

Variable Declaration

var x : T P

Variable Declaration

var x : T · P declare local state variable x with type T and scope P

Variable Declaration

var $x: T \cdot P$ declare local state variable x with type T and scope P

= $\exists x, x': T \cdot P$

Variable Declaration

var $x: T \cdot P$ declare local state variable x with type T and scope P

= $\exists x, x': T \cdot P$

var $x: int \cdot x := 2. y := x + z$

Variable Declaration

var $x: T \cdot P$ declare local state variable x with type T and scope P

= $\exists x, x': T \cdot P$

var $x: int \cdot x:=2. y:=x+z$

= $\exists x, x': int \cdot x'=2 \wedge y' = 2+z \wedge z'=z$

Variable Declaration

var $x: T \cdot P$ declare local state variable x with type T and scope P

= $\exists x, x': T \cdot P$

var $x: int \cdot x:=2. y:=x+z$

= $\exists x, x': int \cdot x'=2 \wedge y' = 2+z \wedge z'=z$ one-point for x' and idempotent for x

= $y' = 2+z \wedge z'=z$

Variable Declaration

var $x: T \cdot P$ declare local state variable x with type T and scope P

= $\exists x, x': T \cdot P$

var $x: int \cdot x:=2. y:=x+z$

= $\exists x, x': int \cdot x'=2 \wedge y' = 2+z \wedge z'=z$ one-point for x' and idempotent for x

= $y' = 2+z \wedge z'=z$

var $x: int \cdot y:=x$

Variable Declaration

var $x: T \cdot P$ declare local state variable x with type T and scope P

$$= \exists x, x': T \cdot P$$

var $x: int \cdot x:=2. y:=x+z$

$$= \exists x, x': int \cdot x'=2 \wedge y' = 2+z \wedge z'=z \quad \text{one-point for } x' \text{ and idempotent for } x$$

$$= y' = 2+z \wedge z'=z$$

var $x: int \cdot y:=x$

$$= \exists x, x': int \cdot x'=x \wedge y'=x \wedge z'=z$$

Variable Declaration

var $x: T \cdot P$ declare local state variable x with type T and scope P

= $\exists x, x': T \cdot P$

var $x: int \cdot x:=2. y:=x+z$

= $\exists x, x': int \cdot x'=2 \wedge y' = 2+z \wedge z'=z$ one-point for x' and idempotent for x

= $y' = 2+z \wedge z'=z$

var $x: int \cdot y:=x$

= $\exists x, x': int \cdot x'=x \wedge y'=x \wedge z'=z$ one-point for x and x'

= $z'=z$

Variable Declaration

var $x: T \cdot P$ declare local state variable x with type T and scope P

= $\exists x, x': T \cdot P$

var $x: int \cdot x:=2. y:=x+z$

= $\exists x, x': int \cdot x'=2 \wedge y'=2+z \wedge z'=z$ one-point for x' and idempotent for x

= $y'=2+z \wedge z'=z$

var $x: int \cdot y:=x$

= $\exists x, x': int \cdot x'=x \wedge y'=x \wedge z'=z$ one-point for x and x'

= $z'=z$

var $x: int \cdot y:=x-x$

Variable Declaration

var $x: T \cdot P$ declare local state variable x with type T and scope P

$$= \exists x, x': T \cdot P$$

var $x: int \cdot x:=2. y:=x+z$

$$= \exists x, x': int \cdot x'=2 \wedge y'=2+z \wedge z'=z \quad \text{one-point for } x' \text{ and idempotent for } x$$

$$= y'=2+z \wedge z'=z$$

var $x: int \cdot y:=x$

$$= \exists x, x': int \cdot x'=x \wedge y'=x \wedge z'=z \quad \text{one-point for } x \text{ and } x'$$

$$= z'=z$$

var $x: int \cdot y:=x-x$

$$= y'=0 \wedge z'=z$$

Variable Declaration

var x : T P

Variable Declaration

var $x: T \cdot P$

= $\exists x: \text{undefined} \cdot \exists x': T, \text{undefined} \cdot P$

Variable Declaration

var $x: T \cdot P$

= $\exists x: \text{undefined} \cdot \exists x': T, \text{undefined} \cdot P$

var $x: T := e \cdot P$

Variable Declaration

var $x: T \cdot P$

= $\exists x: \text{undefined} \cdot \exists x': T, \text{undefined} \cdot P$

var $x: T := e \cdot P$

= $\exists x: e \cdot \exists x': T \cdot P$

Variable Suspension

Suppose the state consists of variables w , x , y , and z .

