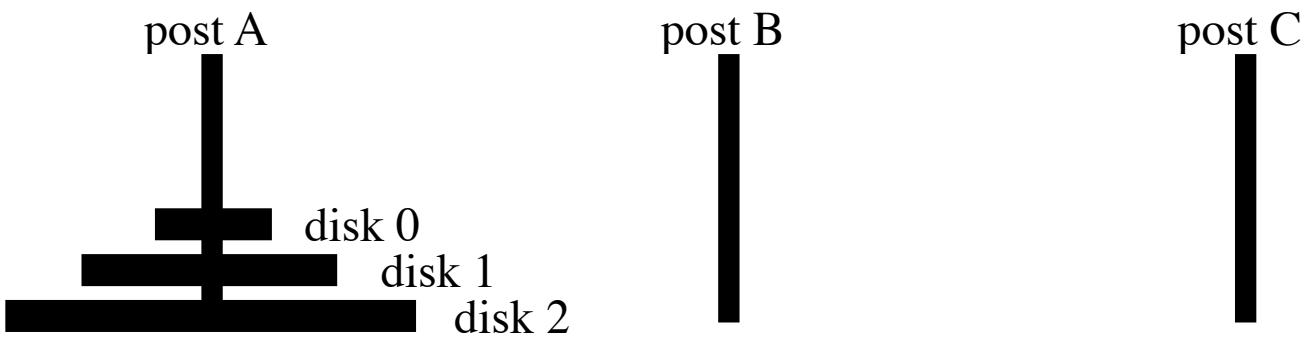
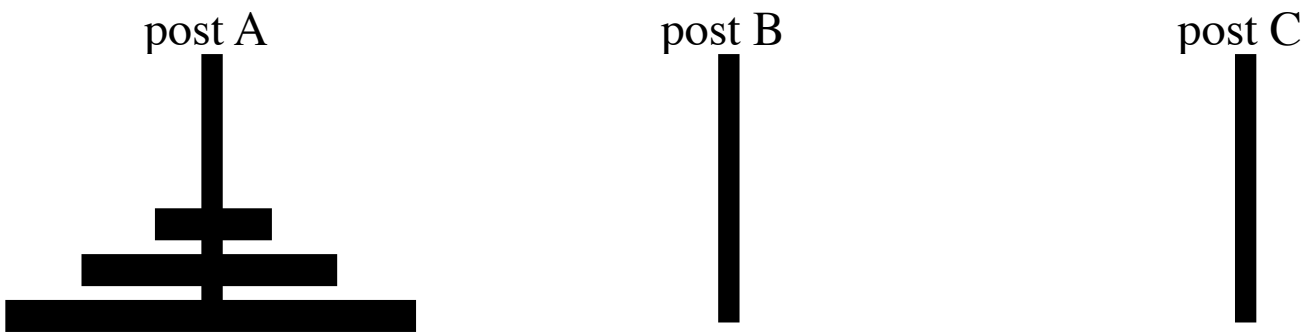
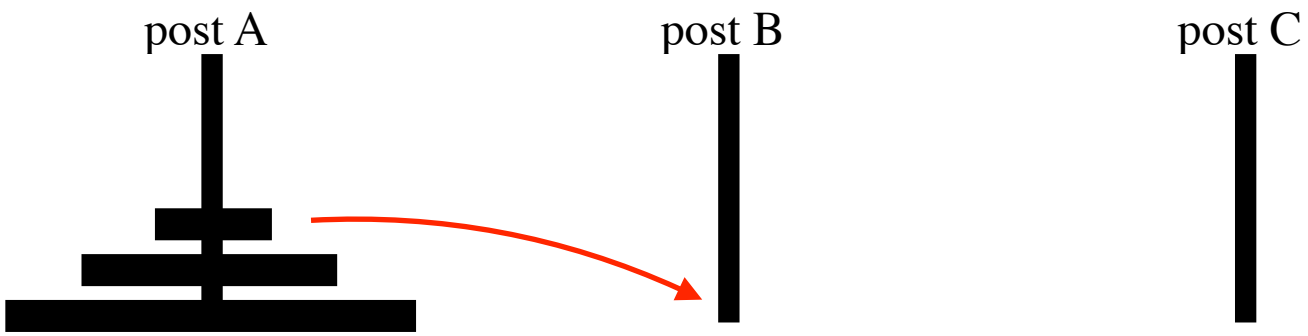
Towers of Hanoi



