zap* **fi**

void zap (void) {if (x==0) y=0; else {x=x–1; zap ( );}}

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Recursive Specification Definition

zap = **if** $x{=}0$ **then** $y{:=}\ 0$ **else** $x{:=}\ x{-}1.\ \ t{:=}\ t{+}1.\ \ zap$ **fi**

solutions

(a) $x{\geq}0 \ \Rightarrow\ x'{=}y'{=}0 \wedge t' = t{+}x$

(b) **if** $x{\geq}0$ **then** $x'{=}y'{=}0 \wedge t' = t{+}x$ **else** $t'{=}\infty$ **fi**

(c) $x'{=}y'{=}0 \wedge (x{\geq}0 \Rightarrow t' = t{+}x)$

(d) $x'{=}y'{=}0 \wedge$ **if** $x{\geq}0$ **then** $t' = t{+}x$ **else** $t'{=}\infty$ **fi**

(e) $x'{=}y'{=}0 \wedge t' = t{+}x$

(f) $x{\geq}0 \wedge x'{=}y'{=}0 \wedge t' = t{+}x$

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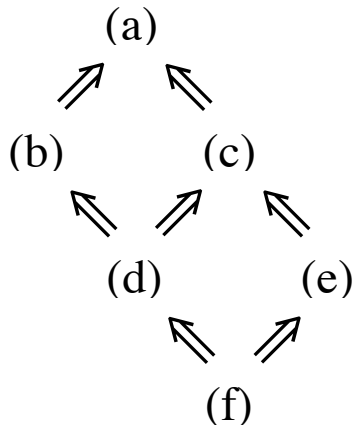

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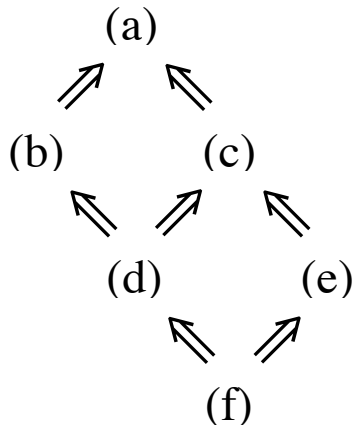

$x{\geq}0 \Rightarrow x'{=}y'{=}0 \wedge t' = t{+}x \Leftarrow zap$

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zap = **if** $x{=}0$ **then** $y{:=}\,0$ **else** $x{:=}\,x{-}1\,.\;\; t{:=}\,t{+}1\,.\;\; zap$ **fi**

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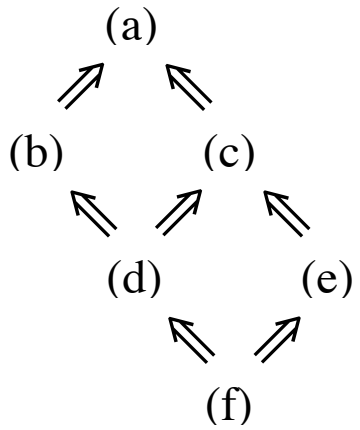

$x{\geq}0 \;\Rightarrow\; x'{=}y'{=}0 \wedge t' = t{+}x \;\Leftarrow\; zap$

$zap \;\Leftarrow$ **if** $x{=}0$ **then** $y{:=}\,0$ **else** $x{:=}\,x{-}1\,.\;\; t{:=}\,t{+}1\,.\;\; zap$ **fi**

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zap construction

$$t' \geq t \quad \Leftarrow \quad zap$$

if $x=0$ **then** $y := 0$ **else** $x := x-1.\ \ t := t+1.\ \ zap$ **fi** $\quad \Leftarrow \quad zap$