frame $w, x \cdot P$

Variable Suspension

Suppose the state consists of variables w , x , y , and z .

frame $w, x \cdot P$ within P , y and z are constants (no y' and z')

Variable Suspension

Suppose the state consists of variables w , x , y , and z .

$$\begin{aligned} & \mathbf{frame} \ w, x \cdot P && \text{within } P, y \text{ and } z \text{ are constants (no } y' \text{ and } z' \text{)} \\ = & P \wedge y'=y \wedge z'=z \end{aligned}$$

Variable Suspension

Suppose the state consists of variables w , x , y , and z .

$$\begin{aligned} & \mathbf{frame } w, x \cdot P && \text{within } P, y \text{ and } z \text{ are constants (no } y' \text{ and } z' \text{)} \\ = & P \wedge y'=y \wedge z'=z \end{aligned}$$

$$x := e = \mathbf{frame } x \cdot x' = e$$

Variable Suspension

Suppose the state consists of variables w , x , y , and z .

$$\begin{aligned} & \mathbf{frame} \ w, x \cdot P && \text{within } P, y \text{ and } z \text{ are constants (no } y' \text{ and } z' \text{)} \\ = & P \wedge y'=y \wedge z'=z \end{aligned}$$

$$x := e = \mathbf{frame} \ x \cdot x' = e$$

$$ok = \mathbf{frame} \cdot \top$$

Variable Suspension

Suppose the state consists of variables w , x , y , and z .

$$\begin{aligned} & \mathbf{frame} \ w, x \cdot P && \text{within } P, y \text{ and } z \text{ are constants (no } y' \text{ and } z' \text{)} \\ = & P \wedge y'=y \wedge z'=z \end{aligned}$$

$$x := e = \mathbf{frame} \ x \cdot x' = e$$

$$ok = \mathbf{frame} \cdot \top$$

$$s := \Sigma L$$

Variable Suspension

Suppose the state consists of variables w , x , y , and z .

$$\begin{aligned} & \mathbf{frame} \ w, x \cdot P && \text{within } P, y \text{ and } z \text{ are constants (no } y' \text{ and } z' \text{)} \\ = & P \wedge y'=y \wedge z'=z \end{aligned}$$

$$x := e = \mathbf{frame} \ x \cdot x' = e$$

$$ok = \mathbf{frame} \cdot \top$$

$$s := \Sigma L \Leftarrow \mathbf{frame} \ s$$

Variable Suspension

Suppose the state consists of variables w , x , y , and z .

$$\begin{aligned} & \mathbf{frame} \ w, x \cdot P && \text{within } P, y \text{ and } z \text{ are constants (no } y' \text{ and } z' \text{)} \\ = & P \wedge y'=y \wedge z'=z \end{aligned}$$

$$x:=e = \mathbf{frame} \ x \cdot x' = e$$

$$ok = \mathbf{frame} \cdot \top$$

$$s:=\Sigma L \Leftarrow \mathbf{frame} \ s \cdot \mathbf{var} \ n: \mathit{nat}$$

Variable Suspension

Suppose the state consists of variables w , x , y , and z .

$$\begin{aligned} & \mathbf{frame} \ w, x \cdot P && \text{within } P, y \text{ and } z \text{ are constants (no } y' \text{ and } z' \text{)} \\ = & P \wedge y'=y \wedge z'=z \end{aligned}$$

$$x := e = \mathbf{frame} \ x \cdot x' = e$$

$$ok = \mathbf{frame} \cdot \top$$

$$s := \Sigma L \Leftarrow \mathbf{frame} \ s \cdot \mathbf{var} \ n: nat \cdot s' = \Sigma L$$

Variable Suspension

Suppose the state consists of variables w , x , y , and z .

