CSC 2534— Decision Making Under Uncertainty
Assignment 3

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Due: December 15, 2014

1. For this game, you are to determine the set of Nash equilibria for two different games.

   (a) Consider the following matrix form game, where Player 1 (row) has two actions A and B, and Player 2 (column) has two actions C and D:

	C	D
A	$10, 12$	$15, 24$
B	$16, 18$	$6, 12$

   Identify all pure strategy equilibria (if any) for this game. Justify your response by explaining why each profile is in fact a pure equilibrium or why none exist.

   What is the (strictly) mixed Nash equilibrium for this game? Explain how you determined it. (There should only be one.)

   (b) Consider the following matrix form game, where Player 1 (row) has two actions A and B, and Player 2 (column) has three actions D, F and G:

	D	F	G
A	$10, 5$	$4, 14$	$5, 11$
B	$4, 10$	$7, 4$	$5, 6$

   Identify all pure strategy equilibria (if any) for this game. Justify your response by explaining why each profile is in fact a pure equilibrium or why none exist.

   What are the (strictly) mixed Nash equilibria for this game? Hint: there are infinitely many (they form a continuous set), but can be described very concisely. Justify your answer.

2. We have two competitors bidding for one of two new machines to install in their new factories. Neither competitor knows what type of factory the other has built; but the *type* of factory dictates how useful a particular machine will be. They must decide how to bid on given this uncertainty. The specifics:

   - The competitors each have one new factory, either of type f_1 or f_2. Neither competitor knows the type of the opponent's factory. However, there is a common prior over factory types: $\langle f_1, f_1\rangle$ and $\langle f_2, f_2\rangle$ each occur with $p = 0.4$; and $\langle f_1, f_2\rangle$ and $\langle f_2, f_1\rangle$ each occur with $p = 0.1$.
   - There are two machines, m_1 and m_2. A factory of type f_1 can use either machine; a factory of type f_2 can only use m_2. The payoffs for each competitor as a function of factory type and machine are: $\langle f_1, m_1\rangle$: 100; $\langle f_1, m_2\rangle$: 45; $\langle f_2, m_1\rangle$: 0; and $\langle f_2, m_2\rangle$: 50.

- The two machines are being sold simultaneously in a first-price sealed bid auction. So a competitor offers a bid on a machine, and if it's higher than the other competitor's bid, they win the machine for the price bid. If not, they pay nothing.
- For simplicity, we'll severely restrict the action space of each competitor. If its factory is of type f_1, it can do one of two things: bid 90 for m_1 or bid 40 for m_2. If its factory is of type f_2, it can do one of two things: bid 40 for m_2 or bid 45 for m_2.
- Each competitor has a quasi-linear utility function. Their utility is given by the value of the machine obtained (if any) less their payment.

(a) Formulate this problem as a Bayesian game. Specify precisely the type space, action space, and payoff functions of each agent.

(b) Convert this Bayesian game to strategic form by considering all strategies that map agent types to actions. Specify this game in matrix form, and include all expected payoffs to each agent in the matrix.

(c) Describe all pure Bayes-Nash equilibria of this game.

3. Consider the following facility location problem on a two-dimensional city grid. We want to build a new park for the citizens of Pleasantville, a town of m people. There are n^2 potential park locations laid out on a discrete $n \times n$ grid with x-coordinates $1, \cdots, n$ and y-coordinates $1, \cdots, n$. Each person i has a single *ideal* location for the park (x_i, y_i). i's preference for park location (x^*, y^*) is given by the negative of its Manhattan distance from i' ideal location. Specifically, i's utility for location (x^*, y^*) is given by
$$u_i(x^*, y^*) = -(|x^* - x_i| + |y^* - y_i|).$$
We decide to poll the town for park locations and use the results as follows: everyone submits their ideal location, and the park is built at location (x_m, y_m) where x_m is the median of all reported x-coordinates, and y_m is the median of all reported y-coordinates.

