Review

Binary Theory	laws	proof	
Number Theory	Character Theory		
Bunches	Sets	Strings	Lists
Functions	Quantifiers		
Specification	Refinement	Program Development	
Time Calculation	real time	recursive time	
Space Calculation	maximum space	average space	
assertions	exact precondition	exact postcondition	invariant
Scope	variable declaration	frame	
Data Structures	array element assignment		
Control Structures	**while**-loop	loop with **exit**	**for**-loop

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Cube

Write a program that cubes using only addition, subtraction, and test for zero.

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Write a program that cubes using only addition, subtraction, and test for zero.

$x' = n^3$

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$$x'=n^3 \quad \Leftarrow \quad x:=n.\ \ x'=x\times n.\ \ x'=x\times n$$

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Write a program that cubes using only addition, subtraction, and test for zero.

constant n: *nat* variable x: *nat*

$x'=n^3 \Leftarrow x:= n.\ x'=x\times n.\ x'=x\times n$

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$x'=n^3 \Leftarrow x:= n.\ x'=x\times n.\ x'=x\times n$

$x'=x\times n \Leftarrow$

Cube

Write a program that cubes using only addition, subtraction, and test for zero.

constant n: *nat* variable x: *nat*

$x' = n^3 \Leftarrow x := n.\ x' = x \times n.\ x' = x \times n$

$x' = x \times n \Leftarrow x := 0.\ x' = x + ?$

Cube

Write a program that cubes using only addition, subtraction, and test for zero.

constant n: *nat* variables x, y: *nat*

$x' = n^3 \Leftarrow x := n.\ x' = x \times n.\ x' = x \times n$

$x' = x \times n \Leftarrow y := x.\ x := 0.\ x' = x + ?$

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$x' = x \times n \Leftarrow y := x.\ x := 0.\ x' = x + y \times n$

$x' = x + y \times n \Leftarrow$ **if** $y = 0$ **then** *ok*

Cube

Write a program that cubes using only addition, subtraction, and test for zero.

constant n: *nat* variables x, y: *nat*

$x'=n^3 \Leftarrow x:= n.\ x'=x\times n.\ x'=x\times n$

$x'=x\times n \Leftarrow y:= x.\ x:= 0.\ x' = x + y\times n$

$x' = x + y\times n \Leftarrow$ **if** $y=0$ **then** *ok* **else** $x:= x+n.\ y:= y-1.\ x' = x + y\times n$ **fi**

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$x' = x + y\times n \Leftarrow$ **if** $y=0$ **then** *ok* **else** $x:= x+n.\ y:= y-1.\ x' = x + y\times n$ **fi**

In C

```
void P (void) { x = n;  Q( );  Q( );}

void Q (void) {y = x;  x = 0;  R( );}

void R (void) { if (y==0); else {x += n;  y--;  R( );}}
```

Cube

Write a program that cubes using only addition, subtraction, and test for zero.

constant n: *nat* variables x, y: *nat*

$x'=n^3 \Leftarrow x:= n.\ x'=x\times n.\ x'=x\times n$

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In C

```
void P (void) { x = n;  Q( );  Q( );}
void Q (void) {y = x;  x = 0;  R: if (y==0); else {x += n;  y--;  goto R;}}
```

Cube

Write a program that cubes using only addition, subtraction, and test for zero.