$$\begin{aligned} & \mathbf{frame} \ w, x \cdot P && \text{within } P, y \text{ and } z \text{ are constants (no } y' \text{ and } z') \\ = & P \wedge y'=y \wedge z'=z \end{aligned}$$

$$x := e = \mathbf{frame} \ x \cdot x' = e$$

$$ok = \mathbf{frame} \cdot \top$$

$$s := \Sigma L \Leftarrow \mathbf{frame} \ s \cdot \mathbf{var} \ n: nat \cdot s' = \Sigma L$$

$$s' = \Sigma L \Leftarrow$$

Array

$A(i) := e$

Array

$A[i] := e$

Array

$A_i := e$

Array

$$A\ i := e \quad = \quad A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots$$

Array

$$A\ i := e \quad = \quad \underline{A'\ i=e} \wedge (\forall j. j \neq i \Rightarrow A'\ j=A\ j) \wedge x'=x \wedge y'=y \wedge \dots$$

Array

$$A\ i := e \quad = \quad A' i = e \wedge \underline{(\forall j. j \neq i \Rightarrow A' j = A j)} \wedge x' = x \wedge y' = y \wedge \dots$$

Array

$$A\ i := e \quad = \quad A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge \underline{x' = x \wedge y' = y \wedge \dots}$$

Array

$$A\ i := e \quad = \quad A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots$$

Array

$$A\ i := e \quad = \quad A'\ i = e \wedge (\forall j. j \neq i \Rightarrow A'\ j = A\ j) \wedge x' = x \wedge y' = y \wedge \dots$$

$A\ 2 := 3. \ i := 2. \ A\ i := 4. \ A\ i = A\ 2$

Array

$$A\ i := e \quad = \quad A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots$$

$$A\ 2 := 3. \ i := 2. \ A\ i := 4. \ A\ i = A\ 2$$

Substitution Law

Array

$$A\ i := e \quad = \quad A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots$$

$$A\ 2 := 3. \ i := 2. \ A\ i := 4. \ A\ i = A\ 2$$

X Substitution Law

Array

$$A\ i := e \quad = \quad A'\ i=e \wedge (\forall j. j \neq i \Rightarrow A'\ j=A\ j) \wedge x'=x \wedge y'=y \wedge \dots$$

$$\begin{aligned} & A\ 2 := 3. \ i := 2. \ A\ i := 4. \ A\ i = A\ 2 \\ = & A\ 2 := 3. \ i := 2. \ 4 = A\ 2 \end{aligned}$$

X Substitution Law

Array

$$A\ i := e \quad = \quad A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots$$

$$A\ 2 := 3. \ i := 2. \ A\ i := 4. \ A\ i = A\ 2$$

 Substitution Law

$$= \quad A\ 2 := 3. \ i := 2. \ 4 = A\ 2$$

 Substitution Law

Array

$$A\ i := e \quad = \quad A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots$$

$$A\ 2 := 3. \ i := 2. \ A\ i := 4. \ A\ i = A\ 2$$

 Substitution Law

$$= \quad A\ 2 := 3. \ i := 2. \ 4 = A\ 2$$

 Substitution Law

$$= \quad A\ 2 := 3. \ 4 = A\ 2$$

Array

$$A\ i := e \quad = \quad A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots$$

$$A\ 2 := 3. \ i := 2. \ A\ i := 4. \ A\ i = A\ 2$$

 Substitution Law

$$= \quad A\ 2 := 3. \ i := 2. \ 4 = A\ 2$$

 Substitution Law

$$= \quad A\ 2 := 3. \ 4 = A\ 2$$

 Substitution Law

Array

$$A\ i := e \quad = \quad A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots$$

$$A\ 2 := 3. \ i := 2. \ A\ i := 4. \ A\ i = A\ 2$$

 Substitution Law

$$= \quad A\ 2 := 3. \ i := 2. \ 4 = A\ 2$$

 Substitution Law

$$= \quad A\ 2 := 3. \ 4 = A\ 2$$

 Substitution Law

$$= \quad 4 = 3$$

Array

$$A\ i := e \quad = \quad A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots$$