Recursive Specification Definition

zap construction

$t' \geq t \Leftarrow zap$

if $x=0$ **then** $y := 0$ **else** $x := x-1.\ t := t+1.\ zap$ **fi** $\Leftarrow zap$

nat construction

$0 : nat$

$nat+1 : nat$

Recursive Specification Definition

zap construction

→ $t' \geq t \Leftarrow zap$

if $x=0$ **then** $y := 0$ **else** $x := x-1.\ t := t+1.\ zap$ **fi** $\Leftarrow zap$

nat construction

→ $0 : nat$

$nat+1 : nat$

Recursive Specification Definition

zap construction

$t' \geq t \Leftarrow zap$

$\rightarrow$ **if** $x=0$ **then** $y := 0$ **else** $x := x-1.\ t := t+1.\ zap$ **fi** $\Leftarrow zap$

nat construction

$0 : nat$

$\rightarrow$ $nat+1 : nat$

Recursive Specification Definition

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$$t' \geq t \wedge \textbf{if } x{=}0 \textbf{ then } y := 0 \textbf{ else } x := x{-}1.\ t := t{+}1.\ zap \textbf{ fi} \Leftarrow zap$$

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$$0, nat{+}1 : nat$$

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$$t' \geq t \wedge \textbf{if } x{=}0 \textbf{ then } y := 0 \textbf{ else } x := x{-}1.\ t := t{+}1.\ zap \textbf{ fi} \Leftarrow zap$$

zap induction

$$\forall \sigma, \sigma' \cdot\ t' \geq t \wedge \textbf{if } x{=}0 \textbf{ then } y := 0 \textbf{ else } x := x{-}1.\ t := t{+}1.\ P \textbf{ fi} \Leftarrow P$$

$$\Rightarrow \quad \forall \sigma, \sigma' \cdot\ zap \Leftarrow P$$

nat construction

$$0, nat{+}1 : nat$$

nat induction

$$0, B{+}1 : B \ \Rightarrow\ nat : B$$

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zap fixed-point construction

$$zap \ = \ t' \geq t \wedge \textbf{if } x{=}0 \textbf{ then } y := 0 \textbf{ else } x := x{-}1.\ t := t{+}1.\ zap \textbf{ fi}$$

zap fixed-point induction

$$\forall \sigma, \sigma' \cdot (P \ = \ t' \geq t \wedge \textbf{if } x{=}0 \textbf{ then } y := 0 \textbf{ else } x := x{-}1.\ t := t{+}1.\ P \textbf{ fi})$$

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zap = **if** x=0 **then** y:= 0 **else** x:= x–1. t:= t+1. zap **fi**

Recursive Specification Construction

$$zap \quad = \quad \textbf{if } x{=}0 \textbf{ then } y{:=}\ 0 \textbf{ else } x{:=}\ x{-}1.\ \ t{:=}\ t{+}1.\ \ zap \textbf{ fi}$$

$$zap_0 \quad = \quad \top$$

Recursive Specification Construction

zap = **if** x=0 **then** y:= 0 **else** x:= x–1. t:= t+1. zap **fi**

zap_0 = $\top$

zap_1 = **if** x=0 **then** y:= 0 **else** x:= x–1. t:= t+1. zap_0 **fi**

Recursive Specification Construction

$$zap \quad = \quad \textbf{if } x{=}0 \textbf{ then } y{:=}\,0 \textbf{ else } x{:=}\,x{-}1.\ \ t{:=}\,t{+}1.\ \ zap \textbf{ fi}$$

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$$\quad = \quad x{=}0 \ \Rightarrow\ x'{=}y'{=}0 \wedge t'{=}t$$

$$zap_2 \quad = \quad \textbf{if } x{=}0 \textbf{ then } y{:=}\,0 \textbf{ else } x{:=}\,x{-}1.\ \ t{:=}\,t{+}1.\ \ zap_1 \textbf{ fi}$$

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$$= \quad 0{\le}x{<}2 \ \Rightarrow\ x'{=}y'{=}0 \wedge t' = t{+}x$$

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$$\quad = \quad 0{\le}x{<}2 \ \Rightarrow\ x'{=}y'{=}0 \wedge t' = t{+}x$$

$$zap_n \quad = \quad 0{\le}x{<}n \ \Rightarrow\ x'{=}y'{=}0 \wedge t' = t{+}x$$

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$$= \quad 0{\le}x{<}2 \ \Rightarrow\ x'{=}y'{=}0 \wedge t' = t{+}x$$

$$zap_n \quad = \quad 0{\le}x{<}n \ \Rightarrow\ x'{=}y'{=}0 \wedge t' = t{+}x$$

$$zap_\infty \quad = \quad 0{\le}x{<}\infty \ \Rightarrow\ x'{=}y'{=}0 \wedge t' = t{+}x$$