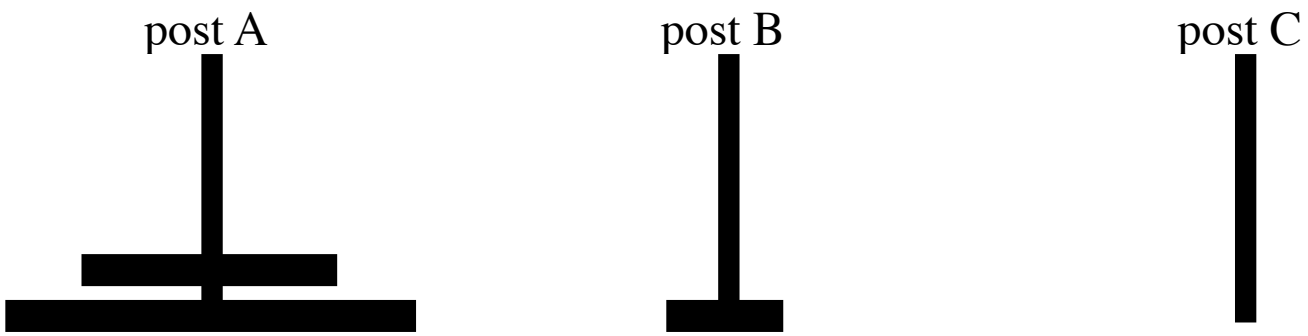
Towers of Hanoi



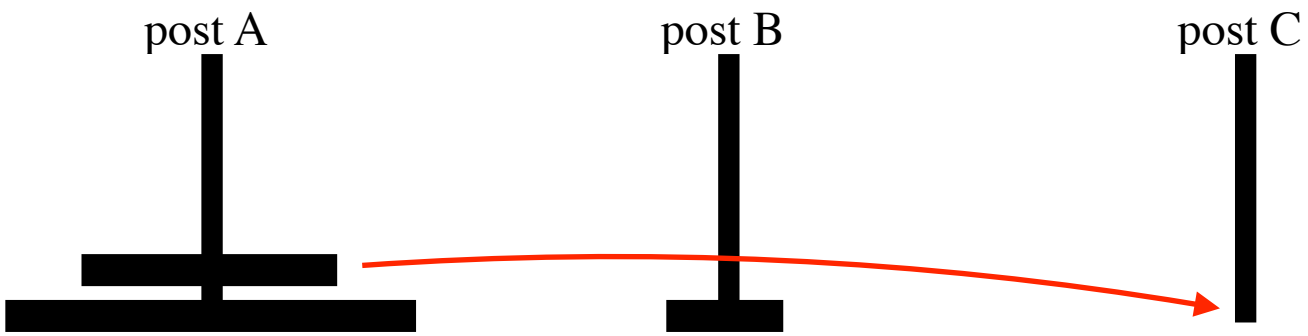
Towers of Hanoi



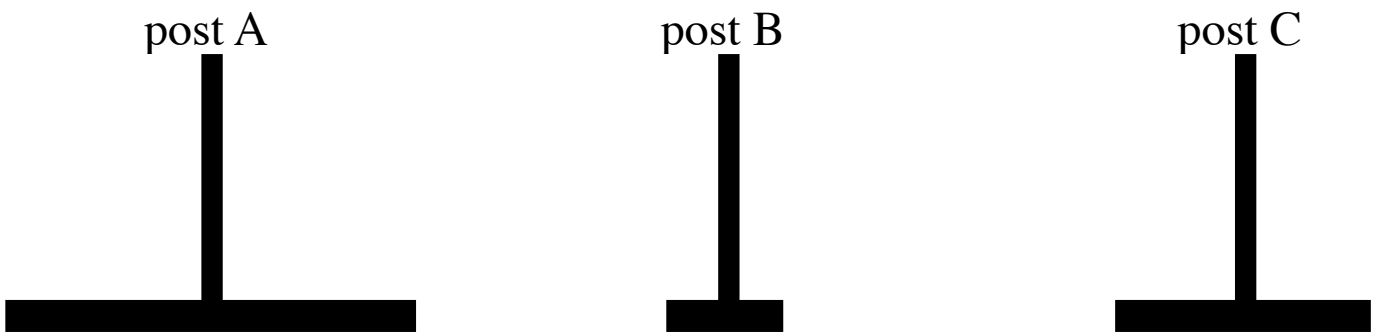
Towers of Hanoi



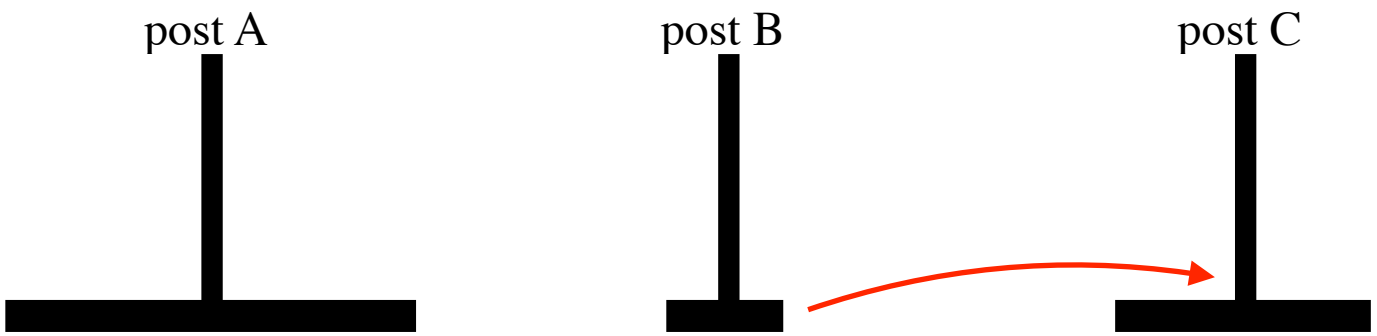
Towers of Hanoi



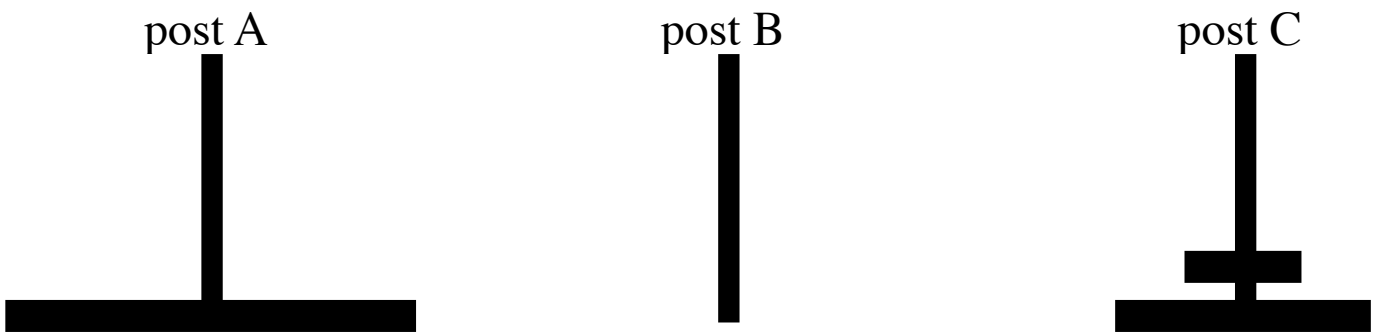
Towers of Hanoi



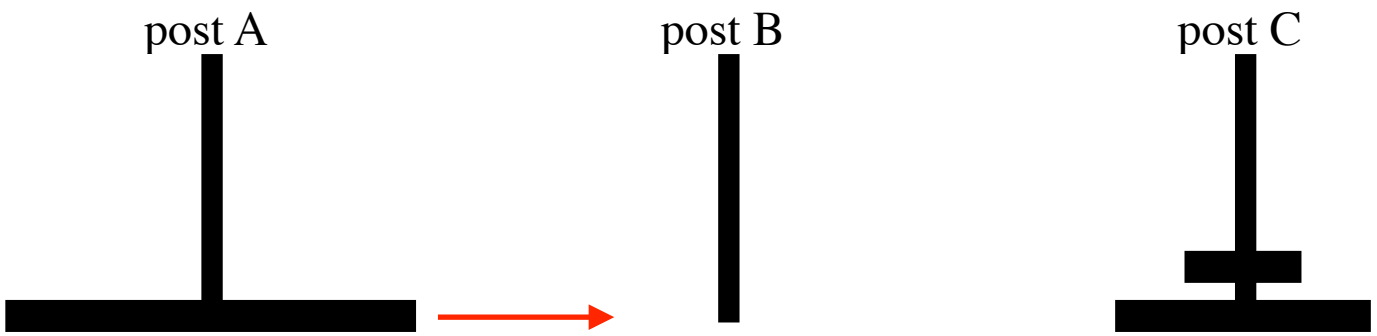
Towers of Hanoi



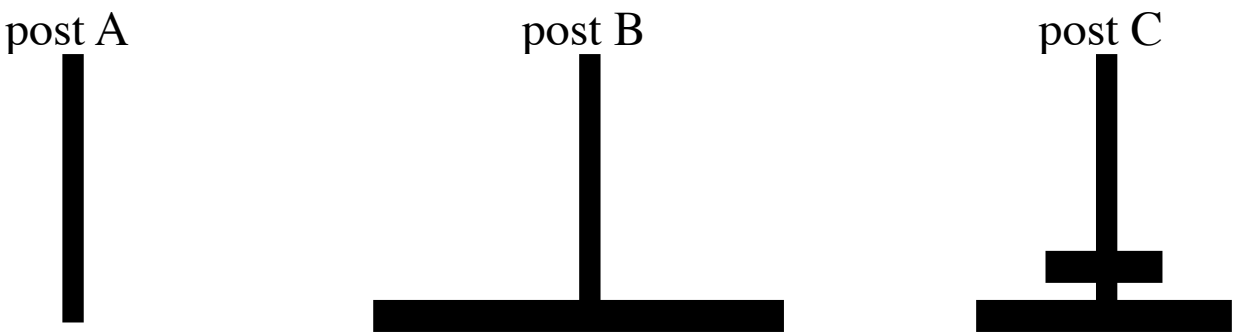
Towers of Hanoi



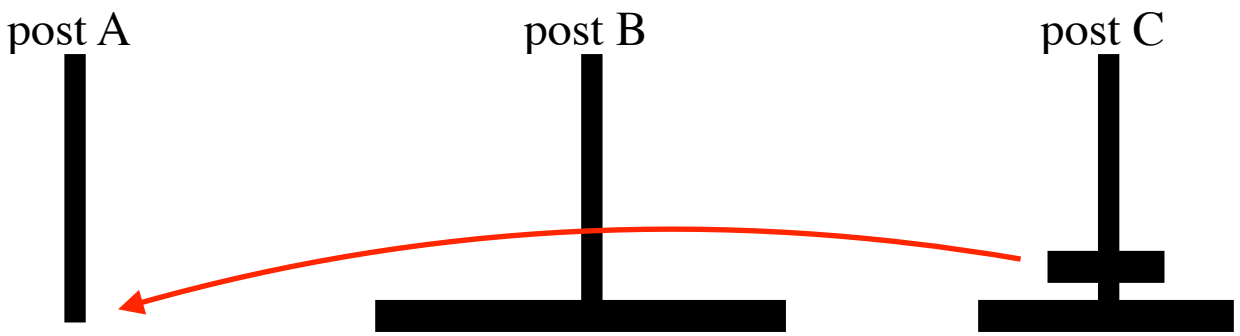
Towers of Hanoi



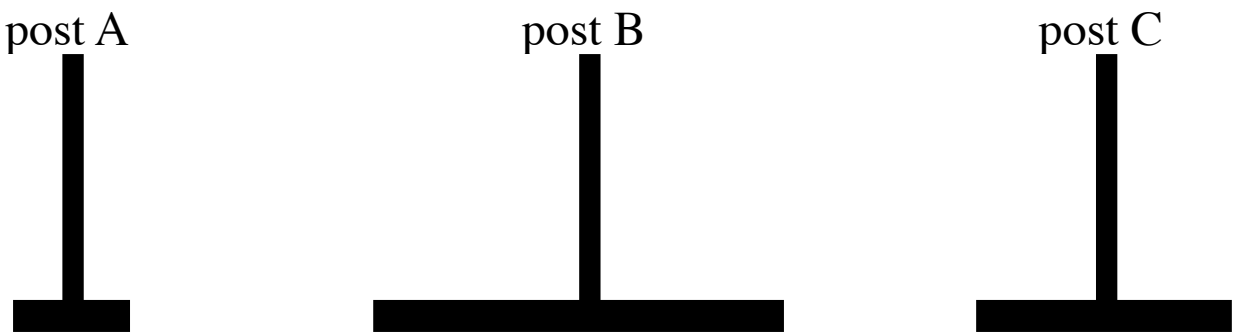
Towers of Hanoi



Towers of Hanoi



Towers of Hanoi