$$A\ 2 := 3. \ i := 2. \ A\ i := 4. \ A\ i = A\ 2$$

 Substitution Law

$$= \quad A\ 2 := 3. \ i := 2. \ 4 = A\ 2$$

 Substitution Law

$$= \quad A\ 2 := 3. \ 4 = A\ 2$$

 Substitution Law

$$= \quad 4 = 3$$

$$= \quad \perp$$

Array

$$A\ i := e \quad = \quad A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots$$

$$A\ 2 := 3. \ i := 2. \ A\ i := 4. \ A\ i = A\ 2$$

 Substitution Law

$$= \quad A\ 2 := 3. \ i := 2. \ 4 = A\ 2$$

 Substitution Law

$$= \quad A\ 2 := 3. \ 4 = A\ 2$$

 Substitution Law

$$= \quad 4 = 3$$

$$= \quad \perp \quad \text{X}$$

Array

$$A\ i := e \quad = \quad A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots$$

$$A\ 2 := 3. \ i := 2. \ A\ i := 4. \ A\ i = A\ 2$$

 Substitution Law

$$= \quad A\ 2 := 3. \ i := 2. \ 4 = A\ 2$$

 Substitution Law

$$= \quad A\ 2 := 3. \ 4 = A\ 2$$

 Substitution Law

$$= \quad 4 = 3$$

$$= \quad \perp \quad \text{X}$$

$$A\ 2 := 2. \ A(A\ 2) := 3. \ A\ 2 = 2$$

Array

$$A\ i := e \quad = \quad A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots$$

$$A\ 2 := 3. \ i := 2. \ A\ i := 4. \ A\ i = A\ 2$$

 Substitution Law

$$= \quad A\ 2 := 3. \ i := 2. \ 4 = A\ 2$$

 Substitution Law

$$= \quad A\ 2 := 3. \ 4 = A\ 2$$

 Substitution Law

$$= \quad 4 = 3$$

$$= \quad \perp \quad \text{X}$$

$$A\ 2 := 2. \ \underline{A(A\ 2) := 3. \ A\ 2 = 2}$$

 Substitution Law

Array

$$A\ i := e \quad = \quad A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots$$

$$A\ 2 := 3. \ i := 2. \ A\ i := 4. \ A\ i = A\ 2$$

 Substitution Law

$$= \quad A\ 2 := 3. \ i := 2. \ 4 = A\ 2$$

 Substitution Law

$$= \quad A\ 2 := 3. \ 4 = A\ 2$$

 Substitution Law

$$= \quad 4 = 3$$

$$= \quad \perp \quad \text{X}$$

$$A\ 2 := 2. \ A(A\ 2) := 3. \ A\ 2 = 2$$

 Substitution Law

$$= \quad A\ 2 := 2. \ A\ 2 = 2$$

Array

$$A\ i := e \quad = \quad A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots$$

$$A\ 2 := 3. \ i := 2. \ A\ i := 4. \ A\ i = A\ 2$$

 Substitution Law

$$= \quad A\ 2 := 3. \ i := 2. \ 4 = A\ 2$$

 Substitution Law

$$= \quad A\ 2 := 3. \ 4 = A\ 2$$

 Substitution Law

$$= \quad 4 = 3$$

$$= \quad \perp \quad \text{X}$$

$$A\ 2 := 2. \ A(A\ 2) := 3. \ A\ 2 = 2$$

 Substitution Law

$$= \quad A\ 2 := 2. \ A\ 2 = 2$$

 Substitution Law

$$= \quad 2 = 2$$

Array

$$A\ i := e \quad = \quad A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots$$

$$A\ 2 := 3. \ i := 2. \ A\ i := 4. \ A\ i = A\ 2$$

 Substitution Law

$$= \quad A\ 2 := 3. \ i := 2. \ 4 = A\ 2$$

 Substitution Law

$$= \quad A\ 2 := 3. \ 4 = A\ 2$$

 Substitution Law

$$= \quad 4 = 3$$

$$= \quad \perp \quad \text{X}$$

$$A\ 2 := 2. \ A(A\ 2) := 3. \ A\ 2 = 2$$

 Substitution Law

$$= \quad A\ 2 := 2. \ A\ 2 = 2$$

 Substitution Law

$$= \quad 2 = 2$$

$$= \quad \top$$

Array

$$A\ i := e \quad = \quad A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots$$