constant n: *nat* variables x, y: *nat*

$x'=n^3 \Leftarrow x:= n.\ x'=x\times n.\ x'=x\times n$

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In C

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void P (void) { x = n;  Q( );  Q( );}
void Q (void) {y = x;  x = 0;  while (y!=0) {x += n;  y--;}}
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In C

```
x = n;

y = x;  x = 0;  while (y!=0) {x += n;  y--;}

y = x;  x = 0;  while (y!=0) {x += n;  y--;}
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Write a program that cubes using only addition, subtraction, and test for zero.

constant n: *nat* variables x, y: *nat* time t: *xnat*

$x'=n^3 \Leftarrow x:= n.\ x'=x\times n.\ x'=x\times n$

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$x' = x + y\times n \Leftarrow$ **if** $y=0$ **then** *ok* **else** $x:= x+n.\ y:= y-1.\ x' = x + y\times n$ **fi**

Cube

Write a program that cubes using only addition, subtraction, and test for zero.

constant n: *nat* variables x, y: *nat* time t: *xnat*

$x'=n^3$ $\Leftarrow$ $x:= n.\ x'=x\times n$. $x'=x\times n$

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proof

$y=0 \wedge ok$

Cube

Write a program that cubes using only addition, subtraction, and test for zero.

constant n: *nat* variables x, y: *nat* time t: *xnat*

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$x'=x\times n \Leftarrow y:= x.\ x:= 0.\ x' = x + y\times n \wedge t'=t+y$

$x' = x + y\times n \wedge t'=t+y \Leftarrow$

if $y=0$ **then** *ok* **else** $x:= x+n.\ y:= y-1.\ t:= t+1.\quad x' = x + y\times n \wedge t'=t+y$ **fi**

proof

$y=0 \wedge ok$ expand *ok*

$= \quad y=0 \wedge x'=x \wedge y'=y \wedge t'=t$

Cube

Write a program that cubes using only addition, subtraction, and test for zero.

constant n: *nat* variables x, y: *nat* time t: *xnat*

$x'=n^3 \Leftarrow x:= n.\ x'=x\times n \quad . \ x'=x\times n$

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$x' = x + y\times n \wedge t'=t+y \Leftarrow$

if $y=0$ **then** *ok* **else** $x:= x+n.\ y:= y-1.\ t:= t+1.\quad x' = x + y\times n \wedge t'=t+y$ **fi**

proof

	$y=0 \wedge ok$	expand *ok*
=	$y=0 \wedge x'=x \wedge y'=y \wedge t'=t$	context
=	$y=0 \wedge x' = x + y\times n \wedge y'=y \wedge t'=t+y$	

Cube

Write a program that cubes using only addition, subtraction, and test for zero.

constant n: *nat* variables x, y: *nat* time t: *xnat*

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$x' = x + y\times n \wedge t'=t+y \Leftarrow$

if $y=0$ **then** *ok* **else** $x:= x+n.\ y:= y-1.\ t:= t+1.\quad x' = x + y\times n \wedge t'=t+y$ **fi**

proof

	$y=0 \wedge ok$	expand *ok*
=	$y=0 \wedge x'=x \wedge y'=y \wedge t'=t$	context
=	$y=0 \wedge x' = x + y\times n \wedge y'=y \wedge t'=t+y$	specialize
$\Rightarrow$	$x' = x + y\times n \wedge t'=t+y$	

Cube

Write a program that cubes using only addition, subtraction, and test for zero.

constant n: *nat* variables x, y: *nat* time t: *xnat*

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$x' = x + y\times n \wedge t'=t+y$ $\Leftarrow$

if $y=0$ **then** *ok* **else** $x:= x+n.\ y:= y-1.\ t:= t+1.\quad x' = x + y\times n \wedge t'=t+y$ **fi**

proof

$y\neq 0 \ \wedge \ (x:= x+n.\ y:= y-1.\ t:= t+1.\ x' = x + y\times n \wedge t'=t+y)$

Cube

Write a program that cubes using only addition, subtraction, and test for zero.

constant n: *nat* variables x, y: *nat* time t: *xnat*

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if $y=0$ **then** *ok* **else** $x:= x+n.\ y:= y-1.\ t:= t+1.\quad x' = x + y\times n \wedge t'=t+y$ **fi**