   Provide a compelling argument (an informal proof) that the resulting mechanism is strategyproof and maximizes social welfare.

4. *This is really a trivial question, don't try to read more into it than there is.* It turns out Pleasantville is a very small town, with three families f_1, f_2, f_3, and only four locations A, B, C, D at which to build a park. Now imagine the families have the following utilities for the four locations: f_1: $u_1(A) = 10; u_1(B) = 6; u_1(C) = 4; u_1(D) = 2$. f_2: $u_2(A) = 2; u_2(B) = 4; u_2(C) = 6; u_2(D) = 10$. f_3: $u_3(A) = 5; u_3(B) = 3; u_3(C) = 9; u_3(D) = 2$. Describe the location chosen and the payments made by each family if the VCG mechanism is used to select the park.

5. **SINCE WE'RE LATE IN THE TERM, YOU ARE NOT REQUIRED TO DO THIS QUESTION AND IT WILL NOT BE GRADED. BUT GIVE IT SOME THOUGHT AND COMPLETE IT ANYWAY TO MAKE SURE YOU'RE COMFORTABLE WITH EXTENSIVE FORM GAMES, SUBGAME PERFECT EQUILIBRIA, ETC.**

   Consider the following sequential *bargaining game* involving three companies A, B and C. The companies have been attempting to form a coalition to win a large goverment contract worth \$60M that will only succeed if they agree to work together. Unable to decide how to split the profits, the call in an arbitrator who gets all three to agree to use the following protocol for dividing the spoils.

- Company A starts the process: it can propose any split of the proceeds (in \$10M chunks) that gives an even share to B and C. Letting $a/b/c$ represent the proposed split among A, B, C, respectively, A can propose either $6/0/0$, $4/1/1$, $2/2/2$, or $0/3/3$ (each number represents the number of \$10M shares).
- Company B can counterpropose, accept A's proposal, or reject it. B's counterproposals are limited to be any redistribution of *i*ts share, as proposed by A, to A and C, as long as both get the same amount of B's share: specifically, if A proposes $0/3/3$ (which means B's proposed share is 3), then B can counterpropose with $1/1/4$.[1] B can also accept A's offer (which can be viewed as a special counterproposal with no redistribution). Finally, B can reject A's proposal. If rejected, the process terminates with no coalition forming.
- If B counterproposes (or accepts), then C is given an opportunity to accept or reject the offer put forward by B. If C accepts, revenue is split according to B's offer. If C rejects, the the process terminates with no coalition forming.

(a) Assume that the companies have no outside options, so if the process terminates without an agreement/coalition, each of them receives a payoff of zero. Furthermore, if an agreement forms, any company who receives a zero share of the profits view this as a marginally worse outcome than the zero payoff obtained by no agreement (e.g., because of reputation effects).

Model this as an extensive form game by drawing the game tree induces by this scenario.

(b) How many pure strategies do each of companies A, B, and C have? Describe them *(for B and C, do not list them all there are a lot—just describe their form)*.

(c) What is the (unique) subgame perfect equilibrium for this game? And what is its value to each company? Which company is in the most powerful position in your opinion (and why)?

(d) Describe a Nash equilibrium that maximizes the payoff to C. (Be sure to give a full description of the equilibrium.) Explain why the game cannot realistically support this equilibrium.

(e) Give an equilibrium that is most equitable; that is, an equilibrium in which the payoffs to the three companies differ the least. Explain why the game cannot realistically support this equilibrium.

(f) Suppose that each company has an outside option that has a value of \$11M; that is, if no agreement is reached, each company's payoff is \$11M. (The structure of game is otherwise unchanged.) What is the subgame perfect equilibrium for this new game? Are there any additional Nash equilibria? If so, give an example; if not, explain why not.

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[1] Note carefully: B can only counterpropose in this very specific way; so in some cases it may not be able to genuinely counterpropose at all.