$$A\ 2 := 3. \ i := 2. \ A\ i := 4. \ A\ i = A\ 2$$

 Substitution Law

$$= \quad A\ 2 := 3. \ i := 2. \ 4 = A\ 2$$

 Substitution Law

$$= \quad A\ 2 := 3. \ 4 = A\ 2$$

 Substitution Law

$$= \quad 4 = 3$$

$$= \quad \perp \quad \text{X}$$

$$A\ 2 := 2. \ A(A\ 2) := 3. \ A\ 2 = 2$$

 Substitution Law

$$= \quad A\ 2 := 2. \ A\ 2 = 2$$

 Substitution Law

$$= \quad 2 = 2$$

$$= \quad \top \quad \text{X}$$

Array

$$A\ i := e \quad = \quad A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots$$

Array

$$\begin{aligned} A\ i := e &= A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots \\ &= A' = i \rightarrow e \mid A \wedge x' = x \wedge y' = y \wedge \dots \end{aligned}$$

Array

$$\begin{aligned} A\ i := e &= A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots \\ &= A' = i \rightarrow e \mid A \wedge x' = x \wedge y' = y \wedge \dots \\ &= A := i \rightarrow e \mid A \end{aligned}$$

Array

$$\begin{aligned} A\ i := e &= A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots \\ &= A' = i \rightarrow e \mid A \wedge x' = x \wedge y' = y \wedge \dots \\ &= A := i \rightarrow e \mid A \end{aligned}$$

$A\ 2 := 3. \ i := 2. \ A\ i := 4. \ A\ i = A\ 2$

Array

$$\begin{aligned} A\ i := e &= A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots \\ &= A' = i \rightarrow e \mid A \wedge x' = x \wedge y' = y \wedge \dots \\ &= A := i \rightarrow e \mid A \end{aligned}$$

$$\begin{aligned} &A\ 2 := 3. \ i := 2. \ A\ i := 4. \ A\ i = A\ 2 \\ = &A := 2 \rightarrow 3 \mid A. \ i := 2. \ A := i \rightarrow 4 \mid A. \ A\ i = A\ 2 \end{aligned}$$

Array

$$\begin{aligned} A\ i := e &= A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots \\ &= A' = i \rightarrow e \mid A \wedge x' = x \wedge y' = y \wedge \dots \\ &= A := i \rightarrow e \mid A \end{aligned}$$

$$\begin{aligned} &A\ 2 := 3. \ i := 2. \ A\ i := 4. \ A\ i = A\ 2 \\ = &A := 2 \rightarrow 3 \mid A. \ i := 2. \ \underline{A := i \rightarrow 4 \mid A. \ A\ i = A\ 2} \end{aligned}$$

Substitution Law

Array

$$\begin{aligned} A\ i := e &= A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots \\ &= A' = i \rightarrow e \mid A \wedge x' = x \wedge y' = y \wedge \dots \\ &= A := i \rightarrow e \mid A \end{aligned}$$

$$\begin{aligned} &A\ 2 := 3. \ i := 2. \ A\ i := 4. \ A\ i = A\ 2 \\ = &A := 2 \rightarrow 3 \mid A. \ i := 2. \ A := i \rightarrow 4 \mid A. \ A\ i = A\ 2 \\ = &A := 2 \rightarrow 3 \mid A. \ i := 2. \ (i \rightarrow 4 \mid A) i = (i \rightarrow 4 \mid A) 2 \end{aligned}$$

Substitution Law

Array

$$\begin{aligned} A\ i := e &= A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots \\ &= A' = i \rightarrow e \mid A \wedge x' = x \wedge y' = y \wedge \dots \\ &= A := i \rightarrow e \mid A \end{aligned}$$

$$A\ 2 := 3. \ i := 2. \ A\ i := 4. \ A\ i = A\ 2$$

$$= A := 2 \rightarrow 3 \mid A. \ i := 2. \ A := i \rightarrow 4 \mid A. \ A\ i = A\ 2$$

Substitution Law

$$= A := 2 \rightarrow 3 \mid A. \ i := 2. \ (i \rightarrow 4 \mid A) i = (i \rightarrow 4 \mid A) 2$$