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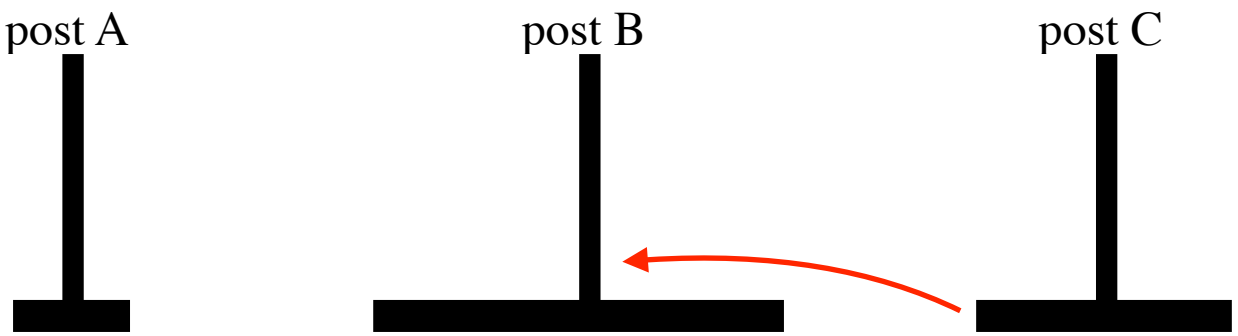
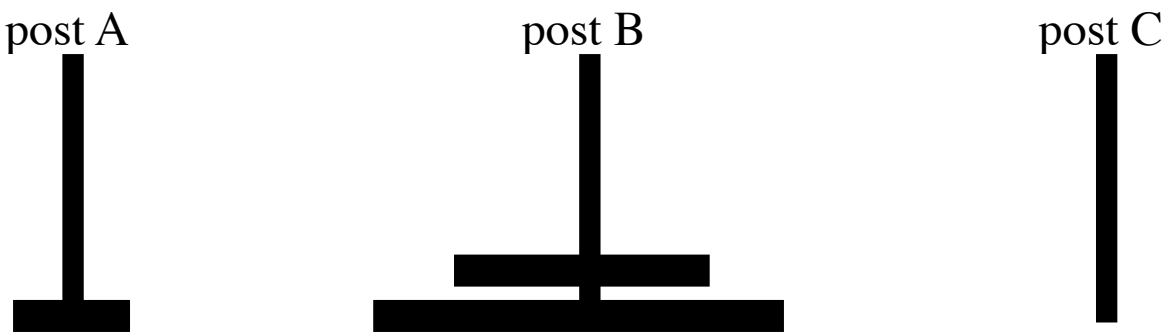
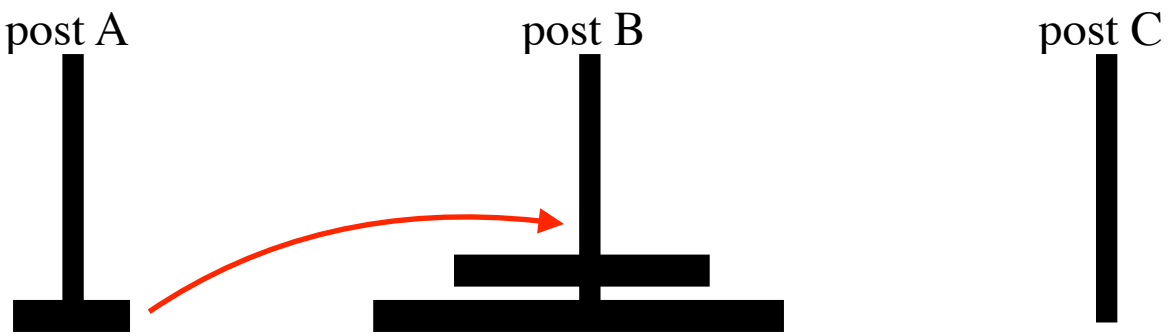


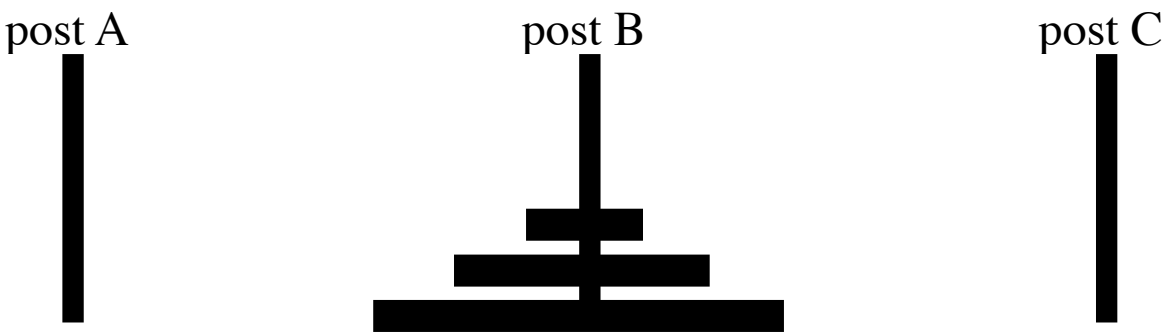
Towers of Hanoi



Towers of Hanoi



Towers of Hanoi



Towers of Hanoi

MovePile from to using $\Leftarrow$

Towers of Hanoi

MovePile from to using $\Leftarrow$

if n=0 **then** *ok*

Towers of Hanoi

MovePile from to using $\Leftarrow$

if n=0 **then** *ok*

else $n := n-1$.

Towers of Hanoi

MovePile from to using $\Leftarrow$

if n=0 **then** *ok*

else $n := n-1$.