proof

$y\neq 0 \ \wedge\ (x:= x+n.\ y:= y-1.\ t:= t+1.\ x' = x + y\times n \wedge t'=t+y)$ substitution law

$=$ $y\neq 0 \ \wedge\ x' = x + n + (y-1)\times n \ \wedge\ t'=t+1+y-1$

Cube

Write a program that cubes using only addition, subtraction, and test for zero.

constant n: *nat* variables x, y: *nat* time t: *xnat*

$x'=n^3 \Leftarrow x:= n.\ x'=x\times n \ .\ x'=x\times n$

$x'=x\times n \Leftarrow y:= x.\ x:= 0.\ x' = x + y\times n \wedge t'=t+y$

$x' = x + y\times n \wedge t'=t+y \Leftarrow$

if $y=0$ **then** *ok* **else** $x:= x+n.\ y:= y-1.\ t:= t+1.\ x' = x + y\times n \wedge t'=t+y$ **fi**

proof

$y\neq 0 \wedge (x:= x+n.\ y:= y-1.\ t:= t+1.\ x' = x + y\times n \wedge t'=t+y)$ substitution law

$=$ $y\neq 0 \wedge x' = x + n + (y-1)\times n \wedge t'=t+1+y-1$ simplify and specialize

$\Rightarrow$ $x' = x + y\times n \wedge t'=t+y$

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Write a program that cubes using only addition, subtraction, and test for zero.

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if $y=0$ **then** *ok* **else** $x:= x+n.\ y:= y-1.\ t:= t+1.\quad x' = x + y\times n \wedge t'=t+y$ **fi**

proof

$y:= x.\ x:= 0.\ x' = x + y\times n \wedge t'=t+y$ substitution law

$= \quad x' = 0 + x\times n \wedge t'=t+x$

Cube

Write a program that cubes using only addition, subtraction, and test for zero.

constant n: *nat* variables x, y: *nat* time t: *xnat*

$x'=n^3 \Leftarrow x:= n.\ x'=x\times n \ .\ x'=x\times n$

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proof

$y:= x.\ x:= 0.\ x' = x + y\times n \wedge t'=t+y$ substitution law

$= \quad x' = 0 + x\times n \wedge t'=t+x$ simplify

$= \quad x'=x\times n \wedge t'=t+x$

Cube

Write a program that cubes using only addition, subtraction, and test for zero.

constant n: *nat* variables x, y: *nat* time t: *xnat*

$x'=n^3 \quad \Leftarrow \quad x:= n.\ x'=x\times n \wedge t'=t+x\ .\ x'=x\times n \wedge t'=t+x$

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if $y=0$ **then** *ok* **else** $x:= x+n.\ y:= y-1.\ t:= t+1.\quad x' = x + y\times n \wedge t'=t+y$ **fi**

proof

$y:= x.\ x:= 0.\ x' = x + y\times n \wedge t'=t+y$ substitution law

$=$ $x' = 0 + x\times n \wedge t'=t+x$ simplify

$=$ $x'=x\times n \wedge t'=t+x$

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$x'=x\times n \wedge t'=t+x \quad \Leftarrow \quad y:= x.\ x:= 0.\ x' = x + y\times n \wedge t'=t+y$

$x' = x + y\times n \wedge t'=t+y \quad \Leftarrow$

if $y=0$ **then** *ok* **else** $x:= x+n.\ y:= y-1.\ t:= t+1.\quad x' = x + y\times n \wedge t'=t+y$ **fi**

proof

Cube

Write a program that cubes using only addition, subtraction, and test for zero.

constant n: *nat* variables x, y: *nat* time t: *xnat*

$x'=n^3 \quad \Leftarrow \quad x:= n.\ x'=x\times n \wedge t'=t+x\ .\ x'=x\times n \wedge t'=t+x$

$x'=x\times n \wedge t'=t+x \quad \Leftarrow \quad y:= x.\ x:= 0.\ x' = x + y\times n \wedge t'=t+y$

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proof

$x:= n.\ x'=x\times n \wedge t'=t+x.\ x'=x\times n \wedge t'=t+x$ substitution law

$= \quad x'=n^2 \wedge t'=t+n.\ x'=x\times n \wedge t'=t+x$

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Write a program that cubes using only addition, subtraction, and test for zero.

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$x:= n.\ x'=x\times n \wedge t'=t+x.\ x'=x\times n \wedge t'=t+x$ substitution law

$= \quad x'=n^2 \wedge t'=t+n.\ x'=x\times n \wedge t'=t+x$ sequential composition