Substitution Law

Array

$$\begin{aligned} A\ i := e &= A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots \\ &= A' = i \rightarrow e \mid A \wedge x' = x \wedge y' = y \wedge \dots \\ &= A := i \rightarrow e \mid A \end{aligned}$$

$$A\ 2 := 3. \ i := 2. \ A\ i := 4. \ A\ i = A\ 2$$

$$= A := 2 \rightarrow 3 \mid A. \ i := 2. \ A := i \rightarrow 4 \mid A. \ A\ i = A\ 2$$

Substitution Law

$$= A := 2 \rightarrow 3 \mid A. \ i := 2. \ (i \rightarrow 4 \mid A) i = (i \rightarrow 4 \mid A) 2$$

Substitution Law

$$= A := 2 \rightarrow 3 \mid A. \ (2 \rightarrow 4 \mid A) 2 = (2 \rightarrow 4 \mid A) 2$$

Array

$$\begin{aligned} A i := e &= A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots \\ &= A' = i \rightarrow e \mid A \wedge x' = x \wedge y' = y \wedge \dots \\ &= A := i \rightarrow e \mid A \end{aligned}$$

$$A 2 := 3. i := 2. A i := 4. A i = A 2$$

$$= A := 2 \rightarrow 3 \mid A. i := 2. A := i \rightarrow 4 \mid A. A i = A 2$$

Substitution Law

$$= A := 2 \rightarrow 3 \mid A. i := 2. (i \rightarrow 4 \mid A) i = (i \rightarrow 4 \mid A) 2$$

Substitution Law

$$= A := 2 \rightarrow 3 \mid A. (2 \rightarrow 4 \mid A) 2 = (2 \rightarrow 4 \mid A) 2$$

= is reflexive

$$= A := 2 \rightarrow 3 \mid A. \top$$

Array

$$\begin{aligned} A i := e &= A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots \\ &= A' = i \rightarrow e \mid A \wedge x' = x \wedge y' = y \wedge \dots \\ &= A := i \rightarrow e \mid A \end{aligned}$$

$$A 2 := 3. i := 2. A i := 4. A i = A 2$$

$$= A := 2 \rightarrow 3 \mid A. i := 2. A := i \rightarrow 4 \mid A. A i = A 2$$

Substitution Law

$$= A := 2 \rightarrow 3 \mid A. i := 2. (i \rightarrow 4 \mid A) i = (i \rightarrow 4 \mid A) 2$$

Substitution Law

$$= A := 2 \rightarrow 3 \mid A. (2 \rightarrow 4 \mid A) 2 = (2 \rightarrow 4 \mid A) 2$$

= is reflexive

$$= A := 2 \rightarrow 3 \mid A. \top$$

Substitution Law

$$= \top$$

Array

$$\begin{aligned} A\ i := e &= A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots \\ &= A' = i \rightarrow e \mid A \wedge x' = x \wedge y' = y \wedge \dots \\ &= A := i \rightarrow e \mid A \end{aligned}$$

$$A\ 2 := 3. \ i := 2. \ A\ i := 4. \ A\ i = A\ 2$$

$$= A := 2 \rightarrow 3 \mid A. \ i := 2. \ A := i \rightarrow 4 \mid A. \ A\ i = A\ 2$$

Substitution Law

$$= A := 2 \rightarrow 3 \mid A. \ i := 2. \ (i \rightarrow 4 \mid A) i = (i \rightarrow 4 \mid A) 2$$

Substitution Law

$$= A := 2 \rightarrow 3 \mid A. \ (2 \rightarrow 4 \mid A) 2 = (2 \rightarrow 4 \mid A) 2$$

= is reflexive

$$= A := 2 \rightarrow 3 \mid A. \ \top$$

Substitution Law

$$= \top \quad \checkmark$$

Array

$$\begin{aligned} A\ i := e &= A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots \\ &= A' = i \rightarrow e \mid A \wedge x' = x \wedge y' = y \wedge \dots \\ &= A := i \rightarrow e \mid A \end{aligned}$$