MovePile from using to.

Towers of Hanoi

MovePile from to using $\Leftarrow$

if n=0 **then** *ok*

else $n := n-1$.

MovePile from using to.

MoveDisk from to.

Towers of Hanoi

MovePile from to using $\Leftarrow$

if n=0 **then** *ok*

else $n := n-1$.

MovePile from using to.

MoveDisk from to.

MovePile using to from.

Towers of Hanoi

MovePile from to using $\Leftarrow$

if n=0 **then** *ok*

else $n := n-1$.

MovePile from using to.

MoveDisk from to.

MovePile using to from.

$n := n+1$ **fi**

Towers of Hanoi

MovePile from to using $\Leftarrow$

if n=0 **then** *ok*

else $n := n-1$.

MovePile from using to.

MoveDisk from to.

MovePile using to from. 

$n := n+1$ **fi**

Towers of Hanoi

MovePile from to using $\Leftarrow$

if n=0 **then** *ok*

else $n := n-1$.

MovePile from using to.

MoveDisk from to.

MovePile using to from.

$n := n+1$ **fi**

Towers of Hanoi

MovePile from to using $\Leftarrow$

if n=0 **then** *ok*

else $n := n-1$.

MovePile from using to.

MoveDisk from to.

MovePile using to from.

$n := n+1$ **fi**

Towers of Hanoi — time

$t := t + 2^n - 1$ $\Leftarrow$

if $n=0$ **then** *ok*

else $n := n-1$.

$t := t + 2^n - 1$.

$t := t+1$.

$t := t + 2^n - 1$.

$n := n+1$ **fi**

Towers of Hanoi — time

$t := t + 2^n - 1$ $\Leftarrow$ 

if $n=0$ **then** *ok*

else $n := n-1$.

$t := t + 2^n - 1$. 

$t := t+1$.

$t := t + 2^n - 1$. 

$n := n+1$ **fi**

Towers of Hanoi — time

$t := t + 2^n - 1$ $\Leftarrow$

if $n=0$ **then** *ok*

else $n := n-1$.

$t := t + 2^n - 1$.

$t := t+1$. 

$t := t + 2^n - 1$.

$n := n+1$ **fi**

Towers of Hanoi — time

$t := t + 2^n - 1 \Leftarrow$

if $n=0$ **then** *ok*

else $n := n-1.$

$t := t + 2^n - 1.$

$t := t+1.$

$t := t + 2^n - 1.$

$n := n+1$ **fi**

Towers of Hanoi — time

$t := t + 2^n - 1 \Leftarrow$

if $n=0$ **then** *ok* 

else $n := n-1.$

$t := t + 2^n - 1.$

$t := t+1.$

$t := t + 2^n - 1.$

$n := n+1$ **fi**

$t := t + 2^n - 1 \Leftarrow n=0 \wedge ok$

Towers of Hanoi — time

$t := t + 2^n - 1 \Leftarrow$

if $n=0$ **then** ok ←

else $n := n-1.$

$t := t + 2^n - 1.$

$t := t+1.$

$t := t + 2^n - 1.$

$n := n+1$ **fi**

$(t := t + 2^n - 1 \Leftarrow n=0 \wedge ok)$ expand assignment and ok

$= \quad t' = t + 2^n - 1 \wedge n'=n \Leftarrow n=0 \wedge t'=t \wedge n'=n$

Towers of Hanoi — time

$t := t + 2^n - 1 \Leftarrow$

if $n=0$ **then** ok ←

else $n := n-1.$

$t := t + 2^n - 1.$

$t := t+1.$

$t := t + 2^n - 1.$

$n := n+1$ **fi**

$(t := t + 2^n - 1 \Leftarrow n=0 \wedge ok)$ expand assignment and ok

$= \quad t' = t + 2^n - 1 \wedge n'=n \Leftarrow n=0 \wedge t'=t \wedge n'=n$ context

$= \quad t = t + 2^0 - 1 \wedge n=n \Leftarrow n=0 \wedge t'=t \wedge n'=n$

Towers of Hanoi — time

$t := t + 2^n - 1 \Leftarrow$

if $n=0$ **then** ok ←

else $n := n-1.$

$t := t + 2^n - 1.$

$t := t+1.$

$t := t + 2^n - 1.$

$n := n+1$ **fi**

$(t := t + 2^n - 1 \Leftarrow n=0 \wedge ok)$ expand assignment and ok

$= \quad t' = t + 2^n - 1 \wedge n'=n \Leftarrow n=0 \wedge t'=t \wedge n'=n$ context

$= \quad t = t + 2^0 - 1 \wedge n=n \Leftarrow n=0 \wedge t'=t \wedge n'=n$

$= \quad \top$

Towers of Hanoi — time

$t := t + 2^n - 1$ $\Leftarrow$

if $n=0$ **then** *ok*

else $n := n-1$.

$t := t + 2^n - 1$.

$t := t+1$.

$t := t + 2^n - 1$.

$n := n+1$ **fi** $\leftarrow$

Towers of Hanoi — time

$t := t + 2^n - 1$ $\Leftarrow$

if $n=0$ **then** *ok*

else $n := n-1$.

$t := t + 2^n - 1$.

$t := t+1$.

$t := t + 2^n - 1$.

$n := n+1$ **fi** $\leftarrow$

$n' = n+1 \quad \wedge \quad t' = t$

Towers of Hanoi — time

$t := t + 2^n - 1$ $\Leftarrow$

if $n=0$ **then** *ok*

else $n := n-1$.

$t := t + 2^n - 1$.

$t := t+1$.

$t := t + 2^n - 1$. $\leftarrow$

$n := n+1$ **fi**

$n' = n+1 \quad \wedge \quad t' = t$

Towers of Hanoi — time

$t := t + 2^n - 1$ $\Leftarrow$

if $n=0$ **then** *ok*

else $n := n-1$.

$t := t + 2^n - 1$.

$t := t+1$.

$t := t + 2^n - 1$. $\leftarrow$

$n := n+1$ **fi**

$n' = n+1 \quad \wedge \quad t' = t \qquad\qquad + 2^n - 1$

Towers of Hanoi — time

$t := t + 2^n - 1$ $\Leftarrow$

if $n=0$ **then** *ok*

else $n := n-1$.

$t := t + 2^n - 1$.

$t := t+1$. 

$t := t + 2^n - 1$.

$n := n+1$ **fi**

$n' = n+1 \quad \wedge \quad t' = t \qquad\qquad + 2^n - 1$

Towers of Hanoi — time

$t := t + 2^n - 1$ $\Leftarrow$

if $n=0$ **then** *ok*

else $n := n-1$.

$t := t + 2^n - 1$.

$t := t+1$. 

$t := t + 2^n - 1$.

$n := n+1$ **fi**

$n' = n+1 \quad \wedge \quad t' = t \qquad\qquad +1 + 2^n - 1$

Towers of Hanoi — time

$t := t + 2^n - 1$ $\Leftarrow$

if $n=0$ **then** *ok*

else $n := n-1$.

$t := t + 2^n - 1$. $\leftarrow$

$t := t+1$.

$t := t + 2^n - 1$.

$n := n+1$ **fi**

$n' = n+1 \quad \wedge \quad t' = t \qquad\qquad +1 + 2^n - 1$

Towers of Hanoi — time

$t := t + 2^n - 1$ $\Leftarrow$

if $n=0$ **then** *ok*

else $n := n-1$.

$t := t + 2^n - 1$. 

$t := t+1$.

$t := t + 2^n - 1$.