$= \quad \exists x'', y'', t''\cdot x''=n^2 \wedge t''=t+n \wedge x'=x''\times n \wedge t'=t''+x''$

Cube

Write a program that cubes using only addition, subtraction, and test for zero.

constant n: *nat* variables x, y: *nat* time t: *xnat*

$x' = n^3 \quad \Leftarrow \quad x := n.\ x' = x \times n \wedge t' = t + x\ .\ x' = x \times n \wedge t' = t + x$

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$= \quad x' = n^2 \wedge t' = t + n.\ x' = x \times n \wedge t' = t + x$ sequential composition

$= \quad \exists x'', y'', t'' \cdot x'' = n^2 \wedge t'' = t + n \wedge x' = x'' \times n \wedge t' = t'' + x''$ 1-pt for x'', t'', idempotent for y''

$= \quad x' = n^3 \wedge t' = t + n^2 + n$

Cube

Write a program that cubes using only addition, subtraction, and test for zero.

constant n: *nat* variables x, y: *nat* time t: *xnat*

$x'=n^3 \wedge t'=t+n^2+n \Leftarrow x:= n.\ x'=x\times n \wedge t'=t+x\ .\ x'=x\times n \wedge t'=t+x$

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variables x, y, n: *nat*

Cube

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variables x, y, n: *nat*

$x'=n^3 \Leftarrow x'=n^3 \wedge y'=n^2$

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$y := y + n + n - 1.$

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if $n=0$ **then** $x := 0.\ y := 0$

else $n := n-1.\ x'=n^3 \wedge y'=n^2.$ ←

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$y:= y + n + n - 1.\ x:= x + y + y + y - n - n - n + 1$ **fi**

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$n^2 = (n-1)^2 + 2\times n - 1$

variables x, y, n: *nat*

$x'=n^3 \wedge t'=t+n \Leftarrow x'=n^3 \wedge y'=n^2 \wedge n'=n \wedge t'=t+n$

$x'=n^3 \wedge y'=n^2 \wedge n'=n \wedge t'=t+n \Leftarrow$

if $n=0$ **then** $x:= 0.\ y:= 0$

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$$A\,0 \Rightarrow A'n \quad \Leftarrow \quad \textbf{for}\ k:=0;..n\ \textbf{do}\ k:\ 0,..n \wedge A\,k \ \Rightarrow\ A'(k+1)\ \textbf{od}$$

$$k:\ 0,..n \wedge A\,k \ \Rightarrow\ A'(k+1) \quad \Leftarrow \quad x:=x+?$$

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$$A\,k \;=\; x=k^3$$

$$A\,k \Rightarrow A'(k+1)$$

$$=\qquad x=k^3 \;\Rightarrow\; x'=(k+1)^3$$

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$$= \qquad x=k^3 \;\Rightarrow\; x'=(k+1)^3$$

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$$A\,k \;=\; x=k^3 \;\wedge\; y = 3\times k^2 + 3\times k + 1$$

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$$= \qquad x=k^3 \;\wedge\; y = 3\times k^2 + 3\times k + 1 \;\Rightarrow\; x'=(k+1)^3 \;\wedge\; y' = 3\times(k+1)^2 + 3\times(k+1) + 1$$

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$$= \quad x=k^3 \;\wedge\; y = 3\times k^2 + 3\times k + 1 \;\Rightarrow\; x' = x+y \;\wedge\; y' = 3\times k^2 + 9\times k + 7$$

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$$A\,k \Rightarrow A'(k{+}1)$$

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Cube

Write a program that cubes using only addition, subtraction, and test for zero.

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$Q \wedge t'=t+n \Leftarrow$

if $n=0$ **then** *ok* **else** $x:=x+y.\ y:=y+z.\ z:=z+6.\ n:=n-1.\ t:=t+1.\ Q \wedge t'=t+n$ **fi**

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$Q \quad = \quad \forall k: nat \cdot\ x=k^3 \ \wedge\ y = 3\times k^2 + 3\times k + 1 \ \wedge\ z = 6\times k + 6 \ \Rightarrow\ x' = (k+n)^3$

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```
    x = 0;  y = 1;  z = 6;
Q:  if (n!=0) {x+=y;  y+=z.  z+=6;  n--; goto Q;}
```