Recursive Specification Construction

Alternative step 0: instead of $\top$ use

$name_0 = whatever$

Recursive Specification Construction

Alternative step 0: instead of $\top$ use

$$name_0 = whatever$$

Alternative step 2: instead of $name_\infty$ use

$$\Updownarrow n \cdot name_n$$

Recursive Specification Construction

zap = **if** x=0 **then** y:= 0 **else** x:= x–1. t:= t+1. zap **fi**

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zap_0 = $t' \geq t$

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$$\quad = \quad \textbf{if } 0{\leq}x{<}2 \textbf{ then } x'{=}y'{=}0 \wedge t' = t{+}x \textbf{ else } t' \geq t{+}2 \textbf{ fi}$$

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$$zap_n \quad = \quad \textbf{if } 0{\leq}x{<}n \textbf{ then } x'{=}y'{=}0 \wedge t'{=}t{+}x \textbf{ else } t' \geq t{+}n \textbf{ fi}$$

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$$zap_0 = t' \geq t$$

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$$zap_n = \textbf{if } 0{\leq}x{<}n \textbf{ then } x'{=}y'{=}0 \wedge t'{=}t{+}x \textbf{ else } t' \geq t{+}n \textbf{ fi}$$

$$zap_\infty = \textbf{if } 0{\leq}x \textbf{ then } x'{=}y'{=}0 \wedge t'{=}t{+}x \textbf{ else } t'{=}\infty \textbf{ fi}$$

Loop Definition

while-loop construction

$t' \geq t \Leftarrow$ **while** b **do** P **od**

if b **then** P. $t := t+1$. **while** b **do** P **od** **else** ok **fi** $\Leftarrow$ **while** b **do** P **od**

Loop Definition

while-loop construction

$$t' \geq t \wedge \textbf{if } b \textbf{ then } P.\ t := t+1.\ \textbf{while } b \textbf{ do } P \textbf{ od else } ok \textbf{ fi} \Leftarrow \textbf{while } b \textbf{ do } P \textbf{ od}$$

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$$t' \geq t \wedge \textbf{if } b \textbf{ then } P.\ t := t+1.\ \textbf{while } b \textbf{ do } P \textbf{ od else } ok \textbf{ fi} \Leftarrow \textbf{while } b \textbf{ do } P \textbf{ od}$$

while-loop induction

$$\forall \sigma, \sigma' \cdot\ t' \geq t \wedge \textbf{if } b \textbf{ then } P.\ t := t+1.\ W \textbf{ else } ok \textbf{ fi} \Leftarrow W$$

$$\Rightarrow \quad \forall \sigma, \sigma' \cdot\ \textbf{while } b \textbf{ do } P \textbf{ od} \Leftarrow W$$

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while-loop induction

$$\forall \sigma, \sigma' \cdot t' \geq t \wedge \textbf{if } b \textbf{ then } P.\ t := t+1.\ W \textbf{ else } ok \textbf{ fi} \Leftarrow W$$

$$\Rightarrow \quad \forall \sigma, \sigma' \cdot \textbf{while } b \textbf{ do } P \textbf{ od} \Leftarrow W$$

while-loop fixed-point construction

$$\textbf{while } b \textbf{ do } P \textbf{ od} = t' \geq t \wedge \textbf{if } b \textbf{ then } P.\ t := t+1.\ \textbf{while } b \textbf{ do } P \textbf{ od else } ok \textbf{ fi}$$

while-loop fixed-point induction

$$\forall \sigma, \sigma' \cdot (W = t' \geq t \wedge \textbf{if } b \textbf{ then } P.\ t := t+1.\ W \textbf{ else } ok \textbf{ fi})$$

$$\Rightarrow \quad \forall \sigma, \sigma' \cdot \textbf{while } b \textbf{ do } P \textbf{ od} \Leftarrow W$$