$n := n+1$ **fi**

$n' = n+1 \quad \wedge \quad t' = t + 2^n - 1 + 1 + 2^n - 1$

Towers of Hanoi — time

$t := t + 2^n - 1 \Longleftarrow$

if $n=0$ **then** *ok*

else $n := n-1$.

$t := t + 2^n - 1$.

$t := t+1$.

$t := t + 2^n - 1$.

$n := n+1$ **fi**

$n' = n+1 \quad \wedge \quad t' = t + 2^n - 1 + 1 + 2^n - 1$

Towers of Hanoi — time

$t := t + 2^n - 1$ $\Leftarrow$

if $n=0$ **then** *ok*

else $n := n-1$.

$t := t + 2^n - 1$.

$t := t+1$.

$t := t + 2^n - 1$.

$n := n+1$ **fi**

$n' = n+1 \quad \wedge \quad t' = t + 2^n \qquad + 2^n - 1$

Towers of Hanoi — time

$t := t + 2^n - 1 \Leftarrow$

if $n=0$ **then** *ok*

else $n := n-1$.

$t := t + 2^n - 1$.

$t := t+1$.

$t := t + 2^n - 1$.

$n := n+1$ **fi**

$n' = n+1 \quad \wedge \quad t' = t + 2^n \qquad + 2^n - 1$

Towers of Hanoi — time

$t := t + 2^n - 1$ $\Leftarrow$

if $n=0$ **then** *ok*

else $n := n-1$.

$t := t + 2^n - 1$.

$t := t+1$.

$t := t + 2^n - 1$.

$n := n+1$ **fi**

$n' = n+1 \quad \wedge \quad t' = t + 2^{n+1} - 1$

Towers of Hanoi — time

$t := t + 2^n - 1$ $\Leftarrow$

if $n=0$ **then** *ok*

else $n := n-1$.

$t := t + 2^n - 1$.

$t := t+1$.

$t := t + 2^n - 1$.

$n := n+1$ **fi**

$n' = n+1 \quad \wedge \quad t' = t + 2^{n+1} - 1$

Towers of Hanoi — time

$t := t + 2^n - 1$ ⟸

if $n=0$ **then** *ok*

else $n := n-1$. ←

$t := t + 2^n - 1$.

$t := t+1$.

$t := t + 2^n - 1$.

$n := n+1$ **fi**

$n' = n \quad \wedge \quad t' = t + 2^n - 1$

Towers of Hanoi — time

$t := t + 2^n - 1 \Leftarrow$ 

if $n=0$ **then** *ok*

else $n := n-1$.

$t := t + 2^n - 1$.

$t := t+1$.

$t := t + 2^n - 1$.

$n := n+1$ **fi**

$n' = n \quad \wedge \quad t' = t + 2^n - 1$

Towers of Hanoi — space

$s'=s \Leftarrow$

if $n=0$ **then** *ok*

else $n := n-1.$

$s := s+1.\ s'=s.\ s := s-1.$

ok.

$s := s+1.\ s'=s.\ s := s-1.$

$n := n+1$ **fi**

Towers of Hanoi — space

$s'=s \Leftarrow$ 

if $n=0$ **then** ok

else $n := n-1.$

$s := s+1.\ s'=s.\ s := s-1.$

$ok.$

$s := s+1.\ s'=s.\ s := s-1.$

$n := n+1$ **fi**

Towers of Hanoi — space

$s'=s \Leftarrow$

if $n=0$ **then** ok

else $n := n-1.$

$s := s+1.\ s'=s.\ s := s-1.$ ←

$ok.$

$s := s+1.\ s'=s.\ s := s-1.$ ←

$n := n+1$ **fi**

Towers of Hanoi — space

$s'=s \Leftarrow$

if $n=0$ **then** ok

else $n := n-1.$

$s := s+1.\ s'=s.\ s := s-1.$

$ok.$ 

$s := s+1.\ s'=s.\ s := s-1.$

$n := n+1$ **fi**

Towers of Hanoi — maximum space

$s \le m \le s+n \;\Rightarrow\; (m{:=}\, s+n) \;\Leftarrow$

if $n{=}0$ **then** *ok*

else $n{:=}\, n{-}1.$

$s{:=}\, s{+}1.\;\; m{:=}\, m{\uparrow}s.\;\; s \le m \le s+n \;\Rightarrow\; (m{:=}\, s+n).\;\; s{:=}\, s{-}1.$

ok.

$s{:=}\, s{+}1.\;\; m{:=}\, m{\uparrow}s.\;\; s \le m \le s+n \;\Rightarrow\; (m{:=}\, s+n).\;\; s{:=}\, s{-}1.$

$n{:=}\, n{+}1$ **fi**

Towers of Hanoi — maximum space

$s \le m \le s+n \;\Rightarrow\; (m{:=}\, s+n) \;\Leftarrow$

if $n{=}0$ **then** *ok*

else $n{:=}\, n{-}1.$

$s{:=}\, s{+}1.\;\; m{:=}\, m{\uparrow}s.\;\; s \le m \le s+n \;\Rightarrow\; (m{:=}\, s+n).\;\; s{:=}\, s{-}1.$

ok.

$s{:=}\, s{+}1.\;\; m{:=}\, m{\uparrow}s.\;\; s \le m \le s+n \;\Rightarrow\; (m{:=}\, s+n).\;\; s{:=}\, s{-}1.$

$n{:=}\, n{+}1$ **fi**

Towers of Hanoi — maximum space

$s \le m \le s+n \;\Rightarrow\; (m := s+n) \Leftarrow$

if $n=0$ **then** *ok*

else $n := n-1.$

$s := s+1.\ m := m \uparrow s.\ s \le m \le s+n \;\Rightarrow\; (m := s+n).\ s := s-1.$

ok.

$s := s+1.\ m := m \uparrow s.\ s \le m \le s+n \;\Rightarrow\; (m := s+n).\ s := s-1.$

$n := n+1$ **fi**

Towers of Hanoi — maximum space

$s \le m \le s+n \;\Rightarrow\; (m := s+n) \;\Leftarrow$

if $n=0$ **then** *ok*

else $n := n-1.$

$s := s+1.\ m := m\uparrow s.\ s \le m \le s+n \;\Rightarrow\; (m := s+n).\ s := s-1.$

ok.

$s := s+1.\ m := m\uparrow s.\ s \le m \le s+n \;\Rightarrow\; (m := s+n).\ s := s-1.$

$n := n+1$ **fi**

Towers of Hanoi — maximum space

$s \le m \le s+n \;\Rightarrow\; (m{:=}\, s+n) \;\Leftarrow$

if $n{=}0$ **then** *ok*

else $n{:=}\, n{-}1.$

$s{:=}\, s{+}1.\;\; m{:=}\, m{\uparrow}s.\;\; s \le m \le s+n \;\Rightarrow\; (m{:=}\, s+n).\;\; s{:=}\, s{-}1.$

ok.

$s{:=}\, s{+}1.\;\; m{:=}\, m{\uparrow}s.\;\; s \le m \le s+n \;\Rightarrow\; (m{:=}\, s+n).\;\; s{:=}\, s{-}1.$

$n{:=}\, n{+}1$ **fi**

Towers of Hanoi — maximum space

$s \le m \le s+n \;\Rightarrow\; (m{:=}\, s+n) \;\Leftarrow$

if $n{=}0$ **then** *ok*

else $n{:=}\, n{-}1.$

$s{:=}\, s{+}1.\;\; m{:=}\, m{\uparrow}s.\;\; s \le m \le s+n \;\Rightarrow\; (m{:=}\, s+n).\;\; s{:=}\, s{-}1.$

ok.