Array

$$\begin{aligned} A\ i := e &= A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots \\ &= A' = i \rightarrow e \mid A \wedge x' = x \wedge y' = y \wedge \dots \\ &= A := i \rightarrow e \mid A \end{aligned}$$

$A\ 2 := 2. A(A\ 2) := 3. A\ 2 = 2$

Array

$$A\ i := e \quad = \quad A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots$$

$$= \quad A' = i \rightarrow e \mid A \wedge x' = x \wedge y' = y \wedge \dots$$

$$= \quad A := i \rightarrow e \mid A$$

$$A\ 2 := 2. \quad A(A\ 2) := 3. \quad A\ 2 = 2$$

$$= \quad A := 2 \rightarrow 2 \mid A. \quad A := A\ 2 \rightarrow 3 \mid A. \quad A\ 2 = 2$$

Array

$$\begin{aligned} A\ i := e &= A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots \\ &= A' = i \rightarrow e \mid A \wedge x' = x \wedge y' = y \wedge \dots \\ &= A := i \rightarrow e \mid A \end{aligned}$$

$$\begin{aligned} &A\ 2 := 2. A(A\ 2) := 3. A\ 2 = 2 \\ = &A := 2 \rightarrow 2 \mid A. A := A\ 2 \rightarrow 3 \mid A. A\ 2 = 2 \\ = &A := 2 \rightarrow 2 \mid A. (A\ 2 \rightarrow 3 \mid A)2 = 2 \end{aligned}$$

Substitution Law

Array

$$\begin{aligned} A\ i := e &= A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots \\ &= A' = i \rightarrow e \mid A \wedge x' = x \wedge y' = y \wedge \dots \\ &= A := i \rightarrow e \mid A \end{aligned}$$

$$\begin{aligned} &A\ 2 := 2. \ A(A\ 2) := 3. \ A\ 2 = 2 \\ = &A := 2 \rightarrow 2 \mid A. \ A := A\ 2 \rightarrow 3 \mid A. \ A\ 2 = 2 \\ = &A := 2 \rightarrow 2 \mid A. \ (A\ 2 \rightarrow 3 \mid A)2 = 2 \\ = &((2 \rightarrow 2 \mid A)2 \rightarrow 3 \mid 2 \rightarrow 2 \mid A)2 = 2 \end{aligned}$$

Substitution Law

Substitution Law

Array

$$\begin{aligned} A\ i := e &= A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots \\ &= A' = i \rightarrow e \mid A \wedge x' = x \wedge y' = y \wedge \dots \\ &= A := i \rightarrow e \mid A \end{aligned}$$

$$A\ 2 := 2. \ A(A\ 2) := 3. \ A\ 2 = 2$$

$$= A := 2 \rightarrow 2 \mid A. \ A := A\ 2 \rightarrow 3 \mid A. \ A\ 2 = 2$$

Substitution Law

$$= A := 2 \rightarrow 2 \mid A. \ (A\ 2 \rightarrow 3 \mid A) 2 = 2$$

Substitution Law

$$= ((2 \rightarrow 2 \mid A) 2 \rightarrow 3 \mid 2 \rightarrow 2 \mid A) 2 = 2$$

Array

$$\begin{aligned} A\ i := e &= A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots \\ &= A' = i \rightarrow e \mid A \wedge x' = x \wedge y' = y \wedge \dots \\ &= A := i \rightarrow e \mid A \end{aligned}$$

$$\begin{aligned} &A\ 2 := 2. \ A(A\ 2) := 3. \ A\ 2 = 2 \\ = &A := 2 \rightarrow 2 \mid A. \ A := A\ 2 \rightarrow 3 \mid A. \ A\ 2 = 2 \\ = &A := 2 \rightarrow 2 \mid A. \ (A\ 2 \rightarrow 3 \mid A)2 = 2 \\ = &((2 \rightarrow 2 \mid A)2 \rightarrow 3 \mid 2 \rightarrow 2 \mid A)2 = 2 \\ = &(2 \rightarrow 3 \mid 2 \rightarrow 2 \mid A)2 = 2 \end{aligned}$$