$s{:=}\, s{+}1.\;\; m{:=}\, m{\uparrow}s.\;\; s \le m \le s+n \;\Rightarrow\; (m{:=}\, s+n).\;\; s{:=}\, s{-}1.$

$n{:=}\, n{+}1$ **fi**

Towers of Hanoi — average space

$p := p + s\times(2^n - 1) + (n-2)\times 2^n + 2 \Leftarrow$

if $n=0$ **then** *ok*

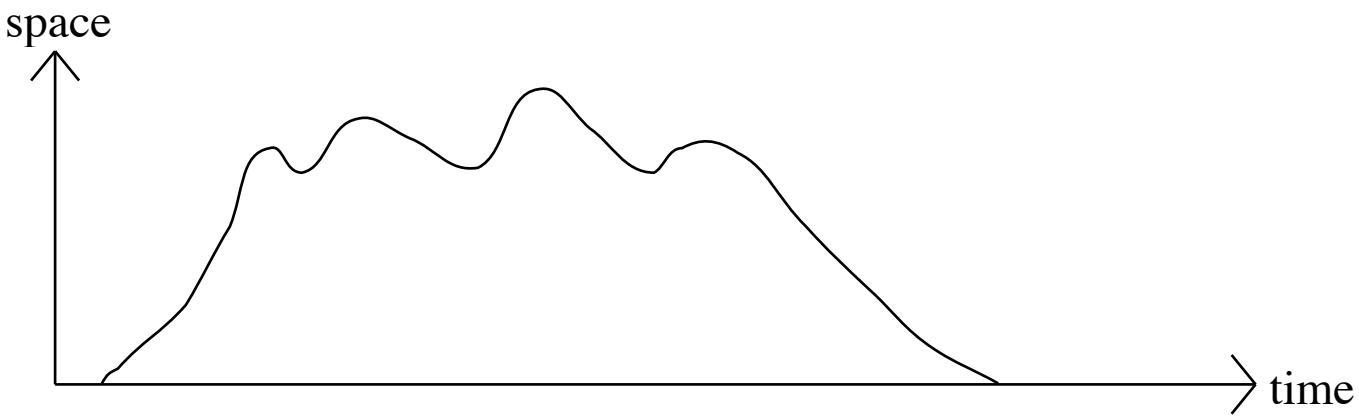
else $n := n-1.$

$s := s+1.\ p := p + s\times(2^n - 1) + (n-2)\times 2^n + 2.\ s := s-1.$

$p := p+s.$

$s := s+1.\ p := p + s\times(2^n - 1) + (n-2)\times 2^n + 2.\ s := s-1.$

$n := n+1$ **fi**



Towers of Hanoi — average space

$p := p + s\times(2^n - 1) + (n-2)\times 2^n + 2 \Leftarrow$

if $n=0$ **then** *ok*

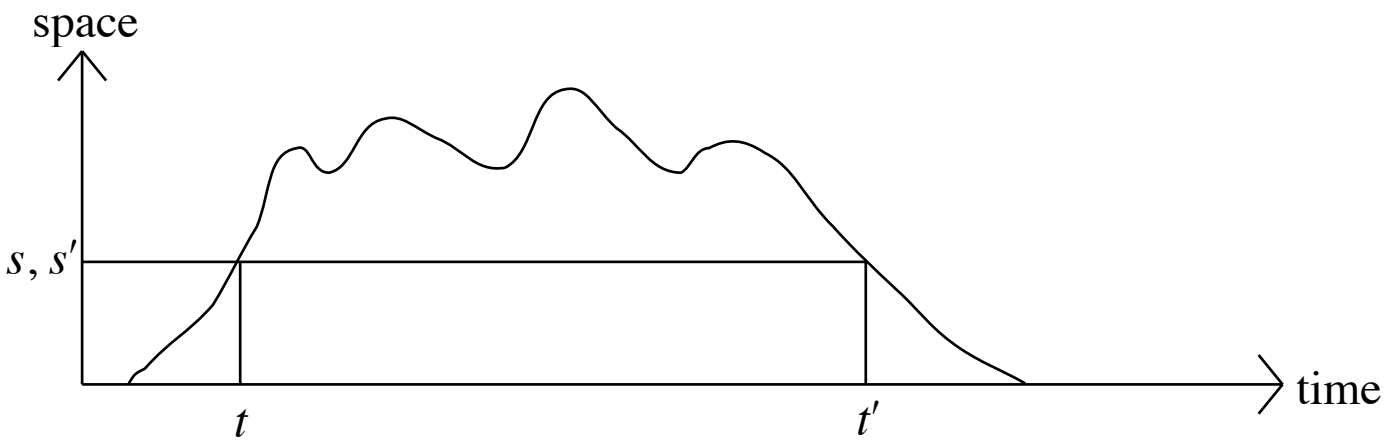
else $n := n-1.$

$s := s+1.\ p := p + s\times(2^n - 1) + (n-2)\times 2^n + 2.\ s := s-1.$

$p := p+s.$

$s := s+1.\ p := p + s\times(2^n - 1) + (n-2)\times 2^n + 2.\ s := s-1.$

$n := n+1$ **fi**



Towers of Hanoi — average space

$p := p + s\times(2^n - 1) + (n-2)\times 2^n + 2 \Leftarrow$

if $n=0$ **then** *ok*

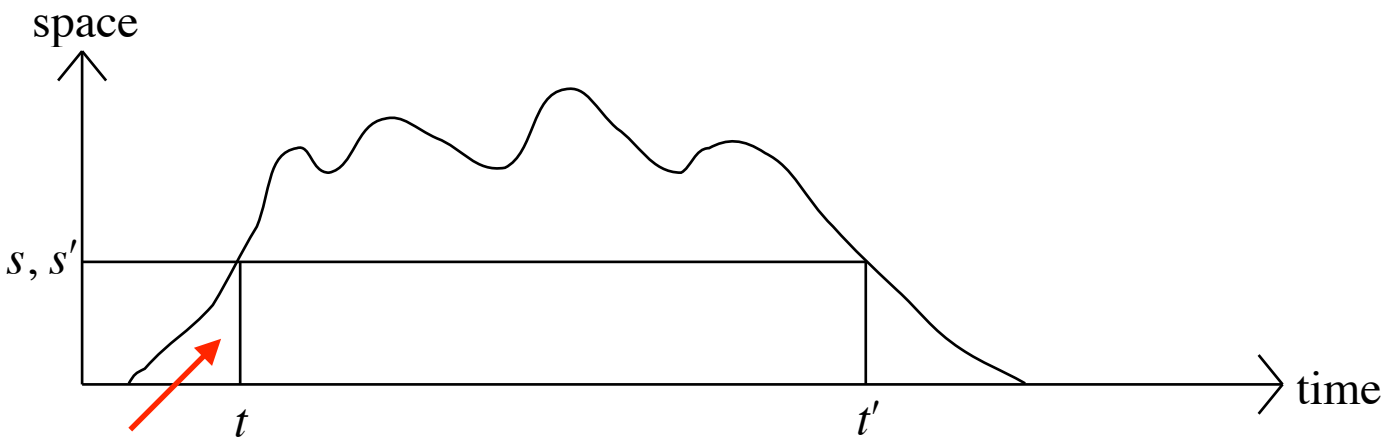
else $n := n-1.$

$s := s+1.\ p := p + s\times(2^n - 1) + (n-2)\times 2^n + 2.\ s := s-1.$

$p := p+s.$

$s := s+1.\ p := p + s\times(2^n - 1) + (n-2)\times 2^n + 2.\ s := s-1.$

$n := n+1$ **fi**



Towers of Hanoi — average space

$p := p + s\times(2^n - 1) + (n-2)\times 2^n + 2 \Leftarrow$

if $n=0$ **then** *ok*

else $n := n-1.$

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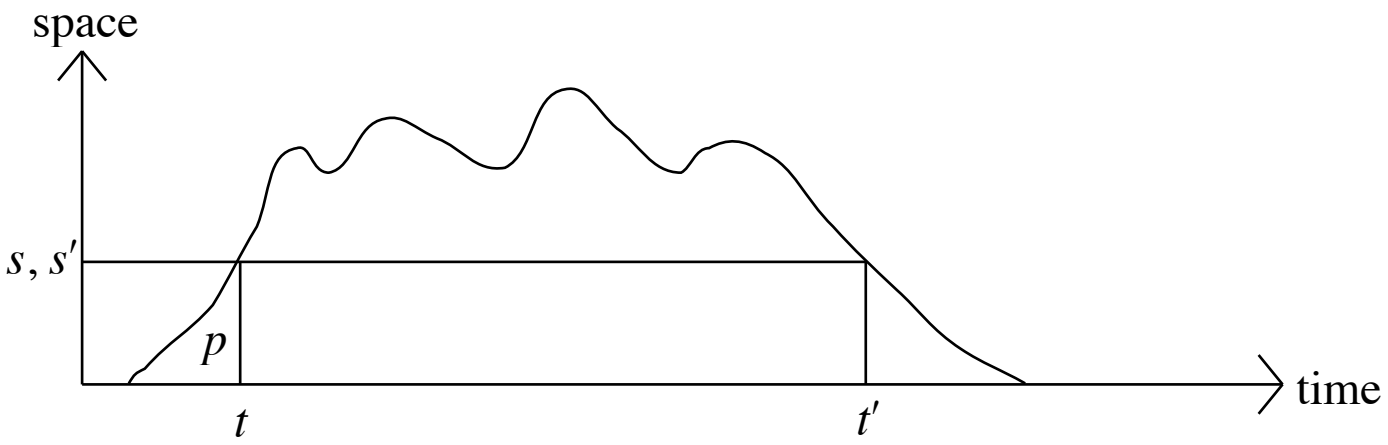

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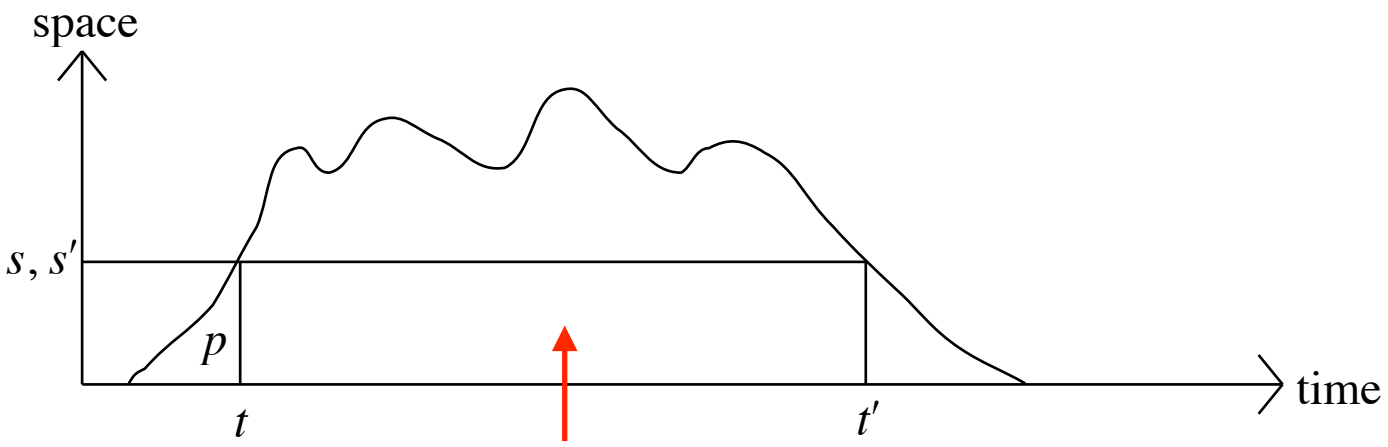

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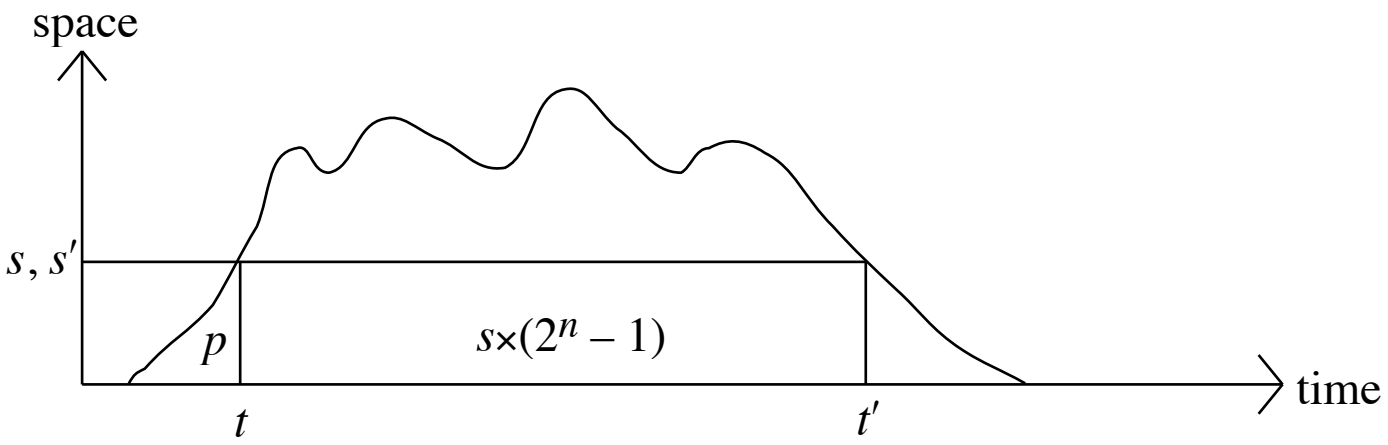

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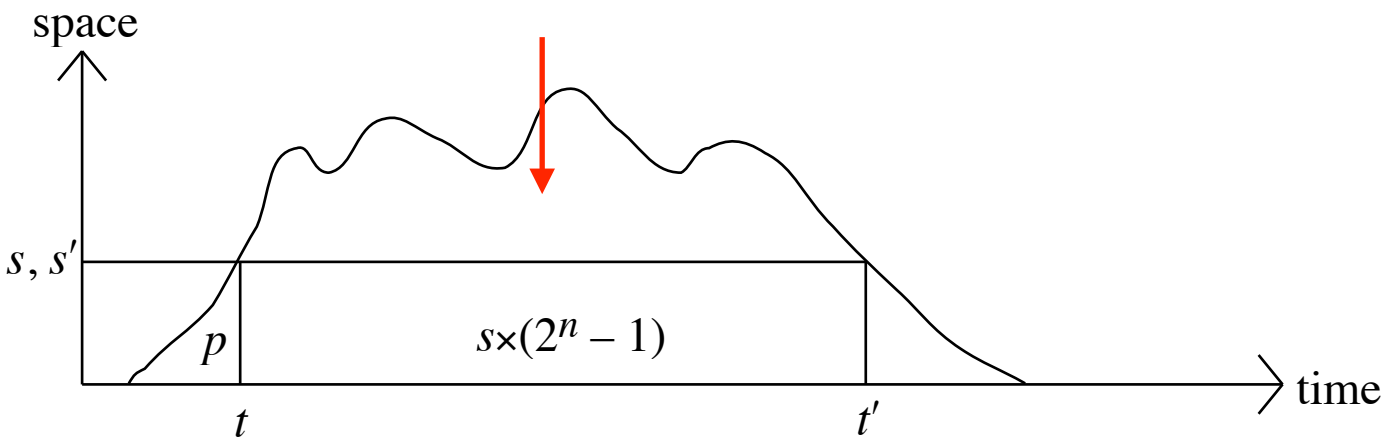