Substitution Law

Substitution Law

Array

$$\begin{aligned} A\ i := e &= A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots \\ &= A' = i \rightarrow e \mid A \wedge x' = x \wedge y' = y \wedge \dots \\ &= A := i \rightarrow e \mid A \end{aligned}$$

$$\begin{aligned} &A\ 2 := 2. \ A(A\ 2) := 3. \ A\ 2 = 2 \\ = &A := 2 \rightarrow 2 \mid A. \ A := A\ 2 \rightarrow 3 \mid A. \ A\ 2 = 2 \\ = &A := 2 \rightarrow 2 \mid A. \ (A\ 2 \rightarrow 3 \mid A)2 = 2 \\ = &((2 \rightarrow 2 \mid A)2 \rightarrow 3 \mid 2 \rightarrow 2 \mid A)2 = 2 \\ = &(2 \rightarrow 3 \mid 2 \rightarrow 2 \mid A)2 = 2 \\ = &3 = 2 \end{aligned}$$

Substitution Law

Substitution Law

Array

$$\begin{aligned} A\ i := e &= A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots \\ &= A' = i \rightarrow e \mid A \wedge x' = x \wedge y' = y \wedge \dots \\ &= A := i \rightarrow e \mid A \end{aligned}$$

$$\begin{aligned} &A\ 2 := 2. \ A(A\ 2) := 3. \ A\ 2 = 2 \\ = &A := 2 \rightarrow 2 \mid A. \ A := A\ 2 \rightarrow 3 \mid A. \ A\ 2 = 2 \\ = &A := 2 \rightarrow 2 \mid A. \ (A\ 2 \rightarrow 3 \mid A)2 = 2 \\ = &((2 \rightarrow 2 \mid A)2 \rightarrow 3 \mid 2 \rightarrow 2 \mid A)2 = 2 \\ = &(2 \rightarrow 3 \mid 2 \rightarrow 2 \mid A)2 = 2 \\ = &3 = 2 \\ = &\perp \end{aligned}$$

Substitution Law

Substitution Law

Array

$$\begin{aligned} A\ i := e &= A' i = e \wedge (\forall j. j \neq i \Rightarrow A' j = A j) \wedge x' = x \wedge y' = y \wedge \dots \\ &= A' = i \rightarrow e \mid A \wedge x' = x \wedge y' = y \wedge \dots \\ &= A := i \rightarrow e \mid A \end{aligned}$$

$$\begin{aligned} &A\ 2 := 2. \ A(A\ 2) := 3. \ A\ 2 = 2 \\ = &A := 2 \rightarrow 2 \mid A. \ A := A\ 2 \rightarrow 3 \mid A. \ A\ 2 = 2 \\ = &A := 2 \rightarrow 2 \mid A. \ (A\ 2 \rightarrow 3 \mid A)2 = 2 \\ = &((2 \rightarrow 2 \mid A)2 \rightarrow 3 \mid 2 \rightarrow 2 \mid A)2 = 2 \\ = &(2 \rightarrow 3 \mid 2 \rightarrow 2 \mid A)2 = 2 \\ = &3 = 2 \\ = &\perp \quad \checkmark \end{aligned}$$

Substitution Law

Substitution Law

Array

Array

remember

$A \ i := e$ becomes $A := i \rightarrow e \mid A$

Array

remember

$A\ i := e$ becomes $A := i \rightarrow e \mid A$

$A\ i\ j := e$ becomes $A := (i; j) \rightarrow e \mid A$

Record

Record

person = “name” → *text*
 | “age” → *nat*

Record

person = “name” → *text*
 | “age” → *nat*

var *p*: *person*

Record

person = “name” → *text*
 | “age” → *nat*

var *p*: *person*

p := “name” → “Josh” | “age” → 17

Record

person = “name” → *text*
 | “age” → *nat*

var *p*: *person*

p := “name” → “Josh” | “age” → 17

p “age” := 18

Record

$person = \begin{array}{l} \text{“name”} \rightarrow text \\ | \text{“age”} \rightarrow nat \end{array}$

var $p: person$

$p := \text{“name”} \rightarrow \text{“Josh”} \mid \text{“age”} \rightarrow 17$

$p \text{ “age”} := 18$

$p := \text{“age”} \rightarrow 18 \mid p$