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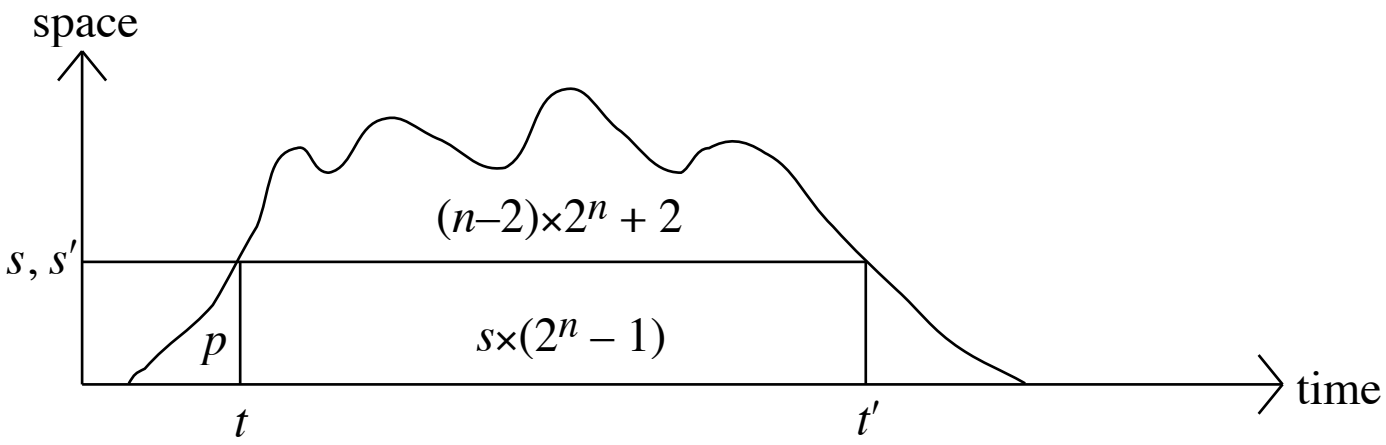

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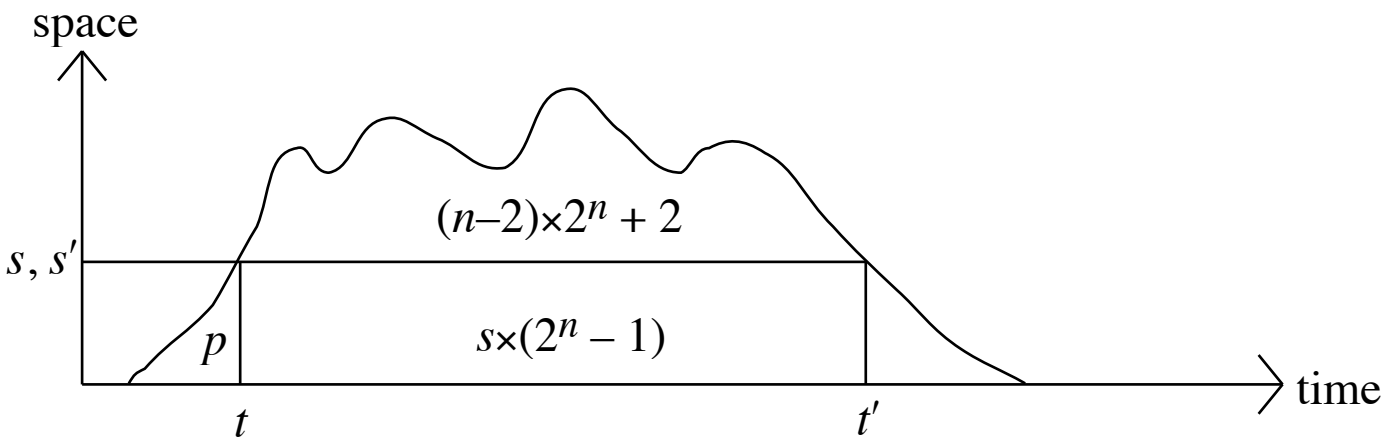

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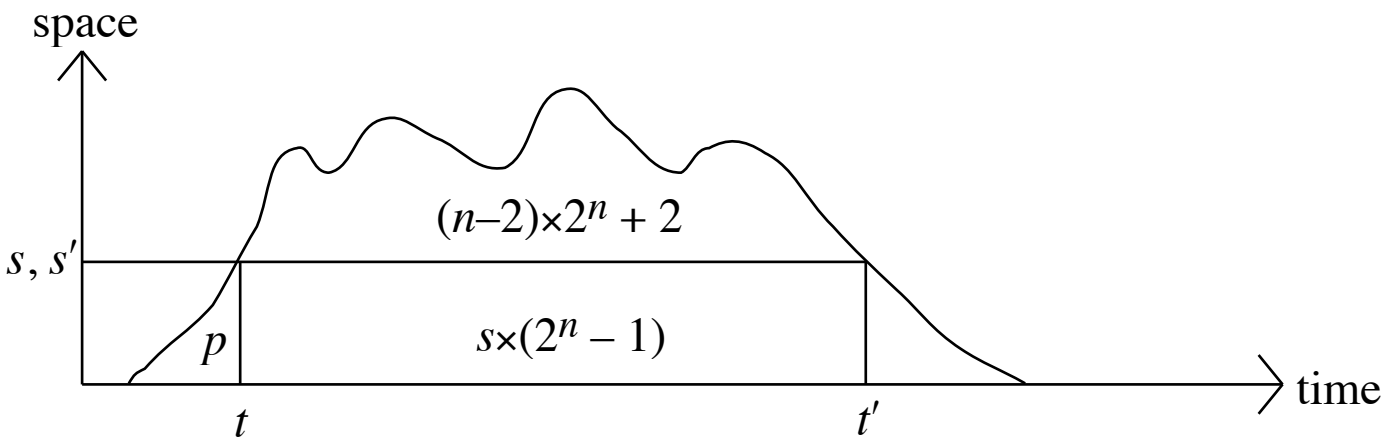

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average space $= ((n-2)\times 2^n + 2) / (2^n - 1)$

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average space $= ((n-2)\times 2^n + 2) / (2^n - 1)$

$= n + n/(2^n - 1) - 2$

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average space $= ((n-2)\times 2^n + 2) / (2^n - 1)$

$= n + n/(2^n - 1) - 2$

Easier: $p' \le p + (s+n)\times(2^n-1)$

average space $\le n$

Towers of Hanoi

MovePile $\Leftarrow$

if $n{=}0$ **then** *ok*

else $n{:=}\,n{-}1.$

$s{:=}\,s{+}1.\ \ m{:=}\,m{\uparrow}s.\ \ MovePile.\ \ s{:=}\,s{-}1.$

$t{:=}\,t{+}1.\ \ p{:=}\,p{+}s.\ \ ok.$

$s{:=}\,s{+}1.\ \ m{:=}\,m{\uparrow}s.\ \ MovePile.\ \ s{:=}\,s{-}1.$

$n{:=}\,n{+}1$ **fi**

$$
\begin{aligned}
MovePile \;=\;\; & n' = n \\
\wedge\;\; & t' = t + 2^n - 1 \\
\wedge\;\; & s' = s \\
\wedge\;\; & (s \le m \le s{+}n \;\Rightarrow\; m' = s{+}n) \\
\wedge\;\; & p' = p + s{\times}(2^n - 1) + (n{-}2){\times}2^n + 2
\end{aligned}
$$